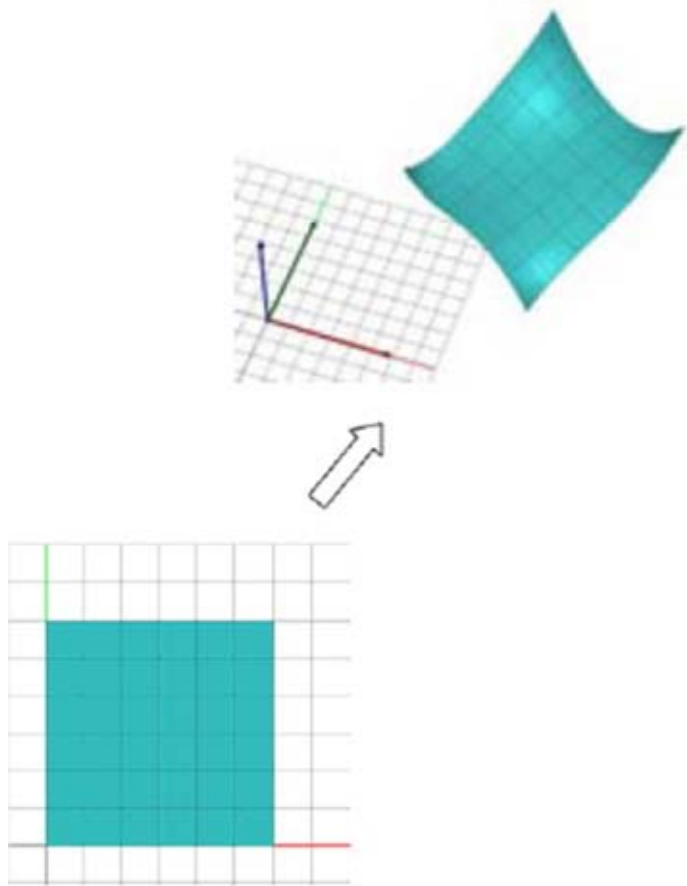


POVRŠI U PROSTORU

POVRŠI U PROSTORU

Parametarske jednačine površi



Obostrano jednoznačno regularno preslikavanje

$$\sigma: M \rightarrow N$$

oblasti $M \subset \mathbb{R}^2$

u neku oblast $N \subset \mathbb{R}^3$

je površ u \mathbb{R}^3 .

POVRŠI U PROSTORU

Parametarske jednačine površi

$$\sigma: M \rightarrow N$$

$$M \subset \mathbb{R}^2 \quad (u, v) \in M$$

$$N \subset \mathbb{R}^3 \quad (x, y, z) \in N$$

$$\sigma: (u, v) \rightarrow (x, y, z)$$

$$x = \sigma_1(u, v)$$

$$y = \sigma_2(u, v)$$

$$z = \sigma_3(u, v)$$

$$(u, v) \in M$$

Parametarske jednačine površi
u Dekartovom pravouglom
koordinatnom sistemu



POVRŠI U PROSTORU

Parametarske jednačine površi

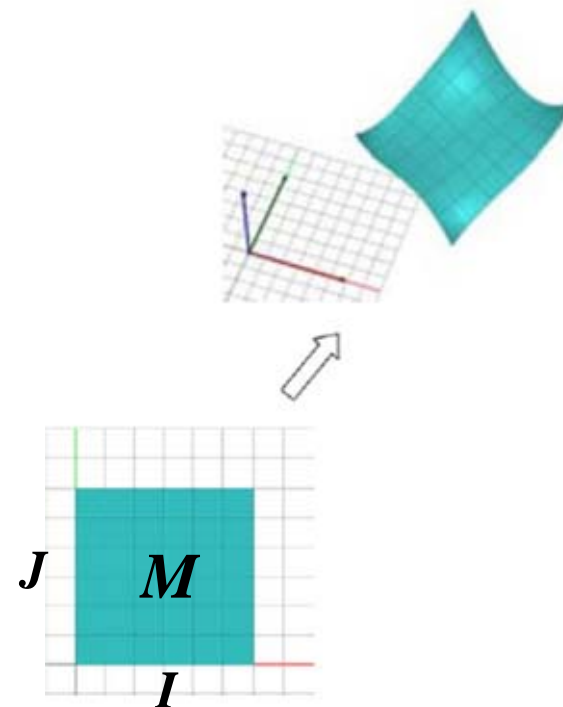
$$\sigma: M \rightarrow N$$

$$M \subset \mathbb{R}^2 \quad (u, v) \in M$$

$$N \subset \mathbb{R}^3 \quad (x, y, z) \in N$$

$$I = [u_1, u_2], \quad J = [v_1, v_2]$$

$$M = I \times J = \left\{ (u, v) \mid u_1 \leq u \leq u_2, v_1 \leq v \leq v_2 \right\}$$



POVRŠI U PROSTORU

Parametarske jednačine površi

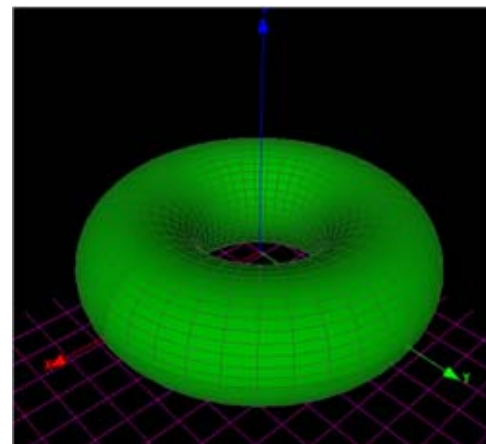
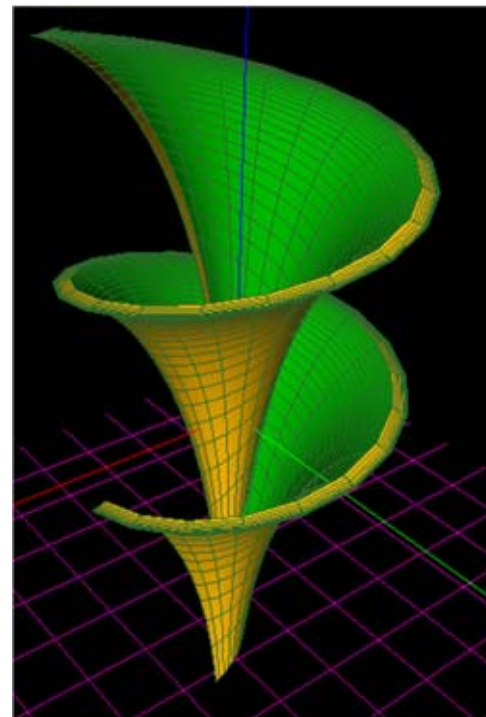
$$x = \sigma_1(u, v)$$

$$y = \sigma_2(u, v)$$

$$z = \sigma_3(u, v)$$

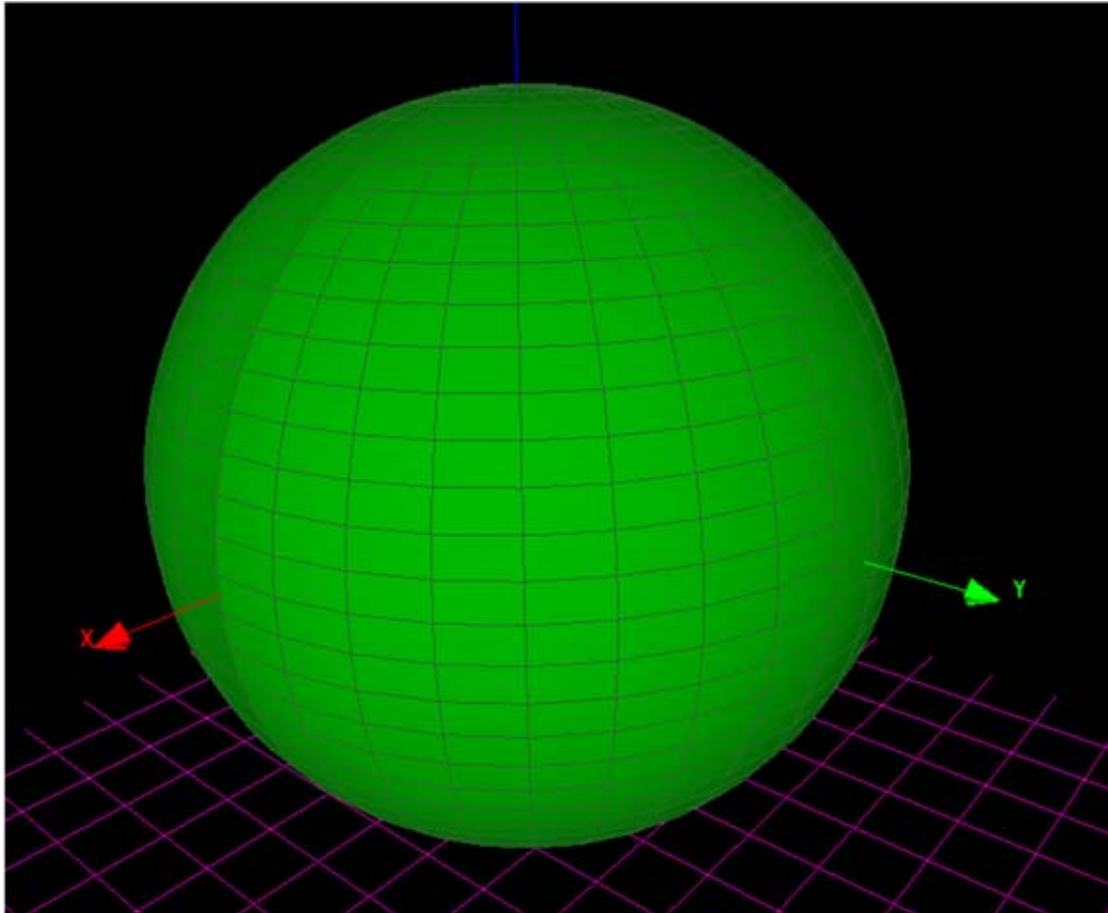
$$u_1 \leq u \leq u_2$$

$$v_1 \leq v \leq v_2$$



POVRŠI U PROSTORU

Primeri



Sfera

$$X(u,v)=r*\cos(u)*\cos(v)$$

$$Y(u,v)=r*\cos(u)*\sin(v)$$

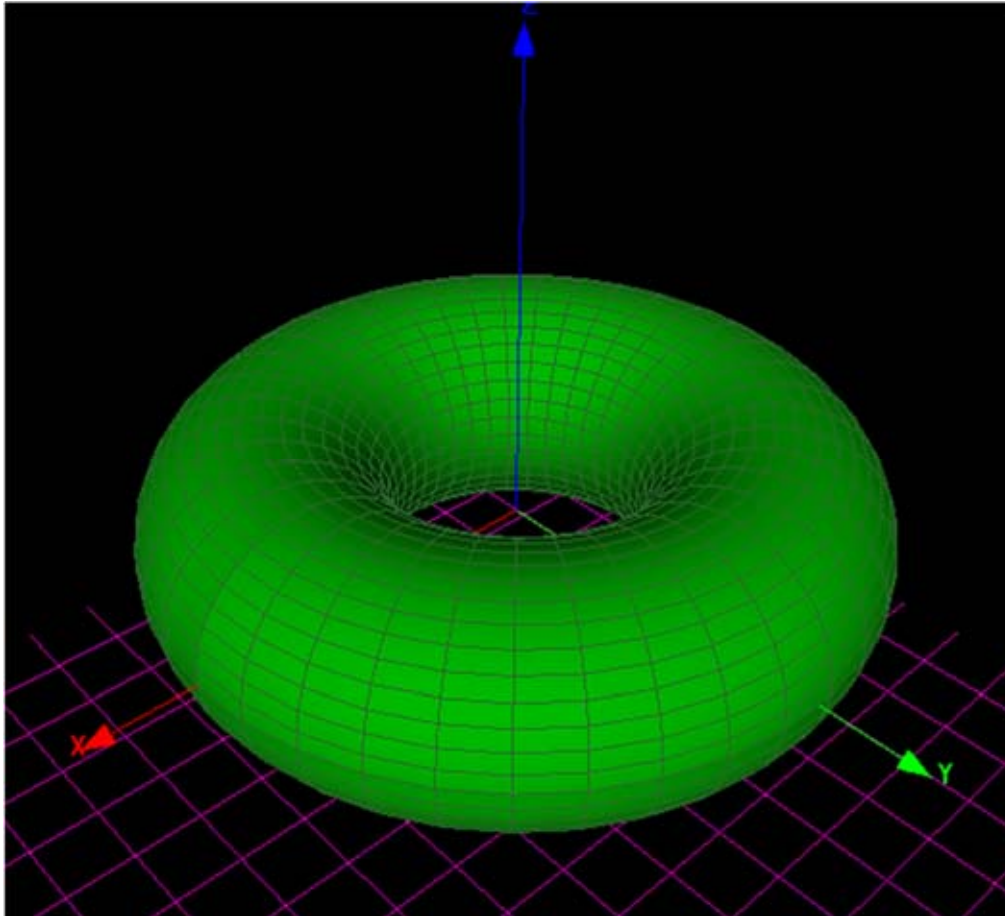
$$Z(u,v)=r*\sin(u)$$

$$-\frac{\pi}{2} \leq u \leq \frac{\pi}{2}$$

$$0 \leq v \leq 2\pi$$

POVRŠI U PROSTORU

Primeri



Torus

$$X(u,v)=(1+0.5*\cos(u))*\cos(v)$$

$$Y(u,v)=(1+0.5*\cos(u))*\sin(v)$$

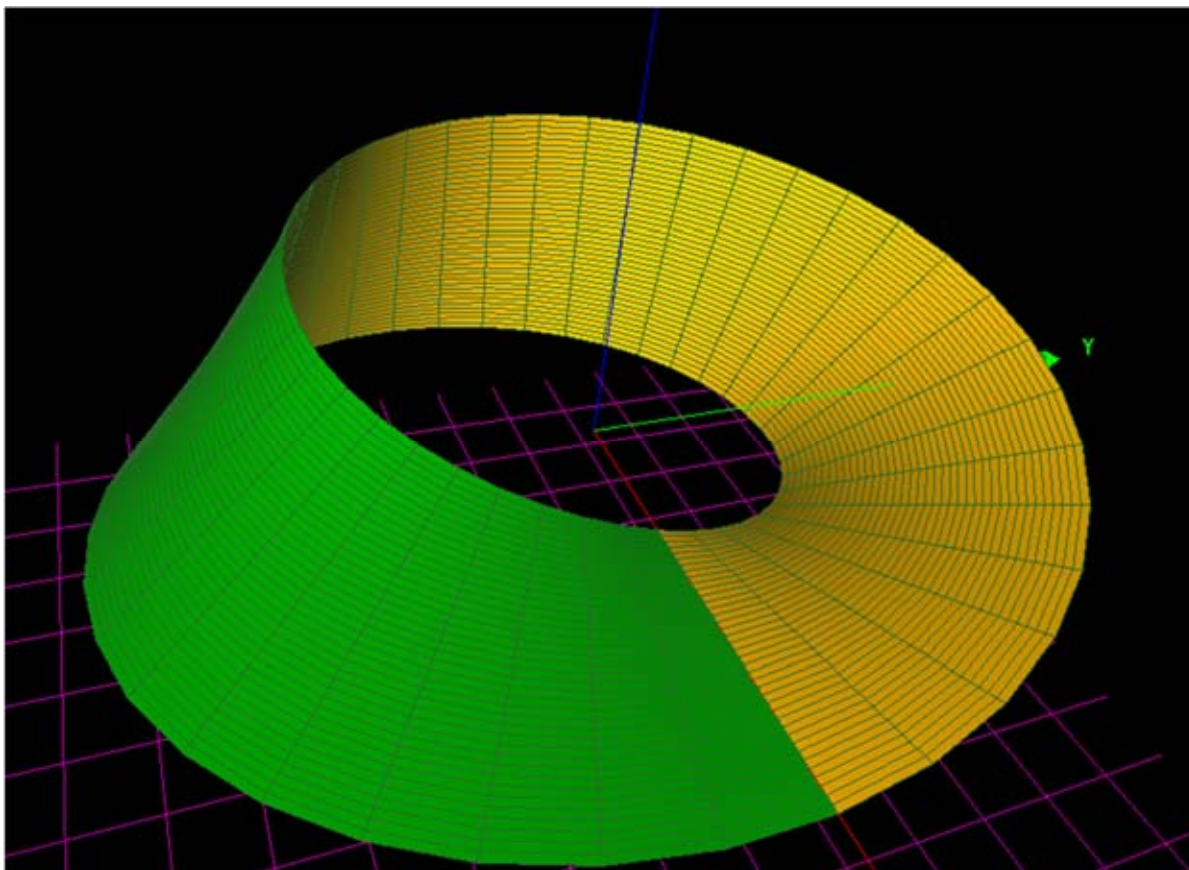
$$Z(u,v)=0.5*\sin(u)$$

$$0 \leq u \leq 2\pi$$

$$0 \leq v \leq 2\pi$$

POVRŠI U PROSTORU

Primeri



$$x(u, v) = \left(1 + \frac{1}{2}v \cos \frac{u}{2}\right) \cos u$$

$$y(u, v) = \left(1 + \frac{1}{2}v \cos \frac{u}{2}\right) \sin u$$

$$z(u, v) = \frac{1}{2}v \sin \frac{u}{2}$$

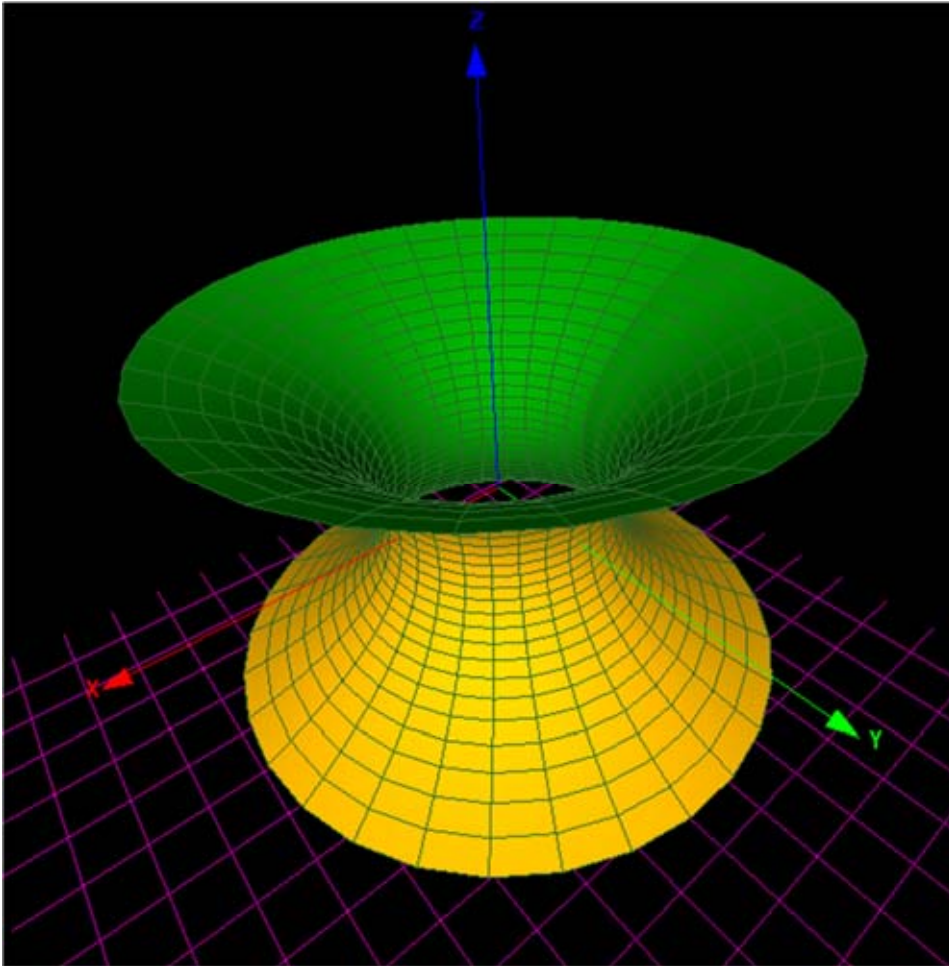
Moebius

$$0 \leq u \leq 2\pi$$

$$-1 \leq v \leq 1$$

POVRŠI U PROSTORU

Primeri



Katenoid

$$X(u,v)=2*\cosh(v/2)*\cos(u)$$

$$Y(u,v)=2*\cosh(v/2)*\sin(u)$$

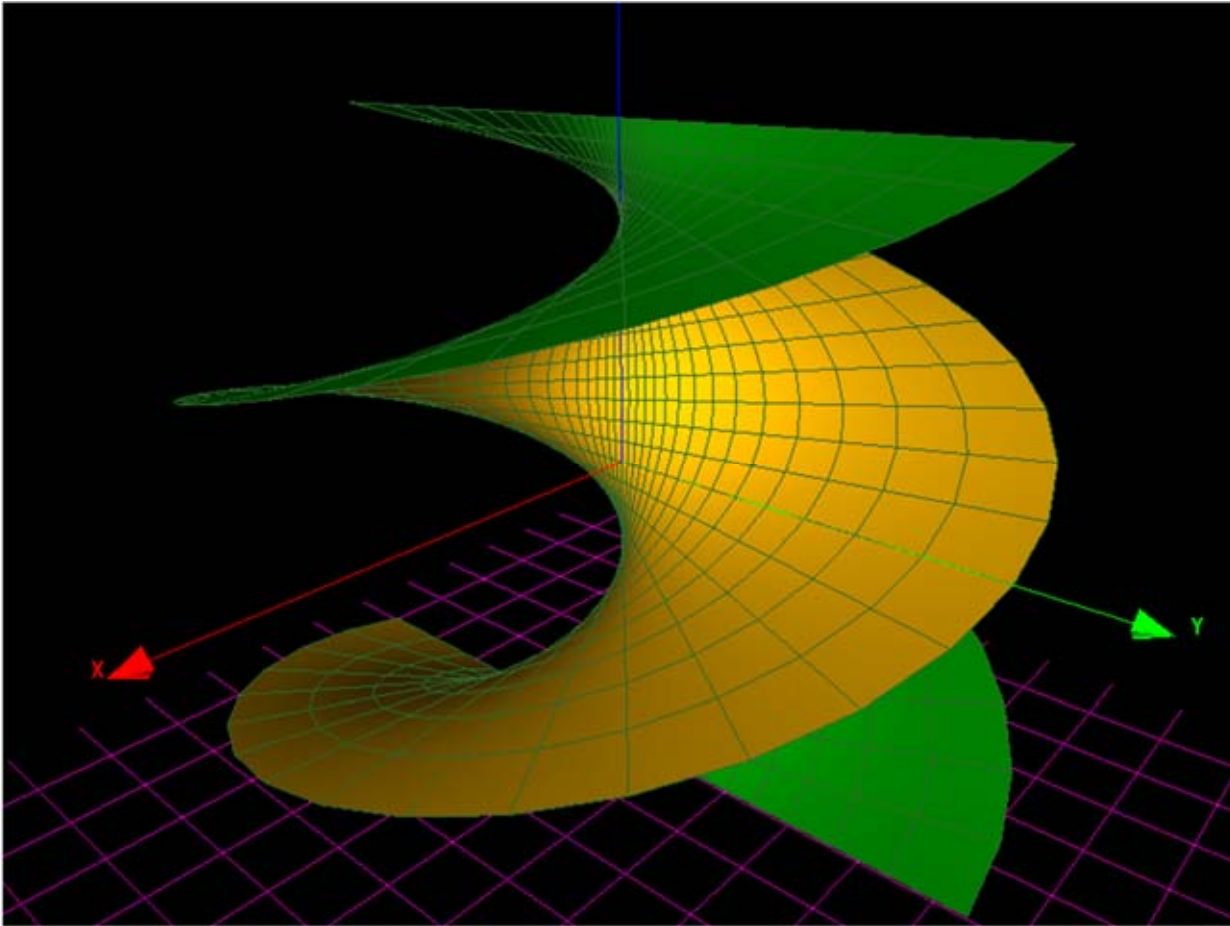
$$Z(u,v)=v$$

$$-\pi \leq u \leq \pi$$

$$-\pi \leq v \leq \pi$$

POVRŠI U PROSTORU

Primeri



Helikoid

$$X(u,v)=\sinh(v)*\cos(u)$$

$$Y(u,v)=-\sinh(v)*\sin(u)$$

$$Z(u,v)=3*u$$

$$-\pi \leq u \leq \pi$$

$$-\pi \leq v \leq \pi$$

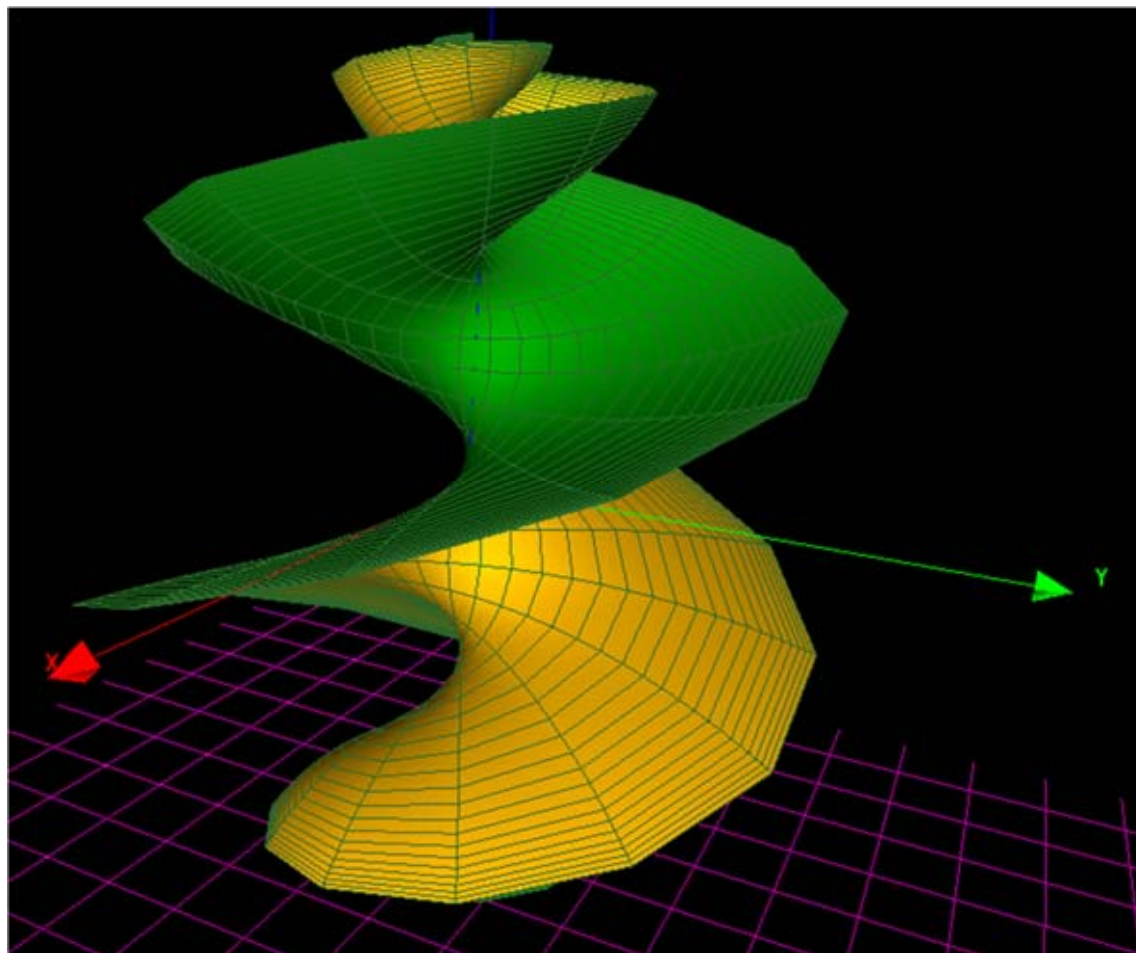
POVRŠI U PROSTORU

Primeri

$$X(u,v) = (\sinh(v) \cdot \cos(3 \cdot u)) / (1 + \cosh(u) \cdot \cosh(v))$$

$$Y(u,v) = (\sinh(v) \cdot \sin(3 \cdot u)) / (1 + \cosh(u) \cdot \cosh(v))$$

$$Z(u,v) = (\cosh(v) \cdot \sinh(u)) / (1 + \cosh(u) \cdot \cosh(v))$$



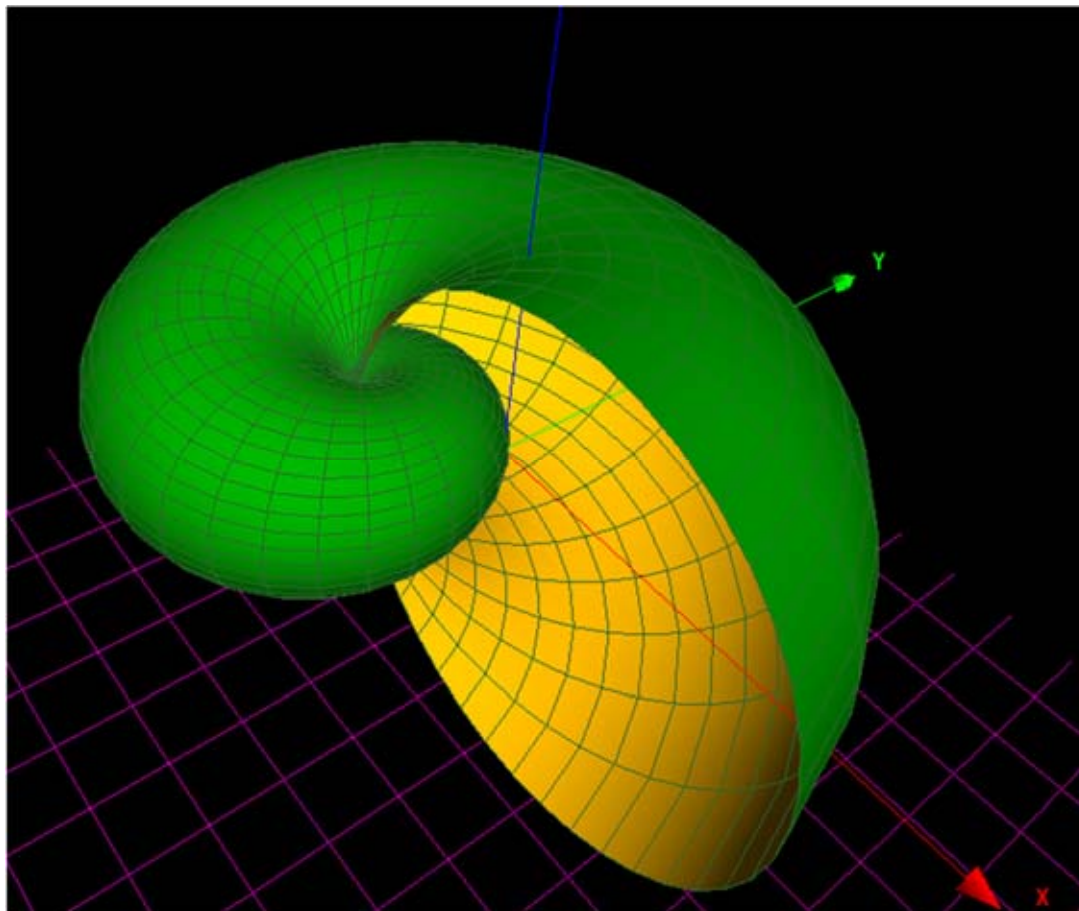
Hiperhelikoid

$$-\pi \leq u \leq \pi$$

$$-\pi \leq v \leq \pi$$

POVRŠI U PROSTORU

Primeri



Shell

$$X(u, v) = 1.2^v (\sin^2 u \cdot \sin v)$$

$$Y(u, v) = 1.2^v (\sin^2 u \cdot \cos v)$$

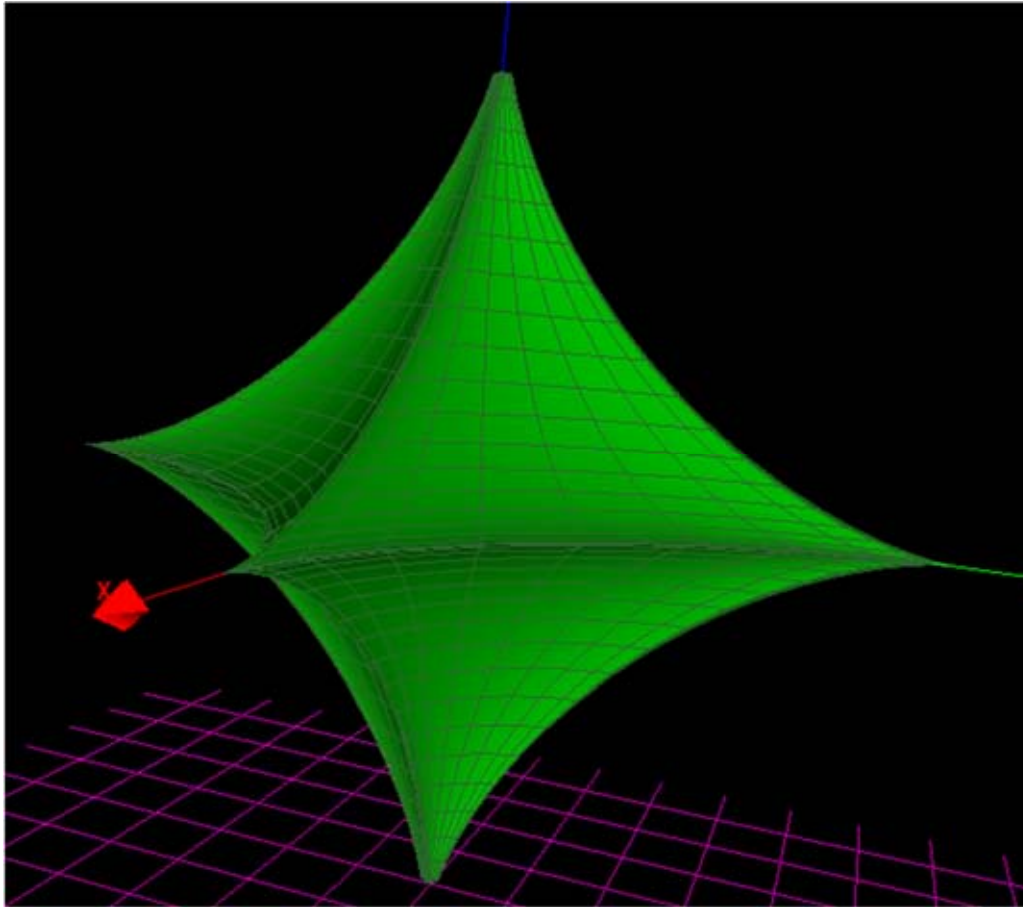
$$Z(u, v) = 1.2^v (\sin u \cdot \cos u)$$

$$0 \leq u \leq \pi$$

$$-\frac{\pi}{4} \leq v \leq \frac{5\pi}{2}$$

POVRŠI U PROSTORU

Primeri



Hexaedron

$$x = \cos^3 u \cdot \cos^3 v$$

$$y = \cos^3 u \cdot \sin^3 v$$

$$z = \sin^3 u$$

$$-1.3 \leq u \leq 1.3$$

$$0 \leq v \leq 2\pi$$

POVRŠI U PROSTORU

Primeri

Dini

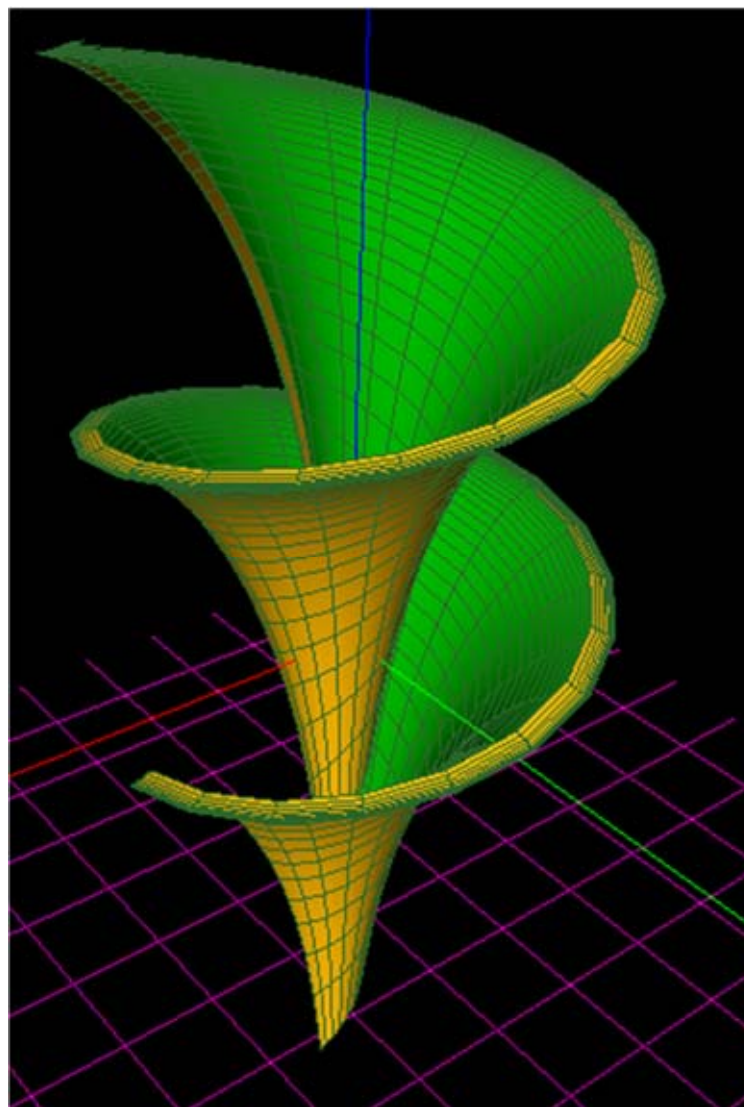
$$X(u, v) = \cos u \cdot \sin v$$

$$Y(u, v) = \sin u \cdot \sin v$$

$$Z(u, v) = \cos v + \log\left(\tan \frac{v}{2}\right) + 0.2u$$

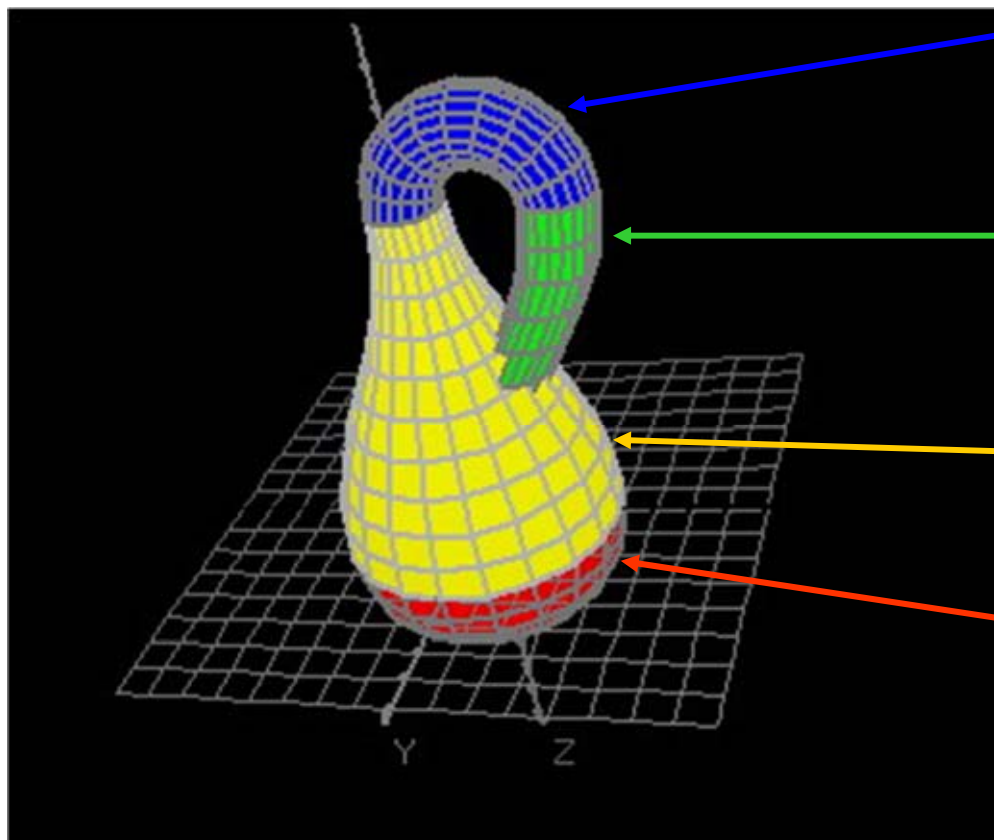
$$0 \leq u \leq 12.4$$

$$0.1 \leq v \leq 2$$



POVRŠI U PROSTORU

Primeri



$$\begin{aligned}x &= 2 + (2 + \cos u) \cos v \\y &= \sin u \\z &= -3\pi + (2 + \cos u) \cdot \sin v\end{aligned}$$

$$\begin{aligned}x &= 2 - 2 \cos v + \sin u \\y &= \cos u \\z &= 3v - 6\pi\end{aligned}$$

$$\begin{aligned}x &= (2.5 + 1.5 \cos v) \cos u \\y &= (2.5 + 1.5 \cos v) \sin u \\z &= 3v - 6\pi\end{aligned}$$

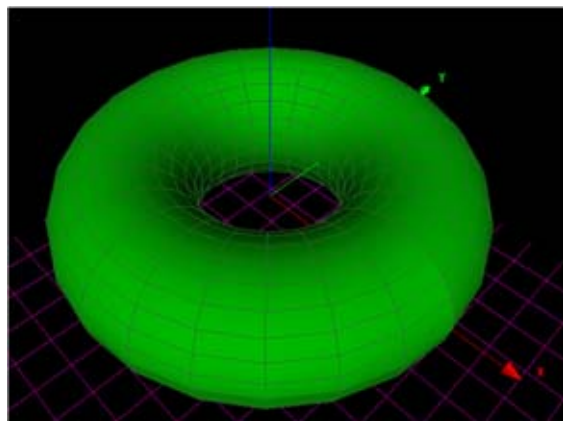
$$\begin{aligned}x &= (2.5 + 1.5 \cos v) \cos u \\y &= (2.5 + 1.5 \cos v) \sin u \\z &= -2.5 \sin v\end{aligned}$$

$$0 \leq u \leq 2\pi$$

$$\pi \leq v \leq 2\pi$$

POVRŠI U PROSTORU

Primeri



$$0 \leq u \leq 2\pi$$

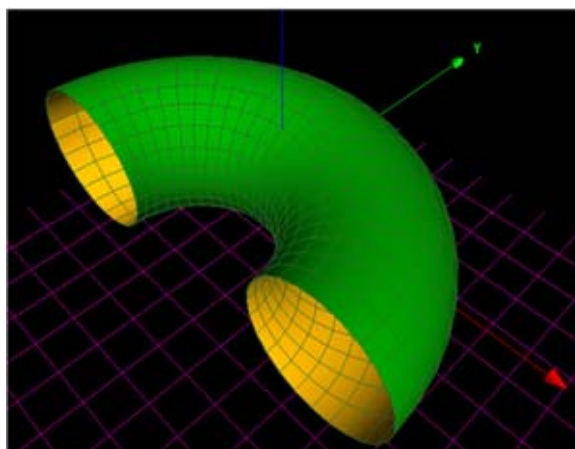
$$0 \leq v \leq 2\pi$$

Torus

$$X(u,v) = (1 + 0.5 \cdot \cos(u)) \cdot \cos(v)$$

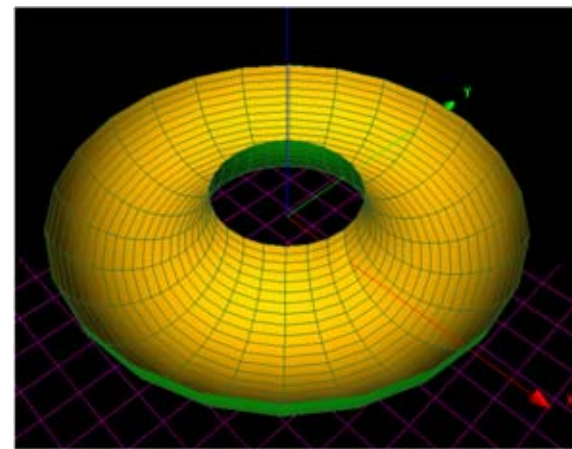
$$Y(u,v) = (1 + 0.5 \cdot \cos(u)) \cdot \sin(v)$$

$$Z(u,v) = 0.5 \cdot \sin(u)$$



$$0 \leq u \leq 2\pi$$

$$0 \leq v \leq \pi$$



$$\pi \leq u \leq 2\pi$$

$$0 \leq v \leq 2\pi$$

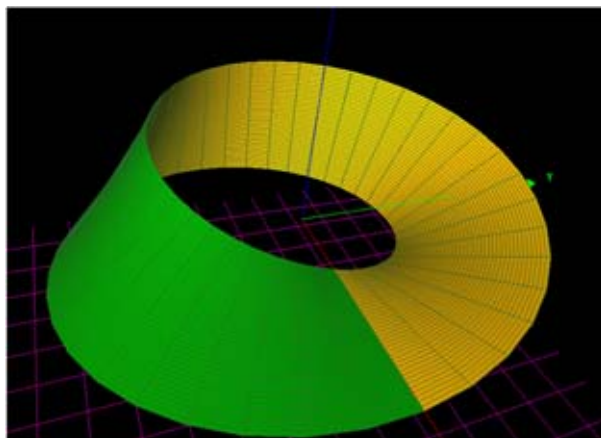
POVRŠI U PROSTORU

Primeri

$$X(u,v)=\cos(v)+u*\cos(v/2)*\cos(v)$$

$$Y(u,v)=\sin(v)+u*\cos(v/2)*\sin(v)$$

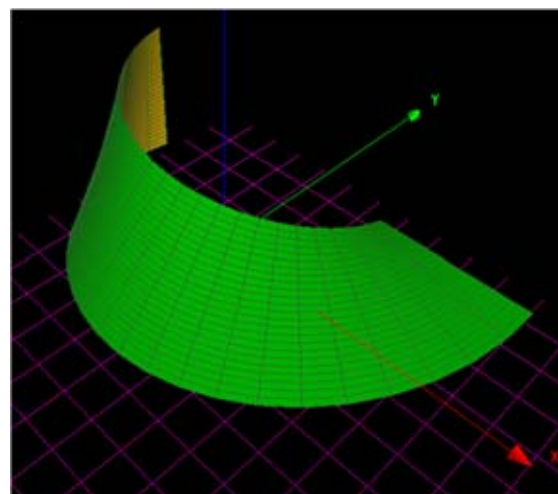
$$Z(u,v)=u*\sin(v/2)$$



$$-0.4 \leq u \leq 0.4$$

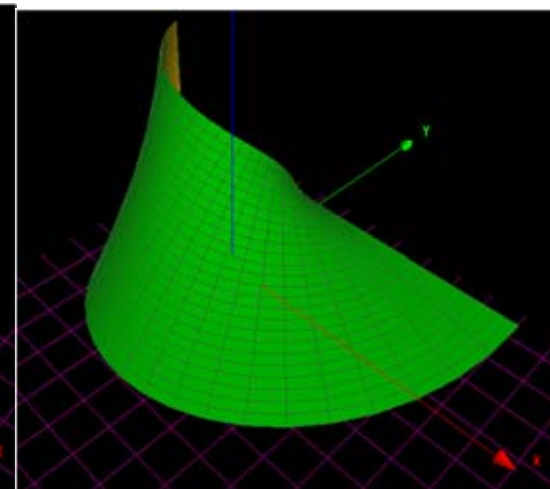
$$0 \leq v \leq 2\pi$$

Moebius



$$-0.4 \leq u \leq 0.4$$

$$\pi \leq v \leq 2\pi$$

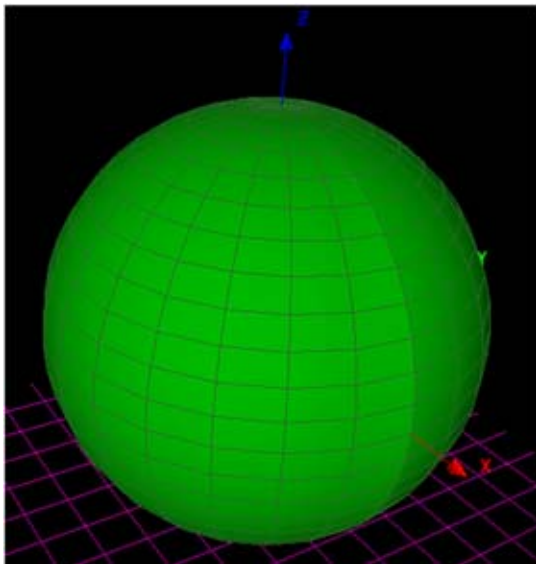


$$-0.8 \leq u \leq 0.8$$

$$\pi \leq v \leq 2\pi$$

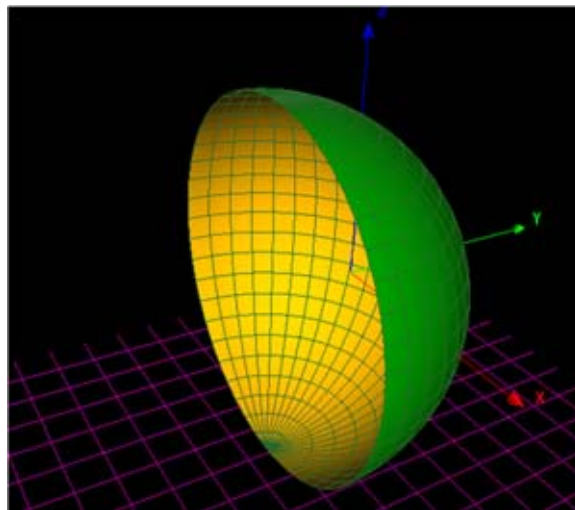
POVRŠI U PROSTORU

Primeri



$$-\frac{\pi}{2} \leq u \leq \frac{\pi}{2}$$

$$0 \leq v \leq 2\pi$$



$$-\frac{\pi}{2} \leq u \leq \frac{\pi}{2}$$

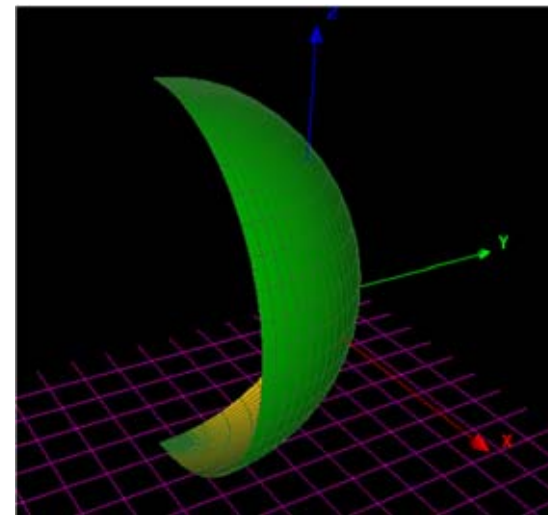
$$0 \leq v \leq \pi$$

Sfera

$$X(u,v) = r \cdot \cos(u) \cdot \cos(v)$$

$$Y(u,v) = r \cdot \cos(u) \cdot \sin(v)$$

$$Z(u,v) = r \cdot \sin(u)$$

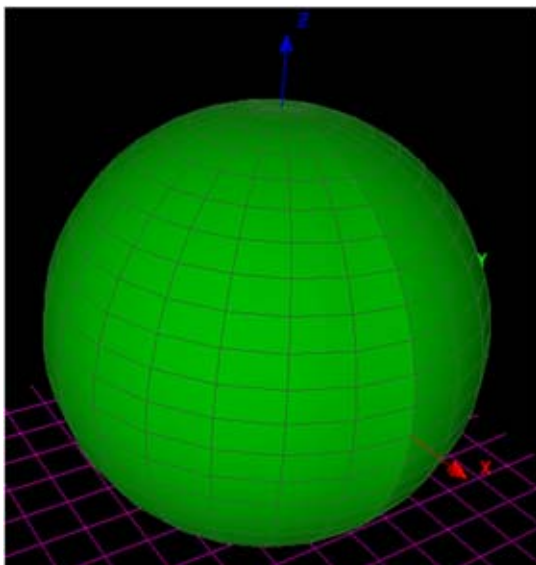


$$-\frac{\pi}{2} \leq u \leq \frac{\pi}{2}$$

$$0 \leq v \leq \frac{\pi}{2}$$

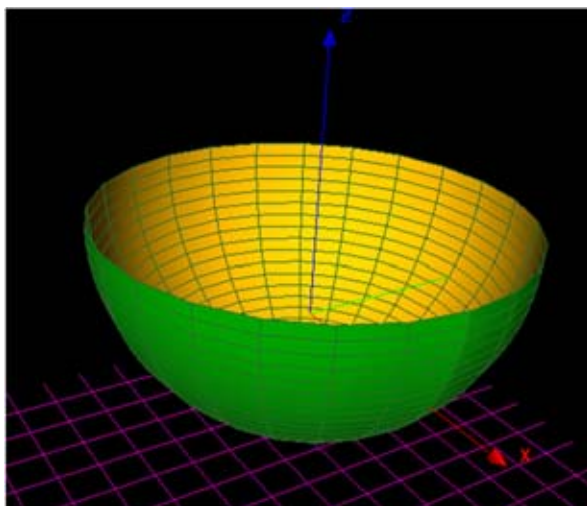
POVRŠI U PROSTORU

Primeri



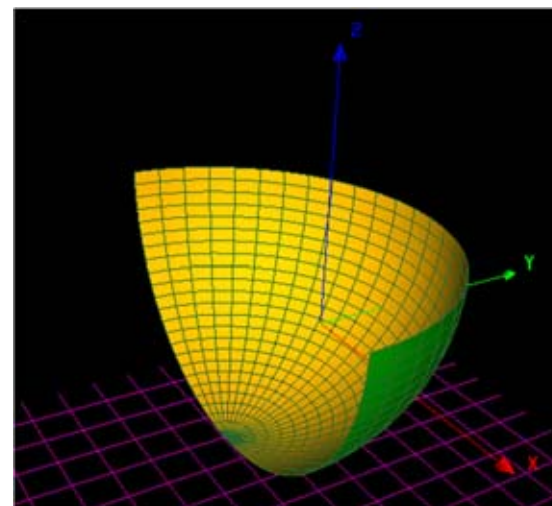
$$-\frac{\pi}{2} \leq u \leq \frac{\pi}{2}$$

$$0 \leq v \leq 2\pi$$



$$-\frac{\pi}{2} \leq u \leq 0$$

$$0 \leq v \leq 2\pi$$



$$-\frac{\pi}{2} \leq u \leq 0$$

$$0 \leq v \leq \pi$$

Sfera

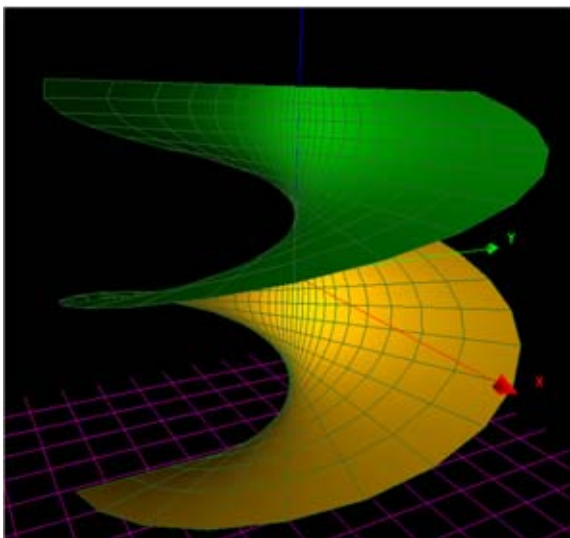
$$X(u,v) = r \cdot \cos(u) \cdot \cos(v)$$

$$Y(u,v) = r \cdot \cos(u) \cdot \sin(v)$$

$$Z(u,v) = r \cdot \sin(u)$$

POVRŠI U PROSTORU

Primeri



$$-\pi \leq u \leq \pi$$

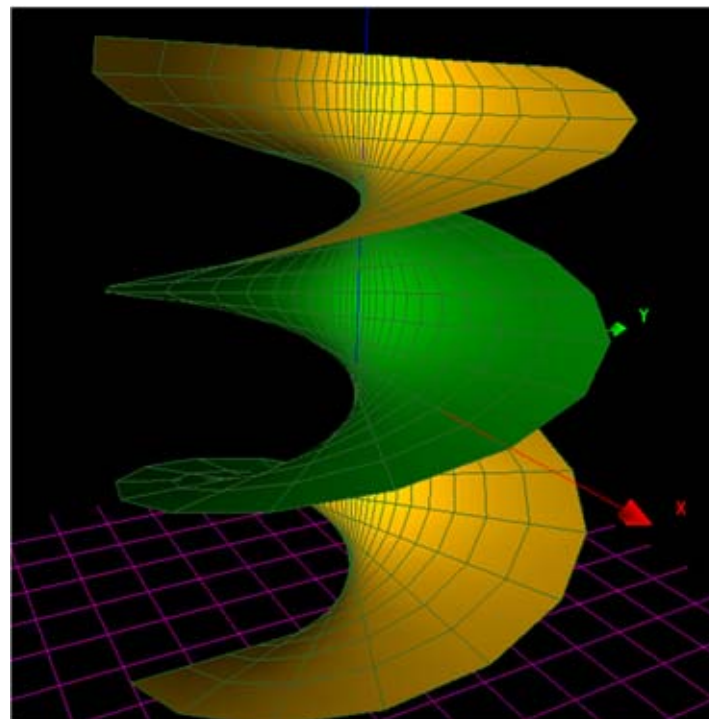
$$-\pi \leq v \leq \pi$$

Helikoid

$$X(u,v) = \sinh(v) \cdot \sin(u)$$

$$Y(u,v) = -\sinh(v) \cdot \cos(u)$$

$$Z(u,v) = 3 \cdot u$$



$$-\pi \leq u \leq 2\pi$$

$$-\pi \leq v \leq \pi$$

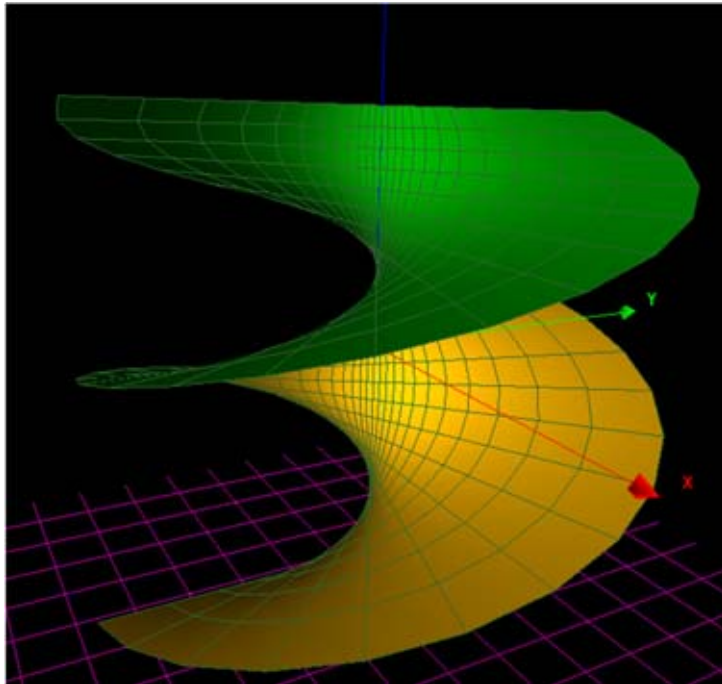
POVRŠI U PROSTORU

Primeri

$$X(u,v)=\sinh(v)*\sin(u)$$

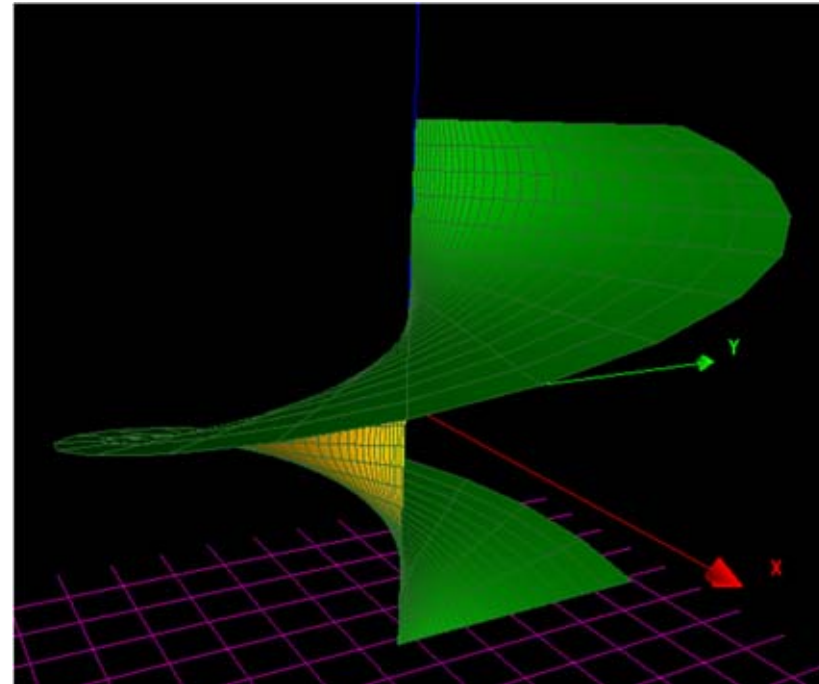
$$Y(u,v)=-\sinh(v)*\cos(u)$$

$$Z(u,v)=3*u$$



$$-\pi \leq u \leq \pi$$

$$-\pi \leq v \leq \pi$$



$$-\pi \leq u \leq \pi$$

$$0 \leq v \leq \pi$$

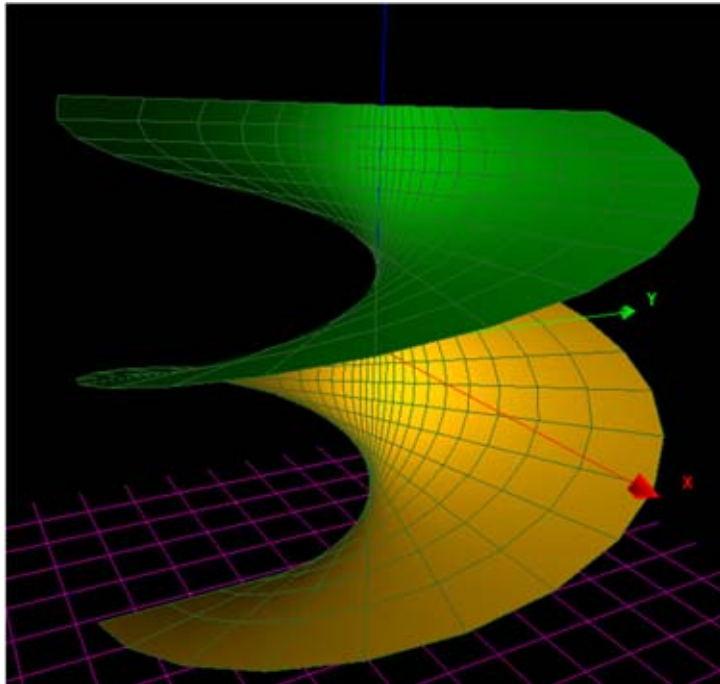
POVRŠI U PROSTORU

Primeri

$$X(u,v)=\sinh(v)*\sin(u)$$

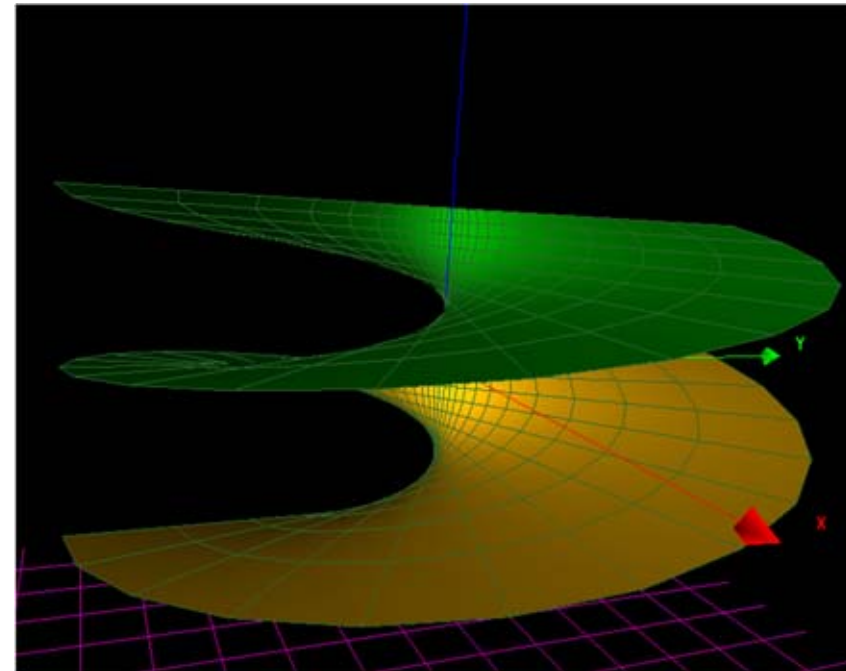
$$Y(u,v)=-\sinh(v)*\cos(u)$$

$$Z(u,v)=3*u$$



$$-\pi \leq u \leq \pi$$

$$-\pi \leq v \leq \pi$$



$$-\pi \leq u \leq \pi$$

$$-1.2 \cdot \pi \leq v \leq 1.2 \cdot \pi$$

POVRŠI U PROSTORU

Primeri

$$X(u,v)=a*\cos(u)*\cos(v)$$

$$Y(u,v)=b*\cos(u)*\sin(v)$$

$$Z(u,v)=c*\sin(u)$$

$$a = 1, b = 1, c = 1$$

$$-\frac{\pi}{2} \leq u \leq \frac{\pi}{2}$$

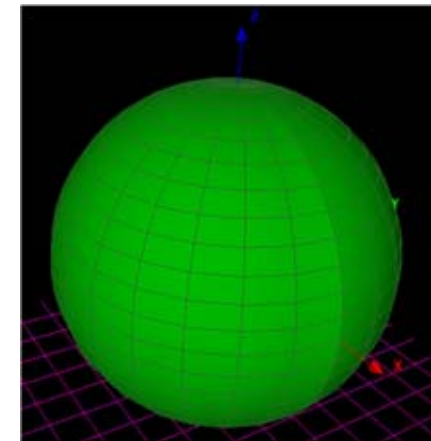
$$0 \leq v \leq 2\pi$$

Sfera

$$X(u,v)=r*\cos(u)*\cos(v)$$

$$Y(u,v)=r*\cos(u)*\sin(v)$$

$$Z(u,v)=r*\sin(u)$$



$$-\frac{\pi}{2} \leq u \leq \frac{\pi}{2}$$

$$0 \leq v \leq 2\pi$$

POVRŠI U PROSTORU

Primeri

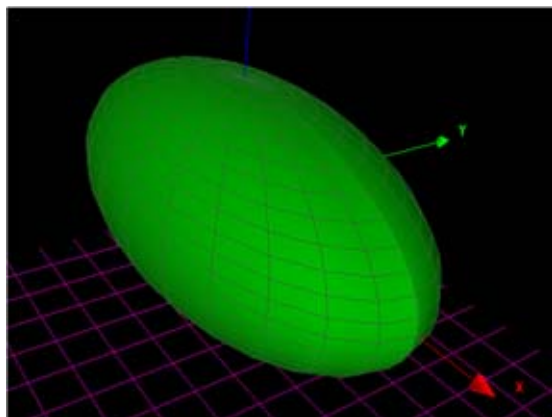
$$X(u,v)=a*\cos(u)*\cos(v)$$

$$Y(u,v)=b*\cos(u)*\sin(v)$$

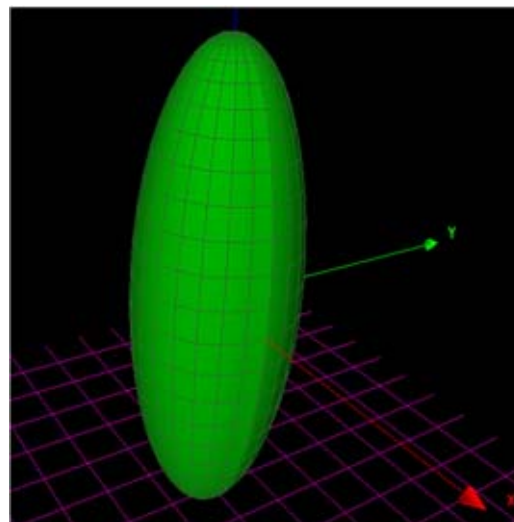
$$Z(u,v)=c*\sin(u)$$

$$-\frac{\pi}{2} \leq u \leq \frac{\pi}{2}$$

$$0 \leq v \leq 2\pi$$



$$a = 2, b = 1, c = 1$$

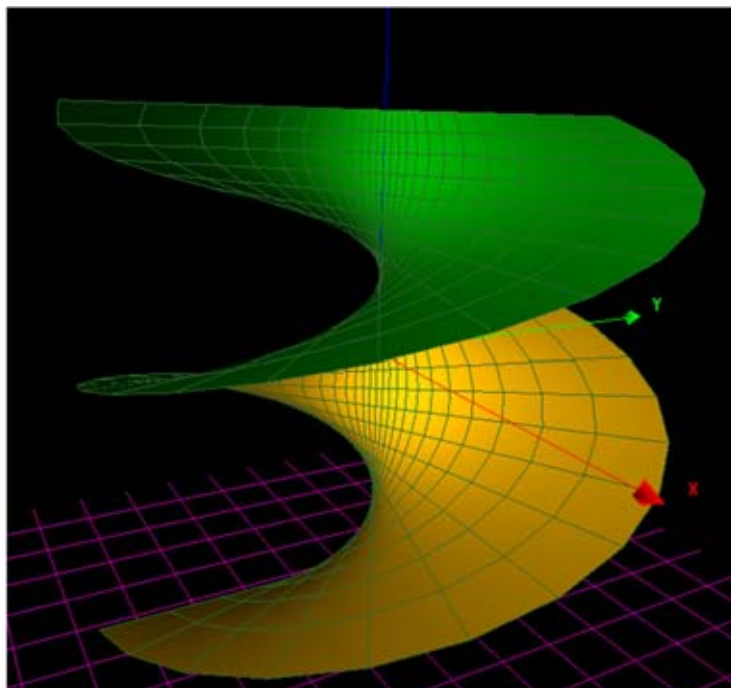


$$a = 1, b = 1, c = 3$$

POVRŠI U PROSTORU

Primeri

Helikoid



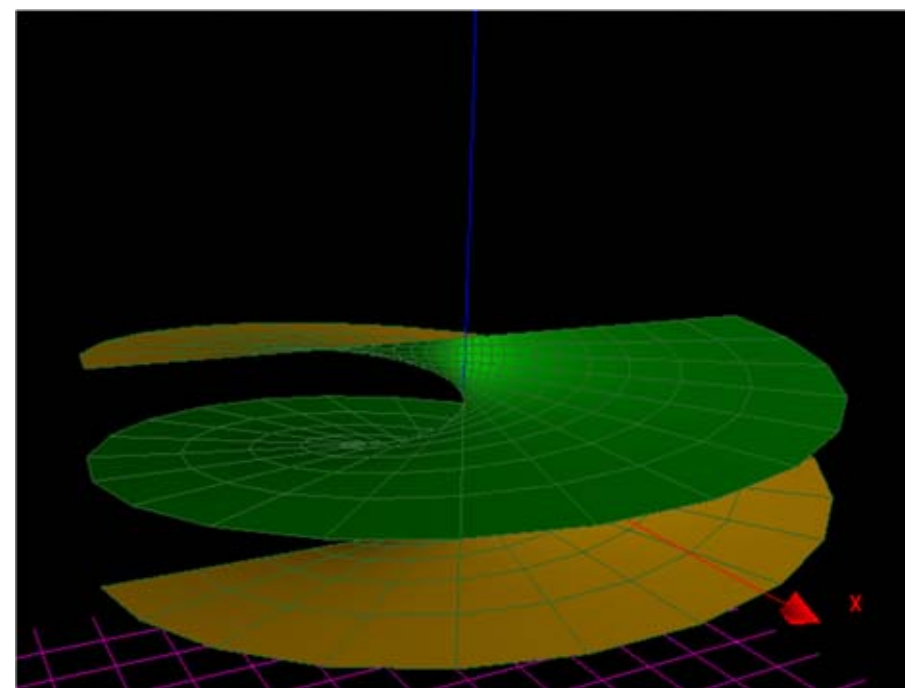
$$X(u,v)=\sinh(v)*\sin(u)$$

$$Y(u,v)=-\sinh(v)*\cos(u)$$

$$Z(u,v)=3*u$$

$$-\pi \leq u \leq \pi$$

$$-\pi \leq v \leq \pi$$



$$X(u,v)=\sinh(v)*\sin(u)$$

$$Y(u,v)=-\sinh(v)*\cos(u)$$

$$Z(u,v)=u$$

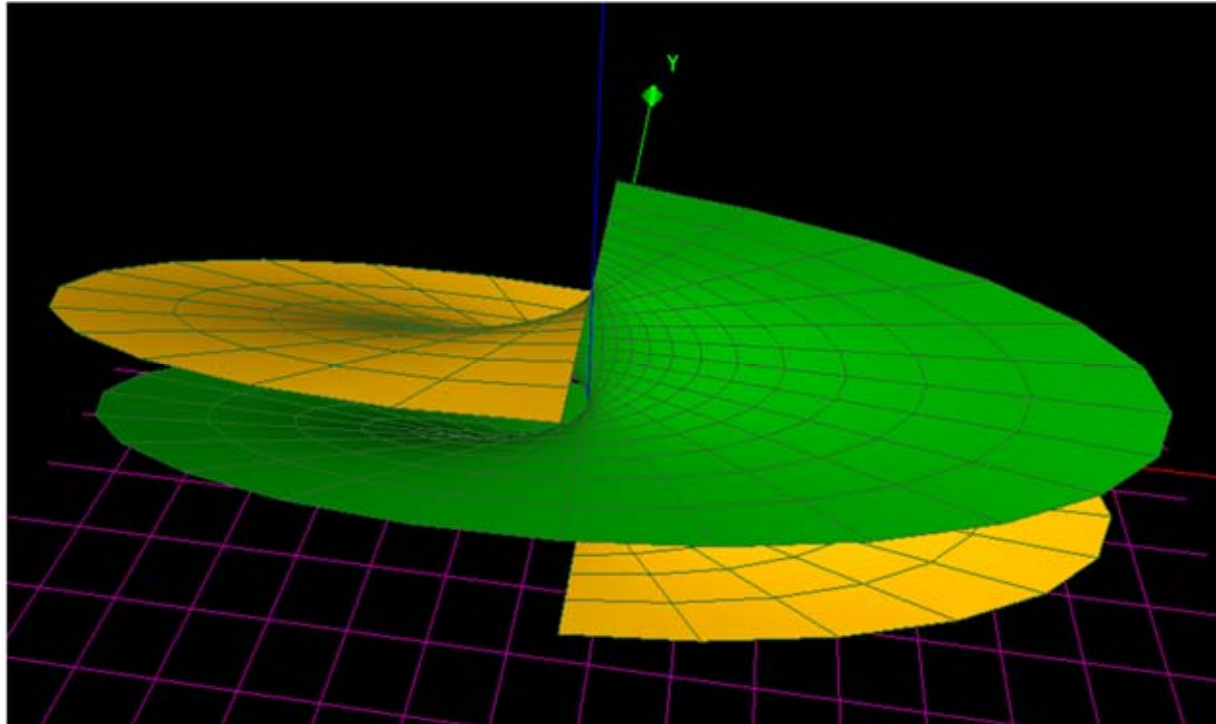
POVRŠI U PROSTORU

Primeri

$$X(u,v)=3*\sinh(v)*\sin(u)$$

$$Y(u,v)=-\sinh(v)*\cos(u)$$

$$Z(u,v)=3*u$$



$$-\pi \leq u \leq \pi$$

$$-\pi \leq v \leq \pi$$

POVRŠI U PROSTORU

Izoparametarske linije

$$\begin{aligned}x &= \sigma_1(u, v) \\y &= \sigma_2(u, v) \\z &= \sigma_3(u, v)\end{aligned}\quad \begin{aligned}u_1 &\leq u \leq u_2 \\v_1 &\leq v \leq v_2\end{aligned}$$

$$u = C = \text{const}$$

Izoparametarska v – linija

$$x = \sigma_1(C, v)$$

za konstantnu vrednost parametra

$$y = \sigma_2(C, v) \quad v_1 \leq v \leq v_2$$

$$u = C$$

$$z = \sigma_3(C, v)$$

Izoparametarske linije pripadaju površi.

POVRŠI U PROSTORU

Izoparametarske linije

$$\begin{aligned}x &= \sigma_1(u, v) \\y &= \sigma_2(u, v) \\z &= \sigma_3(u, v)\end{aligned}\quad \begin{aligned}u_1 &\leq u \leq u_2 \\v_1 &\leq v \leq v_2\end{aligned}$$

$$v = D = \text{const}$$

Izoparametarska u – linija

$$x = \sigma_1(u, D)$$

za konstantnu vrednost parametra

$$y = \sigma_2(u, D) \quad u_1 \leq u \leq u_2$$

$$v = D$$

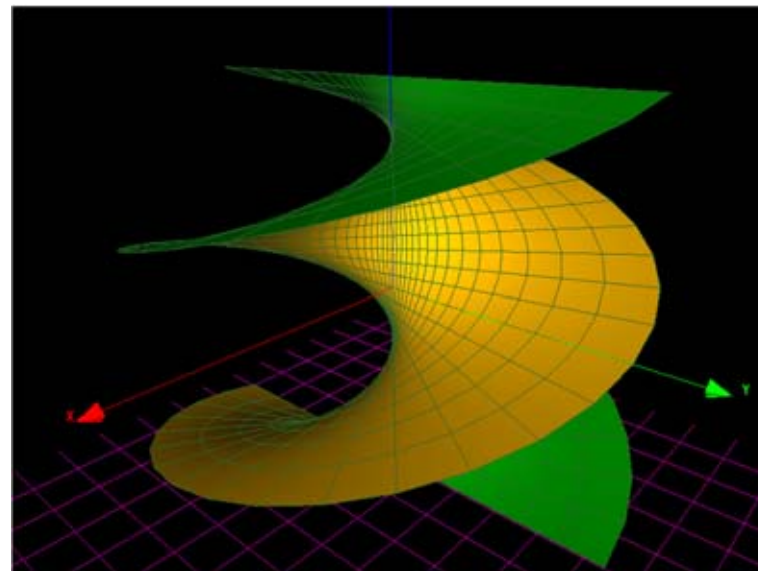
$$z = \sigma_3(u, D)$$

Izoparametarske linije pripadaju površi.

POVRŠI U PROSTORU

Izoparametarske linije

$$\begin{aligned}x &= \sigma_1(u, v) & u_1 \leq u \leq u_2 \\y &= \sigma_2(u, v) & v_1 \leq v \leq v_2 \\z &= \sigma_3(u, v)\end{aligned}$$



$$C = C_i, \quad i = 1, 2, \dots, m$$

$$\begin{aligned}x &= \sigma_1(C_i, v) \\y &= \sigma_2(C_i, v) & v_1 \leq v \leq v_2 \\z &= \sigma_3(C_i, v)\end{aligned}$$

$$D = D_j, \quad j = 1, 2, \dots, n$$

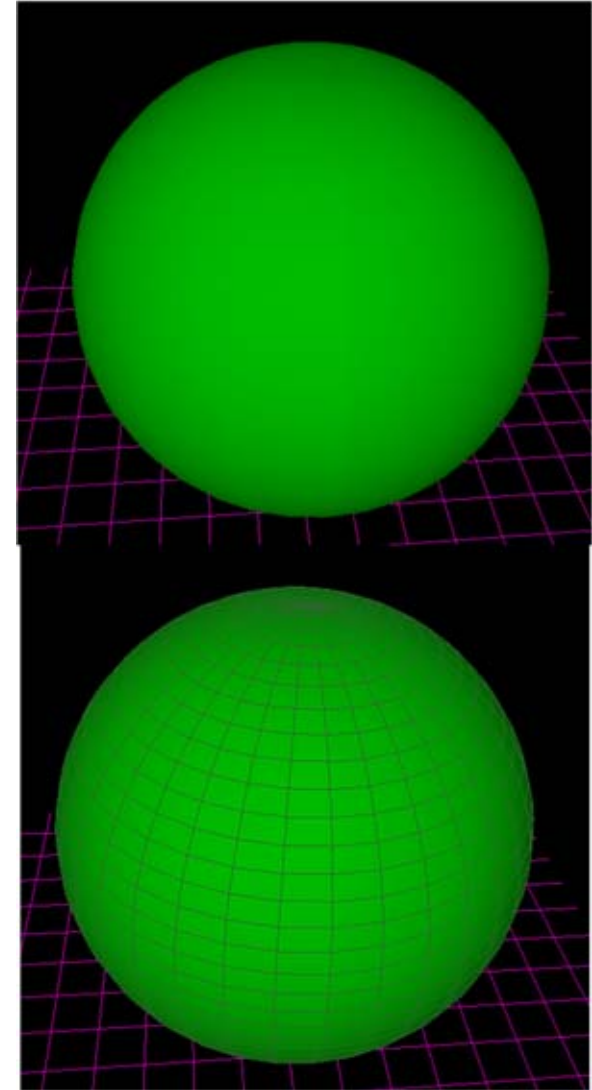
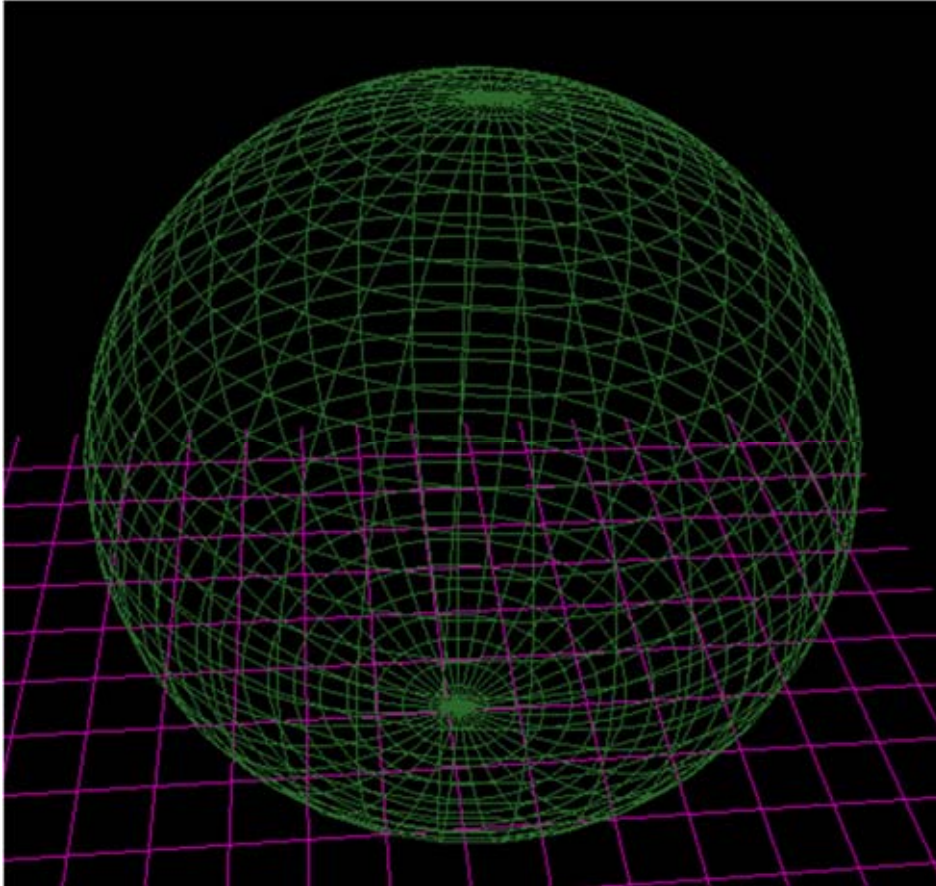
$$\begin{aligned}x &= \sigma_1(u, D_j) \\y &= \sigma_2(u, D_j) & u_1 \leq u \leq u_2 \\z &= \sigma_3(u, D_j)\end{aligned}$$

Dva sistema (skupa) izoparametarskih linija na površi

$$\sigma(u, v) = (\sigma_1(u, v), \sigma_2(u, v), \sigma_3(u, v))$$

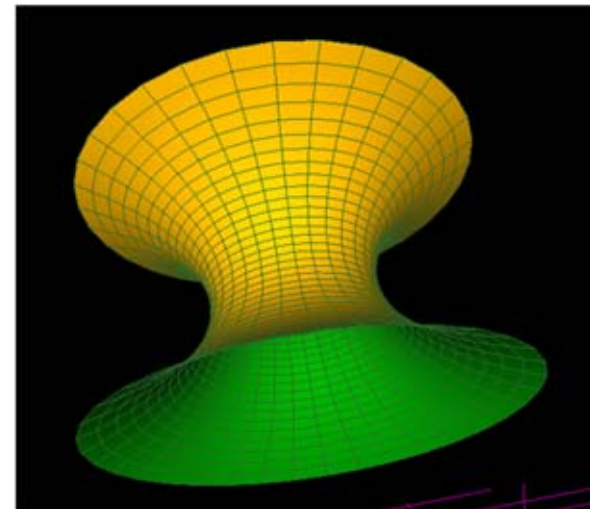
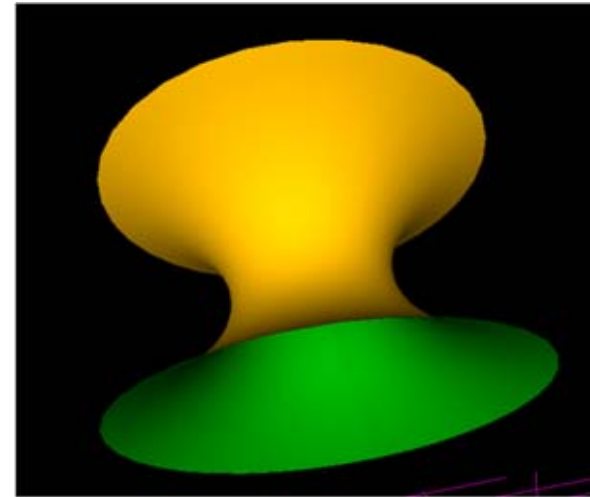
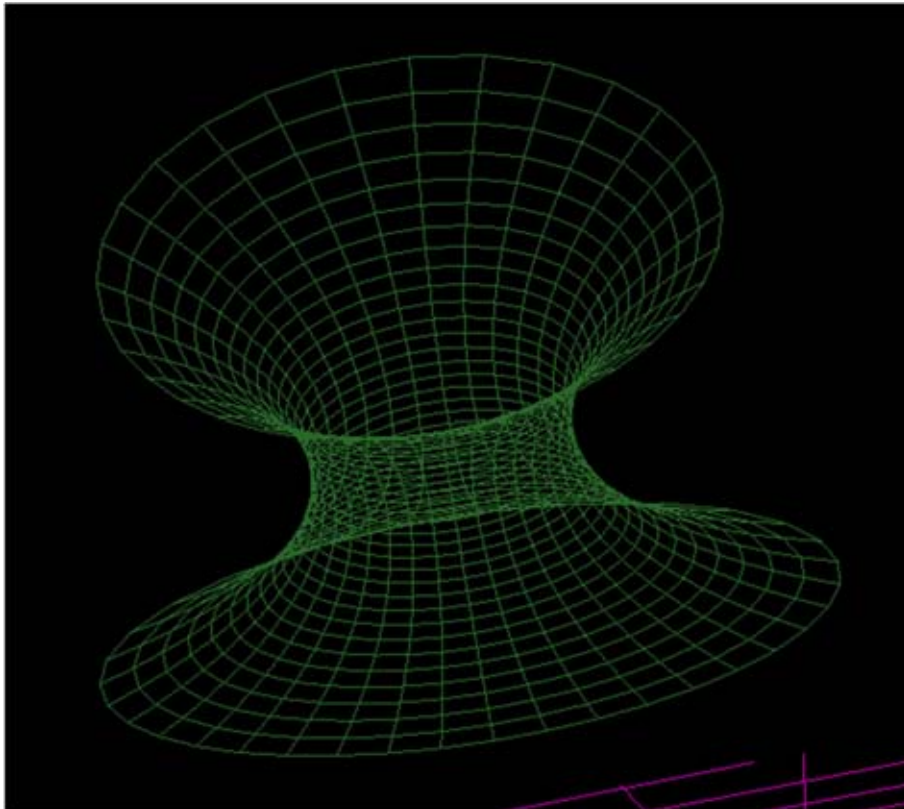
POVRŠI U PROSTORU

Izoparametarske linije



POVRŠI U PROSTORU

Izoparametarske linije



POVRŠI U PROSTORU

Izoparametarske linije

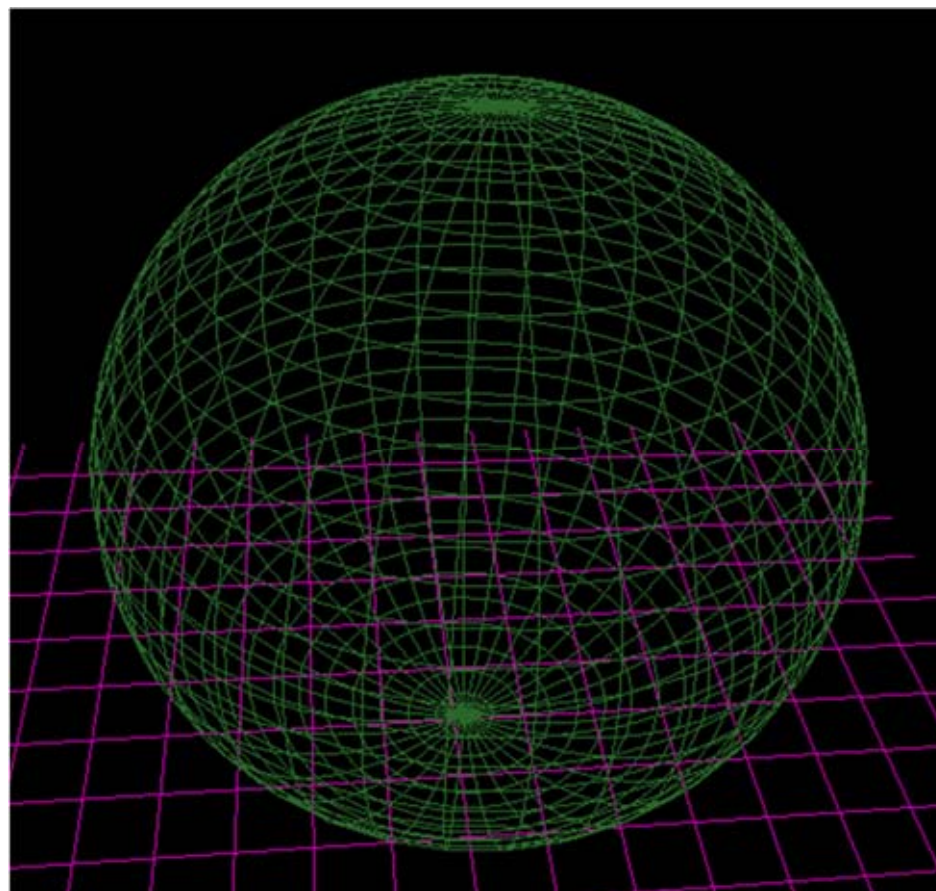
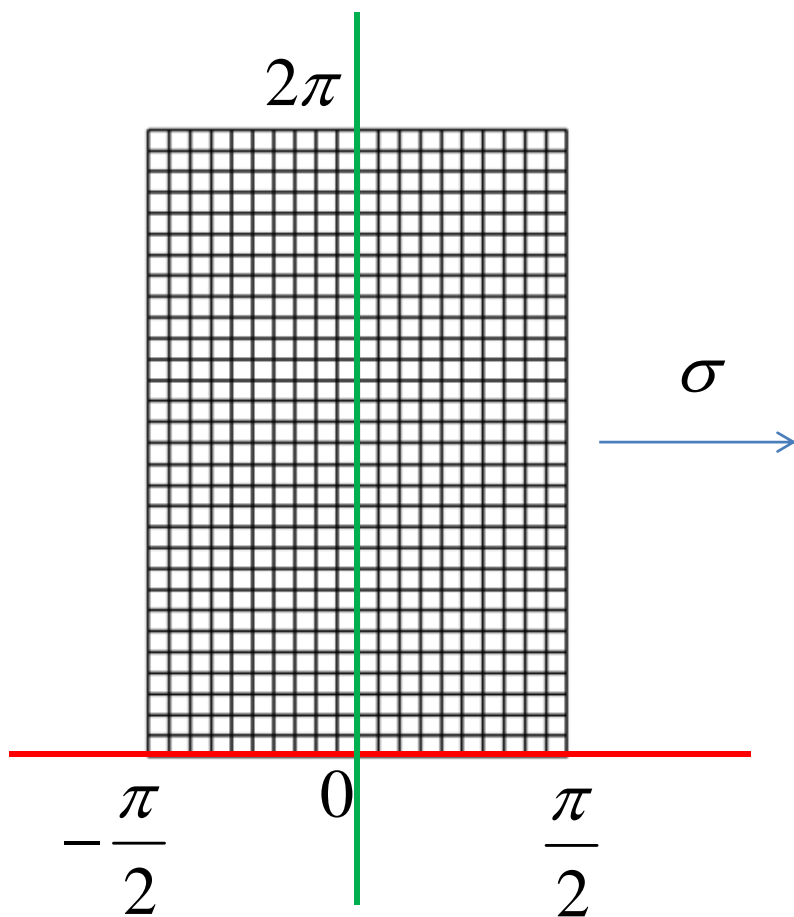
$$X(u,v)=r*\cos(u)*\cos(v)$$

$$Y(u,v)=r*\cos(u)*\sin(v)$$

$$Z(u,v)=r*\sin(u)$$

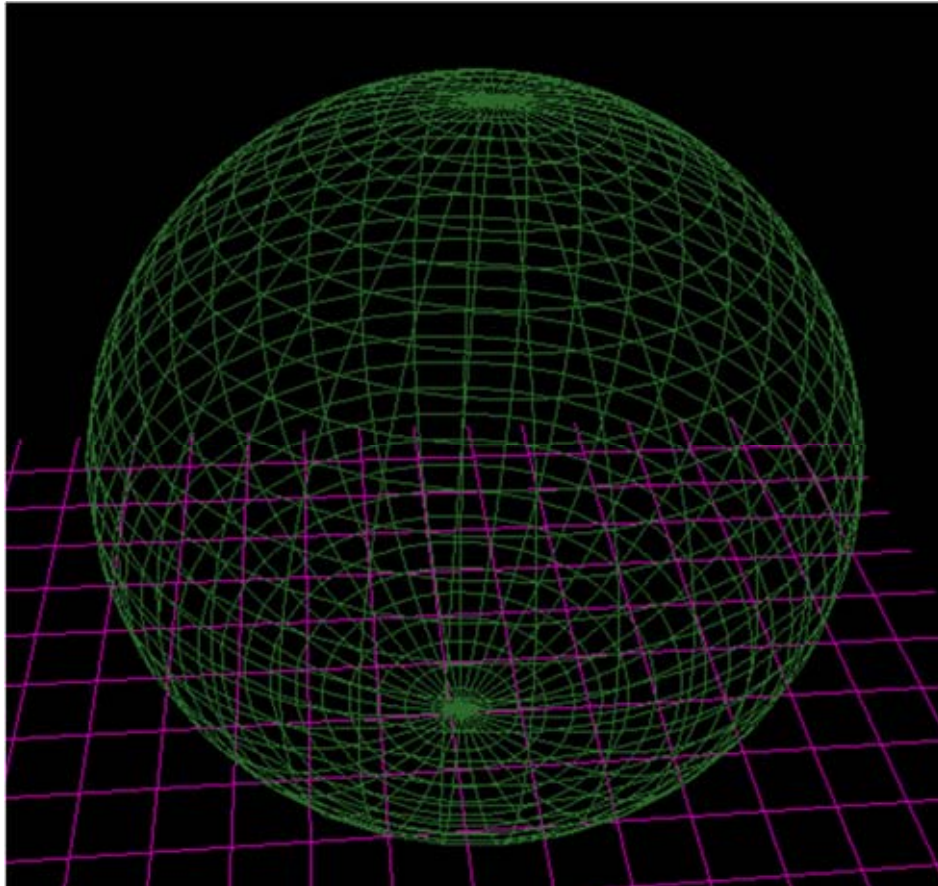
$$-\frac{\pi}{2} \leq u \leq \frac{\pi}{2}$$

$$0 \leq v \leq 2\pi$$



POVRŠI U PROSTORU

Izoparametarske linije



$$X(u,v)=r*\cos(u)*\cos(v)$$

$$Y(u,v)=r*\cos(u)*\sin(v)$$

$$Z(u,v)=r*\sin(u)$$

$$-\frac{\pi}{2} \leq u \leq \frac{\pi}{2}$$

$$0 \leq v \leq 2\pi$$

$$u = c$$

$$x=r*\cos(c)*\cos(v)$$

$$y=r*\cos(c)*\sin(v)$$

$$z=r*\sin(c)$$

$$0 \leq v \leq 2\pi$$

$$v = d$$

$$x=r*\cos(u)*\cos(d)$$

$$Y(u,v)=r*\cos(u)*\sin(d)$$

$$Z(u,v)=r*\sin(u)$$

$$-\frac{\pi}{2} \leq u \leq \frac{\pi}{2}$$

POVRŠI U PROSTORU

Linije na površi

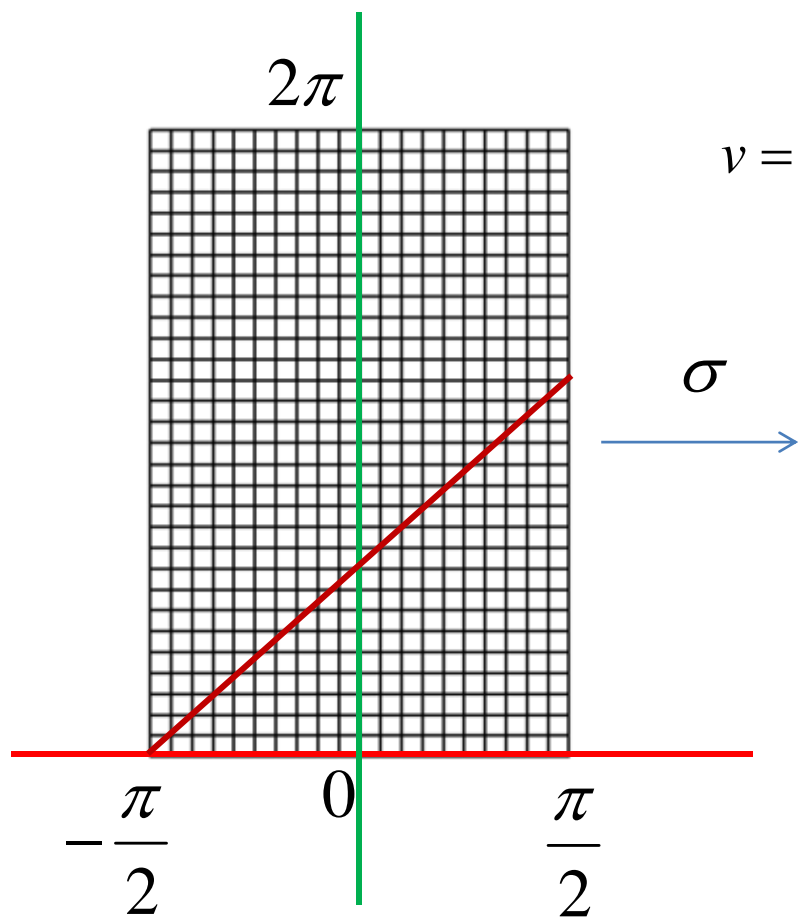
$$X(u,v) = r \cdot \cos(u) \cdot \cos(v)$$

$$Y(u,v) = r \cdot \cos(u) \cdot \sin(v)$$

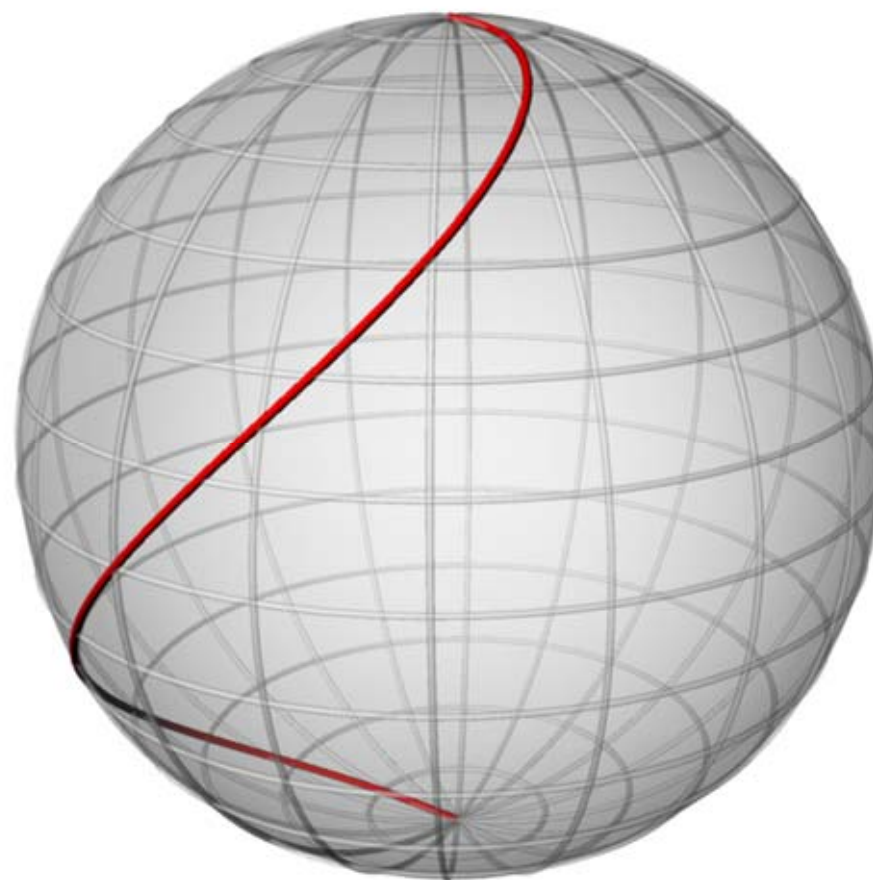
$$Z(u,v) = r \cdot \sin(u)$$

$$-\frac{\pi}{2} \leq u \leq \frac{\pi}{2}$$

$$0 \leq v \leq 2\pi$$



$$v = u$$



POVRŠI U PROSTORU

Linije na površi

$$X(u,v)=r*\cos(u)*\cos(v)$$

$$Y(u,v)=r*\cos(u)*\sin(v)$$

$$Z(u,v)=r*\sin(u)$$

$$-\frac{\pi}{2} \leq u \leq \frac{\pi}{2}$$

$$0 \leq v \leq 2\pi$$

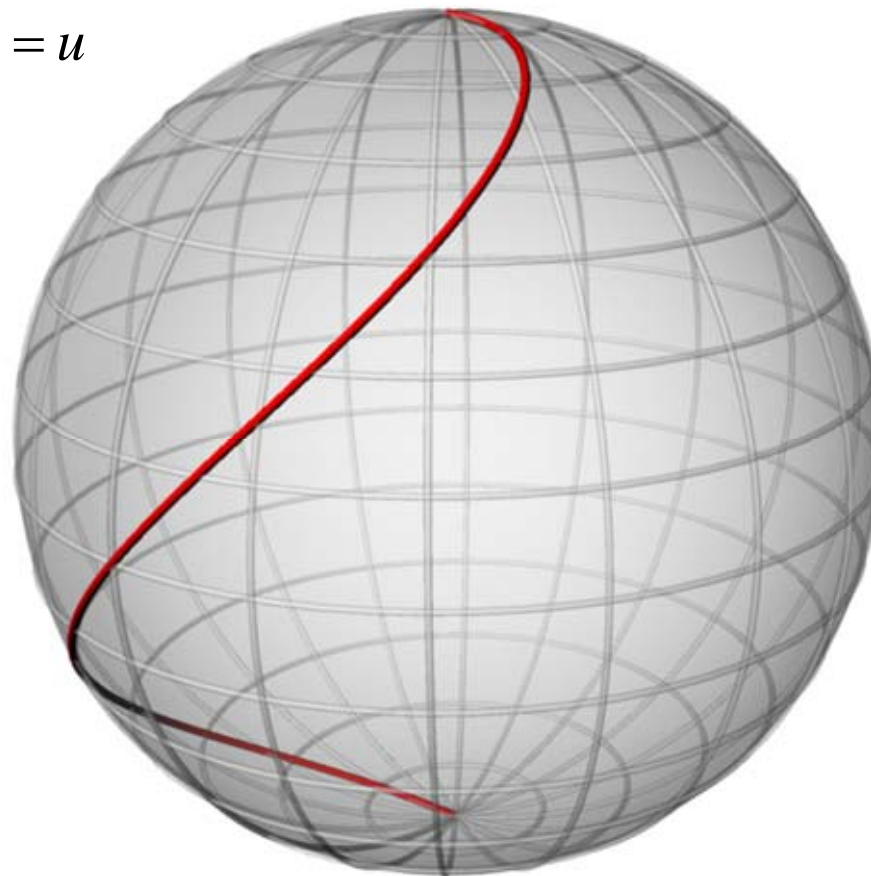
$$v = u$$

$$X(u,v)=r*\cos(u)*\cos(u)$$

$$Y(u,v)=r*\cos(u)*\sin(u)$$

$$Z(u,v)=r*\sin(u)$$

$$-\frac{\pi}{2} \leq u \leq \frac{\pi}{2}$$



POVRŠI U PROSTORU

Linije na površi

$$X(u,v)=r*\cos(u)*\cos(v)$$

$$Y(u,v)=r*\cos(u)*\sin(v)$$

$$Z(u,v)=r*\sin(u)$$

$$-\frac{\pi}{2} \leq u \leq \frac{\pi}{2}$$

$$0 \leq v \leq 2\pi$$

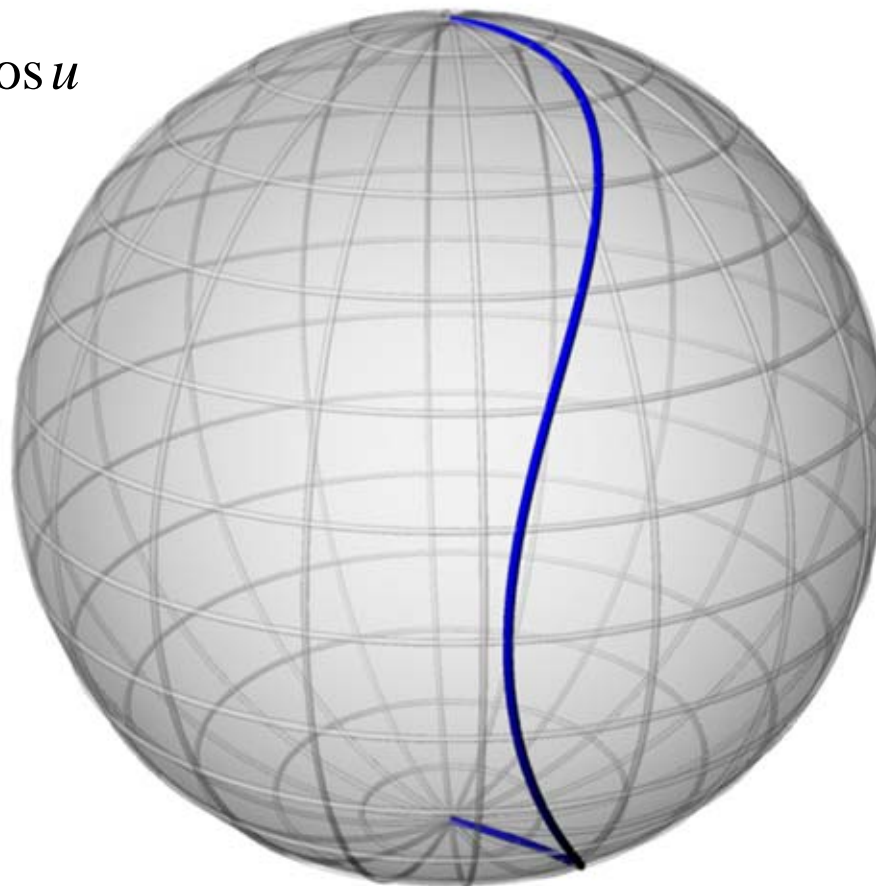
$$v = \cos u$$

$$X(u,v)=r*\cos(u)*\cos(\cos(u))$$

$$Y(u,v)=r*\cos(u)*\sin(\cos(u))$$

$$Z(u,v)=r*\sin(u)$$

$$-\frac{\pi}{2} \leq u \leq \frac{\pi}{2}$$



POVRŠI U PROSTORU

Linije na površi

$$X(u,v)=r*\cos(u)*\cos(u)$$

$$Y(u,v)=r*\cos(u)*\sin(u)$$

$$Z(u,v)=r*\sin(u)$$

$$-\frac{\pi}{2} \leq u \leq \frac{\pi}{2}$$

$$v = u$$

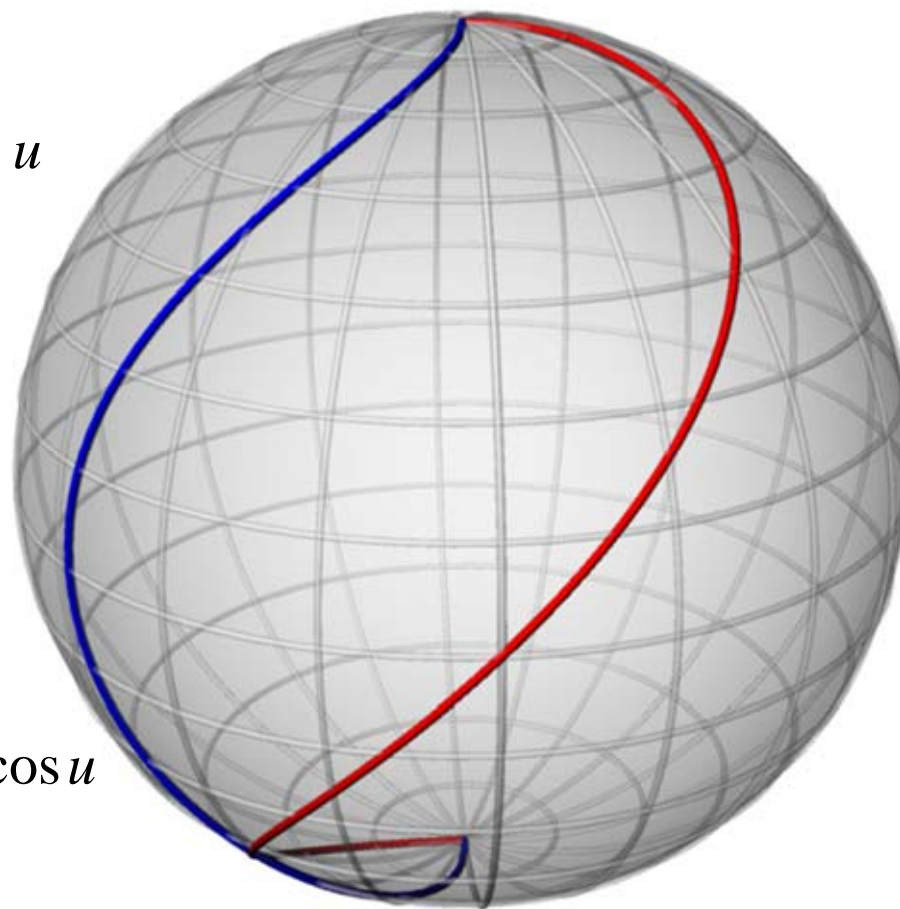
$$X(u,v)=r*\cos(u)*\cos(\cos(u))$$

$$Y(u,v)=r*\cos(u)*\sin(\cos(u))$$

$$Z(u,v)=r*\sin(u)$$

$$-\frac{\pi}{2} \leq u \leq \frac{\pi}{2}$$

$$v = \cos u$$



POVRŠI U PROSTORU

Linije na površi



$$u = c_i$$

$$x = r \cos c_i \cos v$$

$$y = r \cos c_i \sin v$$

$$z = r \sin c_i$$

$$0 \leq v \leq 2\pi$$

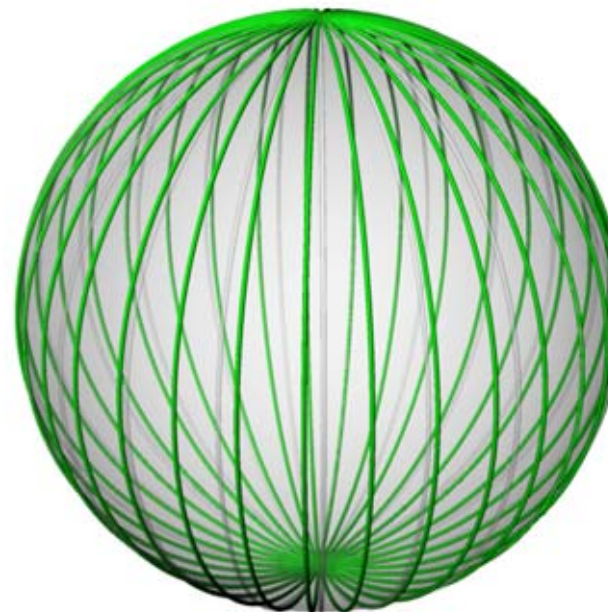
$$x = r \cos u \cos v$$

$$y = r \cos u \sin v$$

$$z = r \sin u$$

$$-\frac{\pi}{2} \leq u \leq \frac{\pi}{2}$$

$$0 \leq v \leq 2\pi$$



$$v = d_i$$

$$x = r \cos u \cos d_i$$

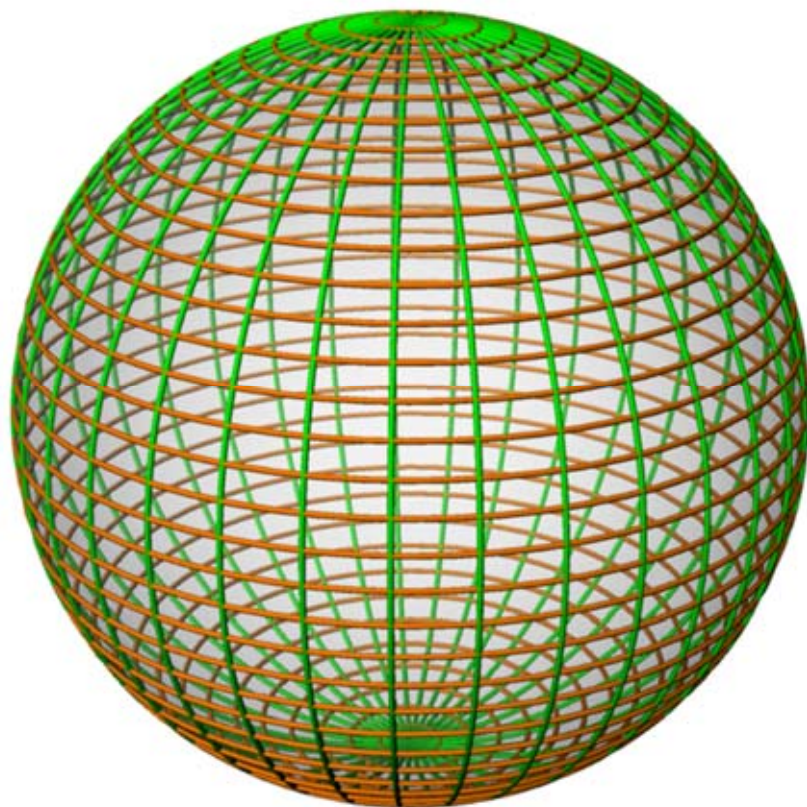
$$y = r \cos u \sin d_i$$

$$z = r \sin u$$

$$-\frac{\pi}{2} \leq u \leq \frac{\pi}{2}$$

POVRŠI U PROSTORU

Linije na površi



$$x = r \cos u \cos v$$

$$y = r \cos u \sin v$$

$$z = r \sin u$$

$$-\frac{\pi}{2} \leq u \leq \frac{\pi}{2}$$

$$0 \leq v \leq 2\pi$$

Izoparametarske linije za

$$u = c_i$$

$$v = d_i$$

$$x = r \cos c_i \cos v$$

$$x = r \cos u \cos d_i$$

$$y = r \cos c_i \sin v$$

$$y = r \cos u \sin d_i$$

$$z = r \sin c_i$$

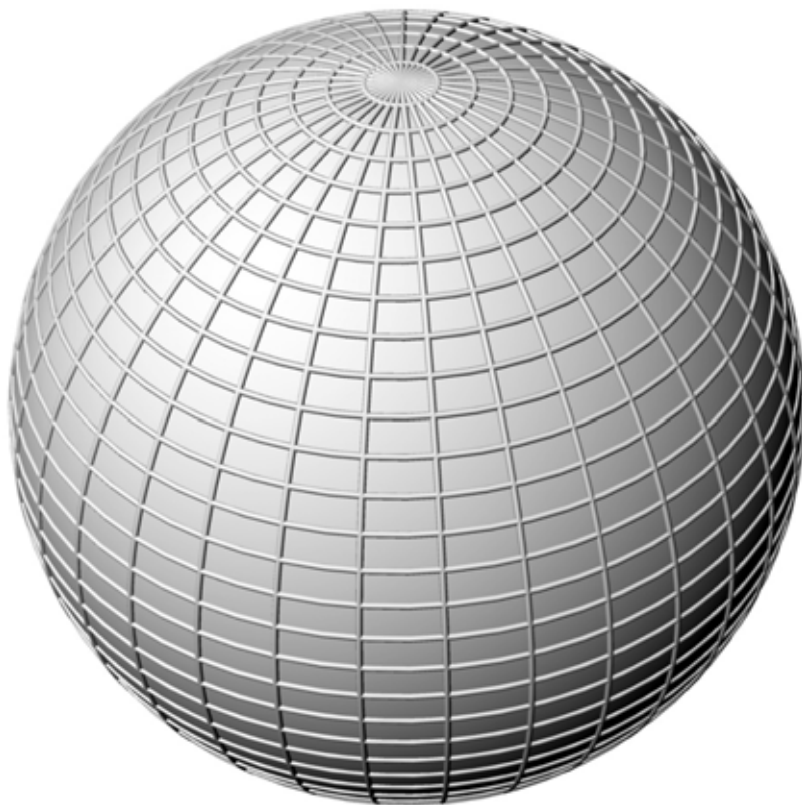
$$z = r \sin u$$

$$0 \leq v \leq 2\pi$$

$$-\frac{\pi}{2} \leq u \leq \frac{\pi}{2}$$

POVRŠI U PROSTORU

Linije na površi



$$x = r \cos u \cos v$$

$$y = r \cos u \sin v$$

$$z = r \sin u$$

$$-\frac{\pi}{2} \leq u \leq \frac{\pi}{2}$$

$$0 \leq v \leq 2\pi$$

Izoparametarske linije za

$$u = c_i$$

$$v = d_i$$

$$x = r \cos c_i \cos v$$

$$y = r \cos c_i \sin v$$

$$z = r \sin c_i$$

$$0 \leq v \leq 2\pi$$

$$x = r \cos u \cos d_i$$

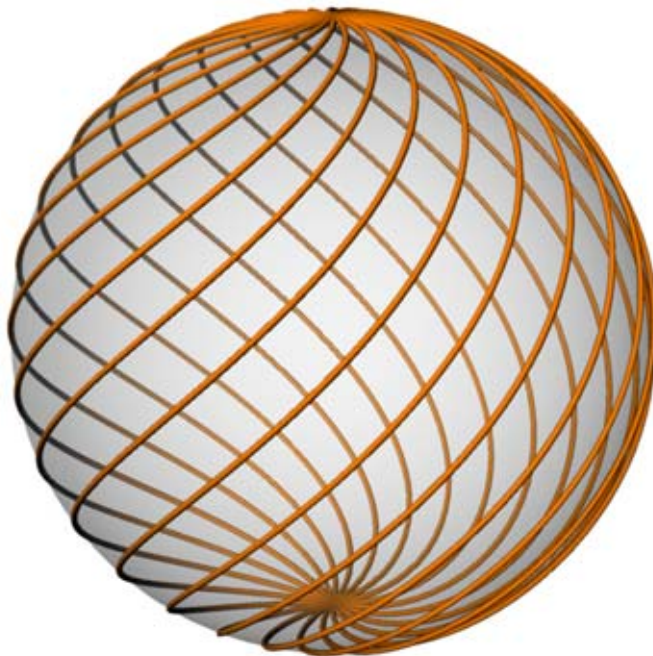
$$y = r \cos u \sin d_i$$

$$z = r \sin u$$

$$-\frac{\pi}{2} \leq u \leq \frac{\pi}{2}$$

POVRŠI U PROSTORU

Linije na površi



$$v = u + c_i \quad x = r \cos u \cos(u + c_i)$$

$$y = r \cos u \sin(u + c_i)$$

$$z = r \sin u$$

$$-\frac{\pi}{2} \leq u \leq \frac{\pi}{2}$$

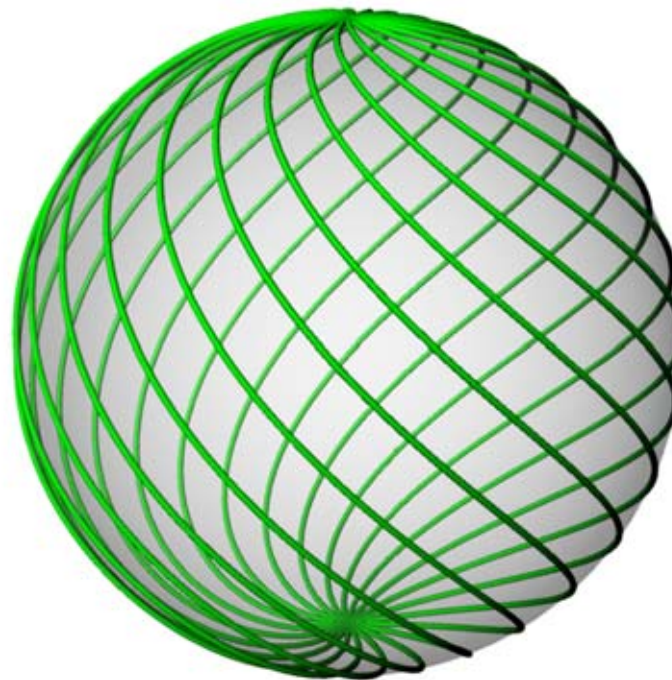
$$x = r \cos u \cos v$$

$$y = r \cos u \sin v$$

$$z = r \sin u$$

$$-\frac{\pi}{2} \leq u \leq \frac{\pi}{2}$$

$$0 \leq v \leq 2\pi$$



$$x = r \cos u \cos(-u + c_i)$$

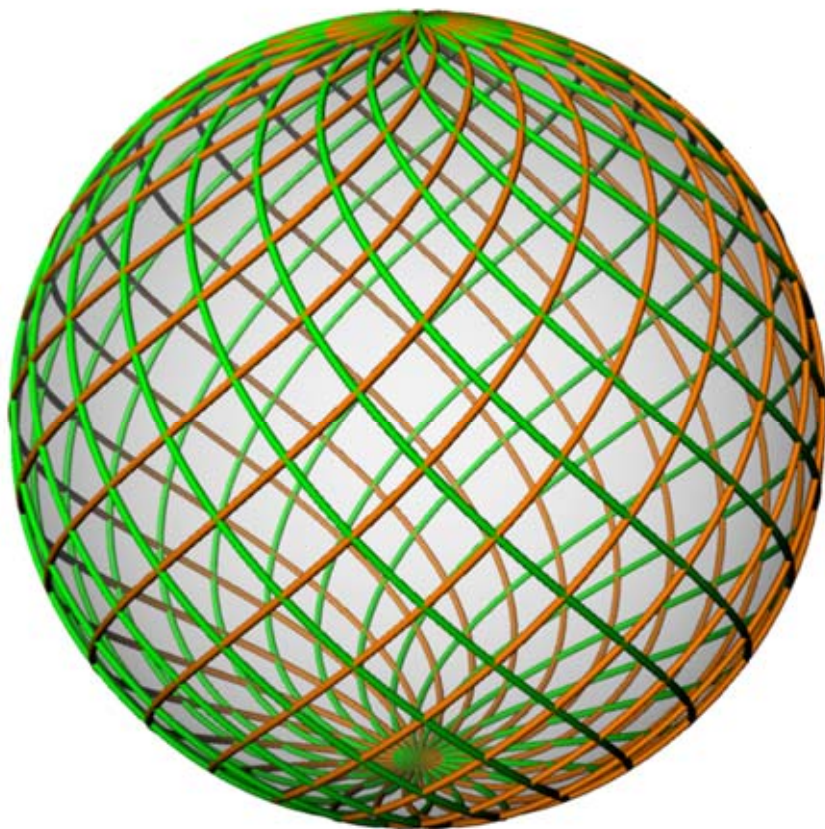
$$v = -u + c_i \quad y = r \cos u \sin(-u + c_i)$$

$$z = r \sin u$$

$$-\frac{\pi}{2} \leq u \leq \frac{\pi}{2}$$

POVRŠI U PROSTORU

Linije na površi



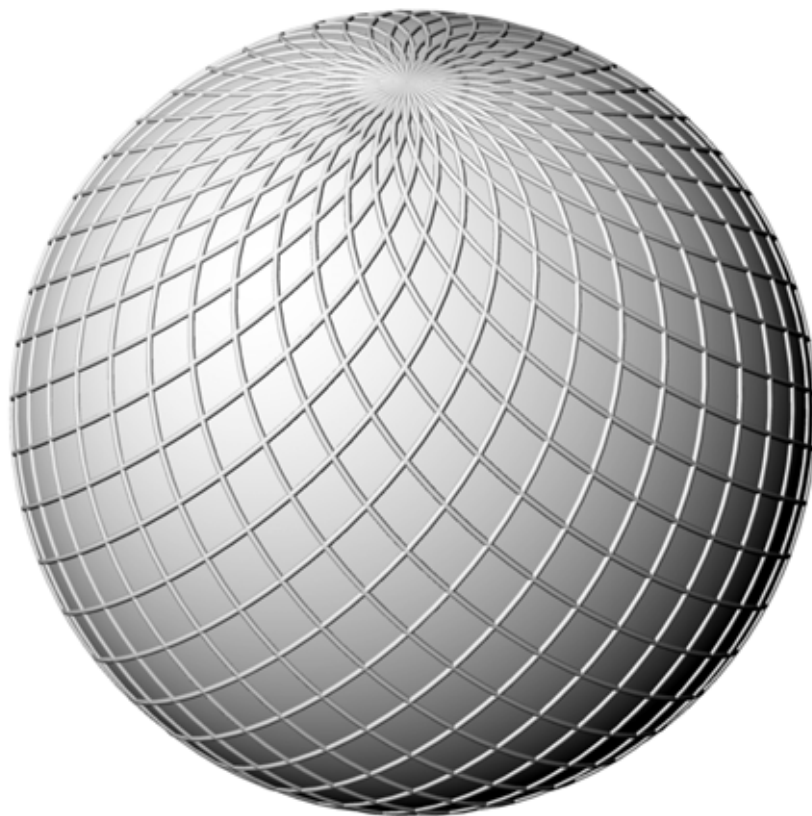
$$\begin{aligned}x &= r \cos u \cos v & -\frac{\pi}{2} \leq u \leq \frac{\pi}{2} \\y &= r \cos u \sin v & 0 \leq v \leq 2\pi \\z &= r \sin u\end{aligned}$$

$$\begin{aligned}v &= u + c_i & x &= r \cos u \cos(u + c_i) \\& & y &= r \cos u \sin(u + c_i) \\& & z &= r \sin u \\& & -\frac{\pi}{2} \leq u \leq \frac{\pi}{2}\end{aligned}$$

$$\begin{aligned}v &= -u + c_i & x &= r \cos u \cos(-u + c_i) \\& & y &= r \cos u \sin(-u + c_i) \\& & z &= r \sin u \\& & -\frac{\pi}{2} \leq u \leq \frac{\pi}{2}\end{aligned}$$

POVRŠI U PROSTORU

Linije na površi



$$\begin{aligned}x &= r \cos u \cos v & -\frac{\pi}{2} \leq u \leq \frac{\pi}{2} \\y &= r \cos u \sin v & 0 \leq v \leq 2\pi \\z &= r \sin u\end{aligned}$$

$$\begin{aligned}v &= u + c_i & x &= r \cos u \cos(u + c_i) \\y &= r \cos u \sin(u + c_i) \\z &= r \sin u \\-\frac{\pi}{2} &\leq u \leq \frac{\pi}{2}\end{aligned}$$

$$\begin{aligned}v &= -u + c_i & x &= r \cos u \cos(-u + c_i) \\y &= r \cos u \sin(-u + c_i) \\z &= r \sin u \\-\frac{\pi}{2} &\leq u \leq \frac{\pi}{2}\end{aligned}$$

POVRŠI U PROSTORU

Linije na površi

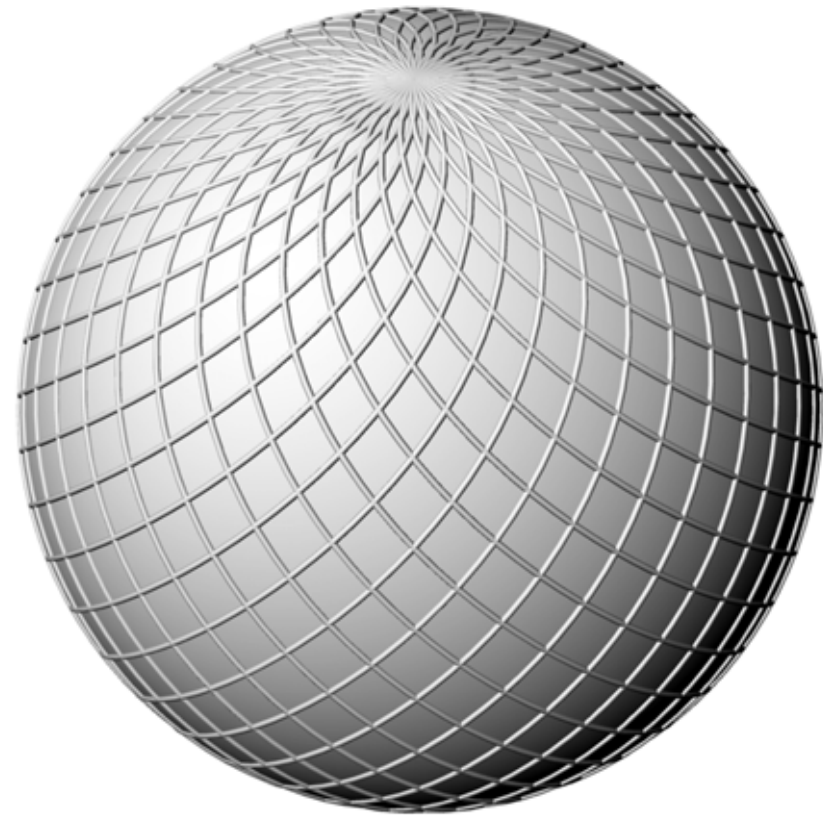
$$x = r \cos u \cos v$$

$$y = r \cos u \sin v$$

$$z = r \sin u$$

$$-\frac{\pi}{2} \leq u \leq \frac{\pi}{2}$$

$$0 \leq v \leq 2\pi$$



POVRŠI U PROSTORU

Linije na površi

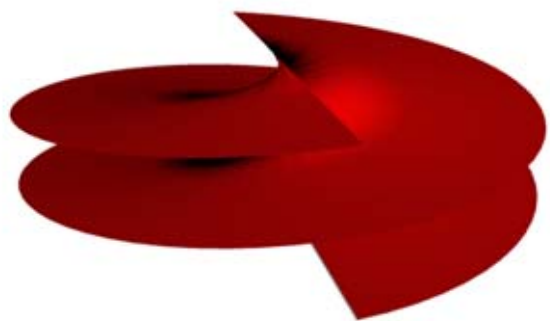
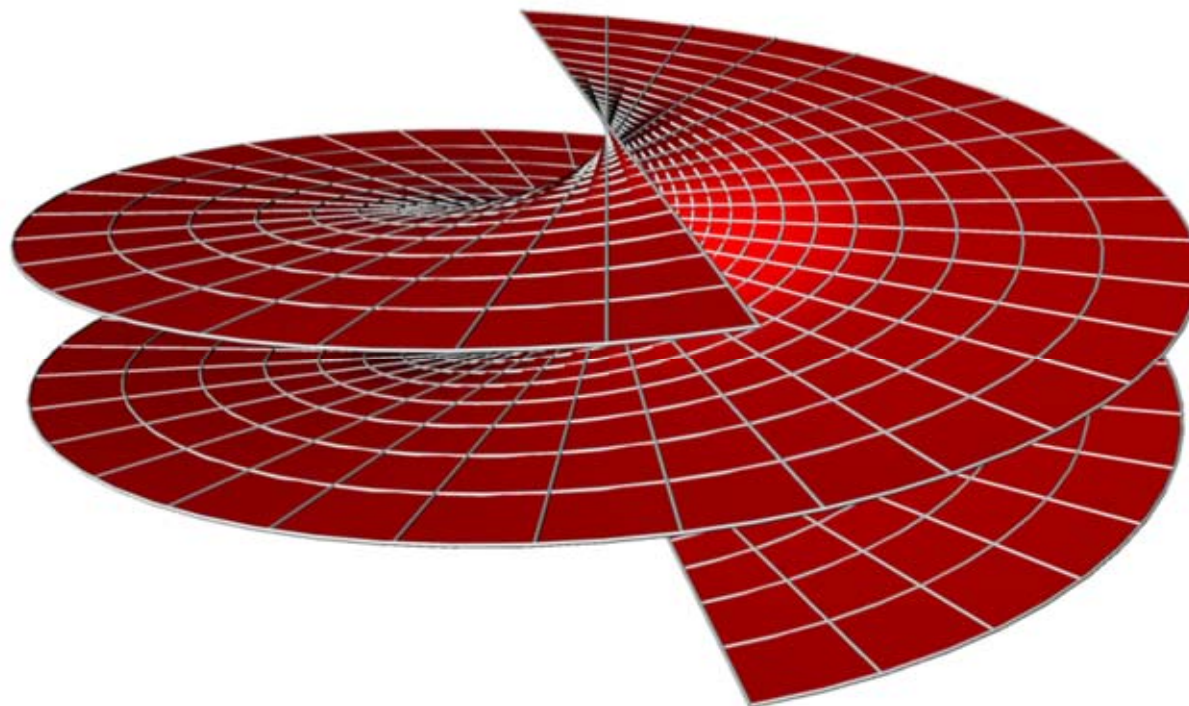
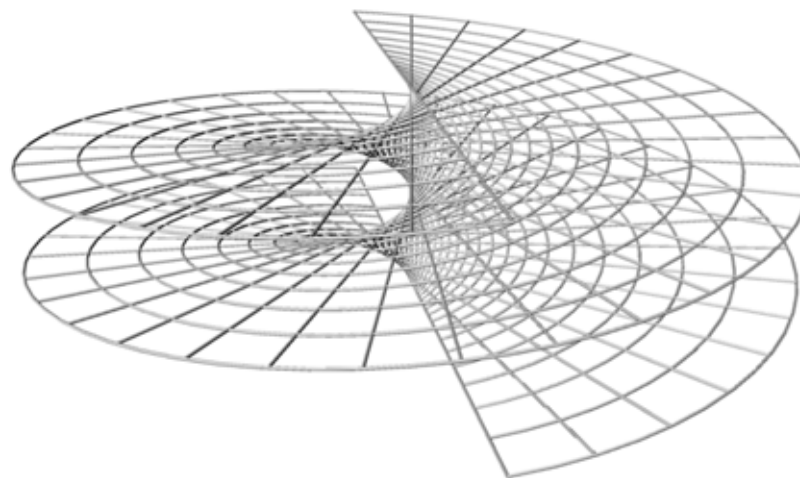
$$X(u, v) = \sinh v \sin u$$

$$Y(u, v) = \sinh v \cos u$$

$$Z(u, v) = u$$

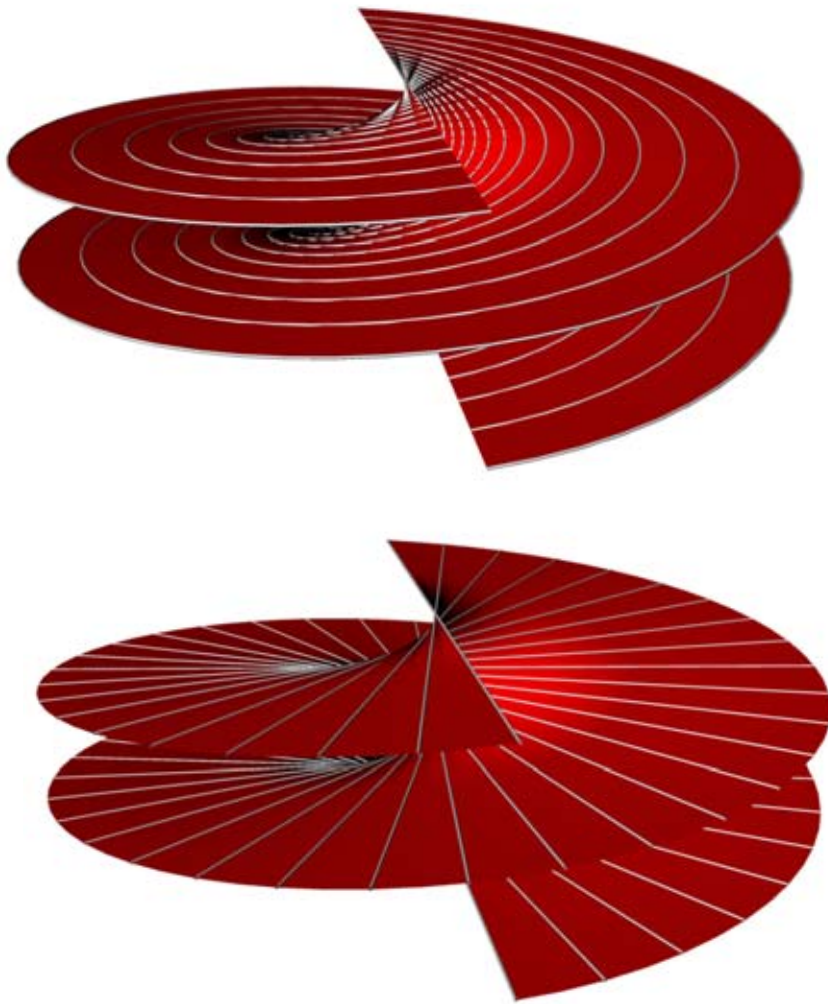
$$-\pi \leq u \leq \pi$$

$$-\pi \leq v \leq \pi$$



POVRŠI U PROSTORU

Linije na površi



Izoparametarske linije

$$v = c_i \quad -\pi \leq c_i \leq \pi$$

$$X(u, v) = \sinh c_i \sin u$$

$$Y(u, v) = \sinh c_i \cos u$$

$$Z(u, v) = u$$

$$-\pi \leq u \leq \pi$$

$$u = d_j \quad -\pi \leq d_j \leq \pi$$

$$X(u, v) = \sinh v \sin d_j$$

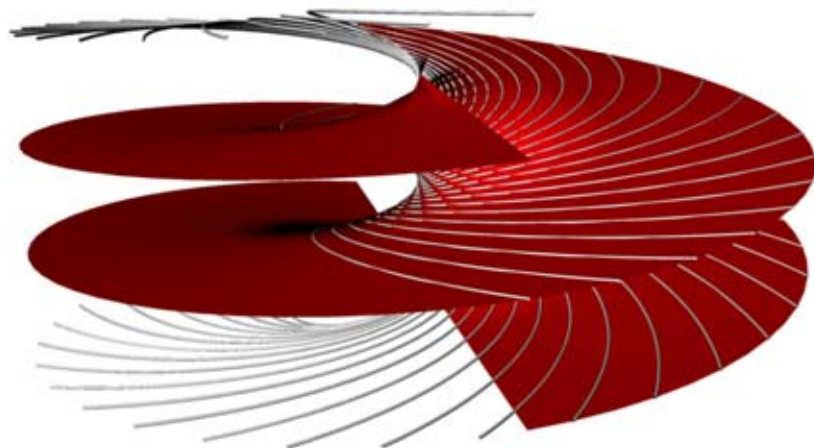
$$Y(u, v) = \sinh v \cos d_j$$

$$Z(u, v) = d_j$$

$$-\pi \leq v \leq \pi$$

POVRŠI U PROSTORU

Linije na površi



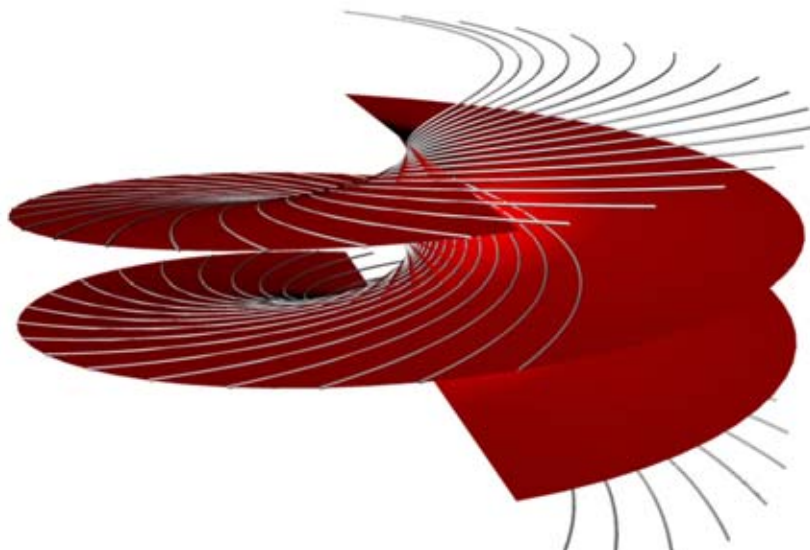
$$v = u + c_i \quad -\pi \leq c_i \leq \pi$$

$$X(u, v) = \sinh(u + c_i) \sin u$$

$$Y(u, v) = \sinh(u + c_i) \cos u$$

$$Z(u, v) = u$$

$$-\pi \leq u \leq \pi$$



$$v = -u + d_j \quad -\pi \leq d_j \leq \pi$$

$$X(u, v) = \sinh(-u + d_j) \sin u$$

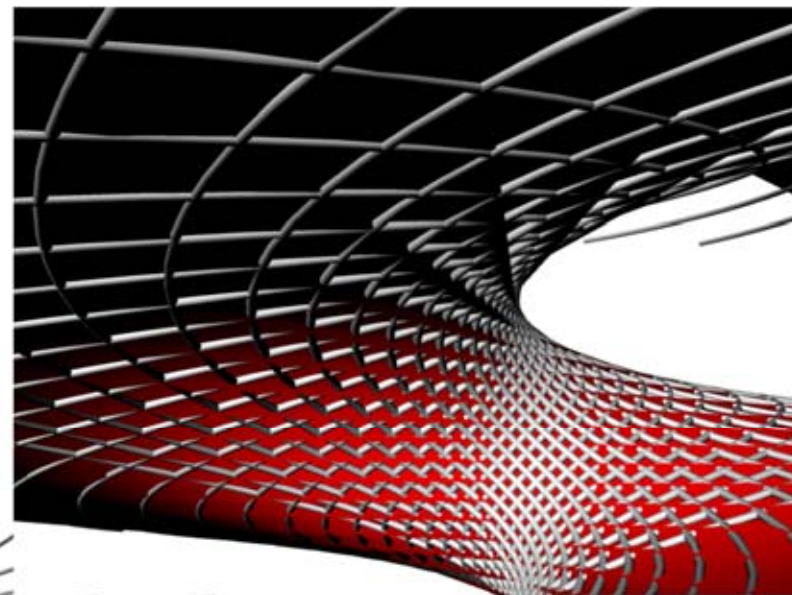
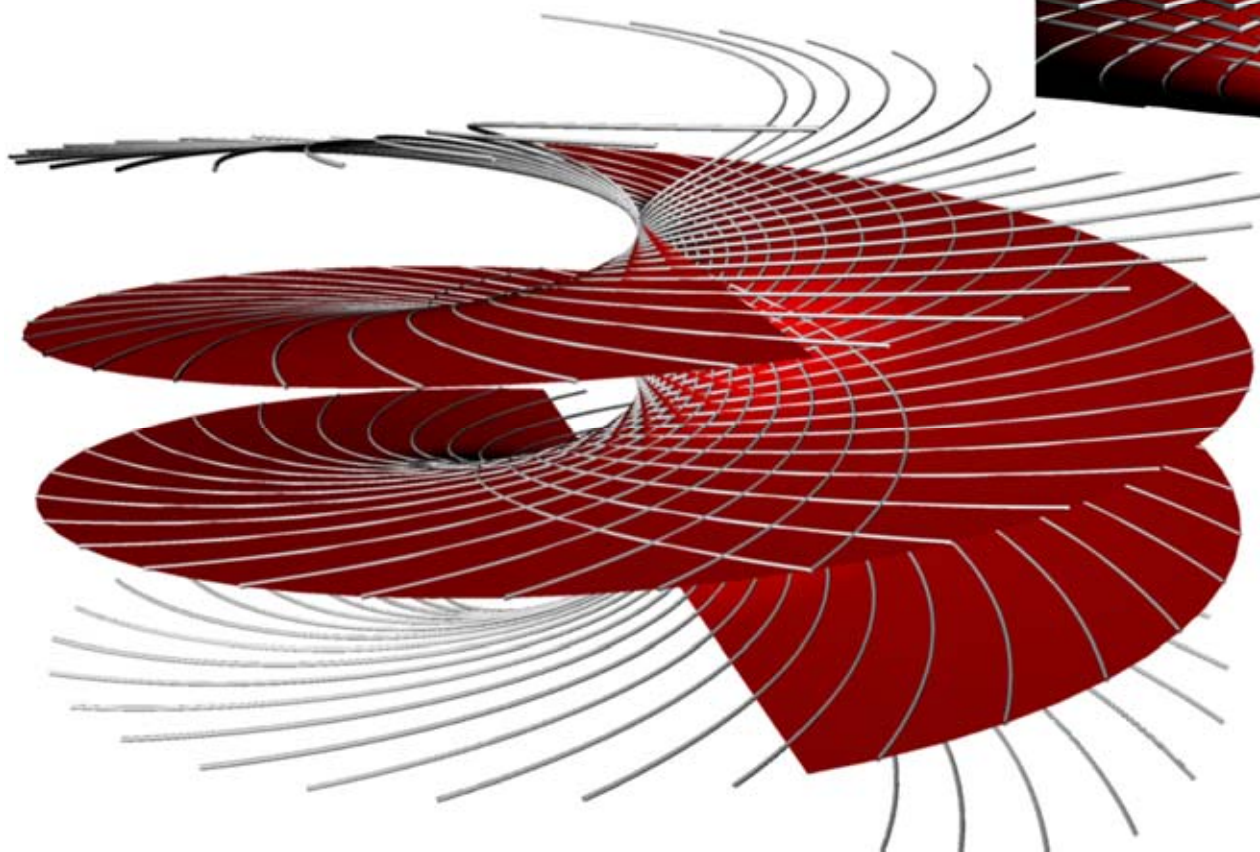
$$Y(u, v) = \sinh(-u + d_j) \cos u$$

$$Z(u, v) = u$$

$$-\pi \leq u \leq \pi$$

POVRŠI U PROSTORU

Linije na površi



POVRŠI U PROSTORU

Izoparametarske linije - reparametrizacija

Helikoid

$$u \rightarrow u$$

$$v \rightarrow \sinh v$$

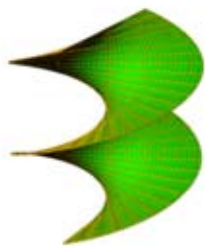
$$X(u, v) = v \sin u$$

$$Y(u, v) = v \cos u$$

$$Z(u, v) = u$$

$$-\pi \leq u \leq \pi$$

$$-\pi \leq v \leq \pi$$



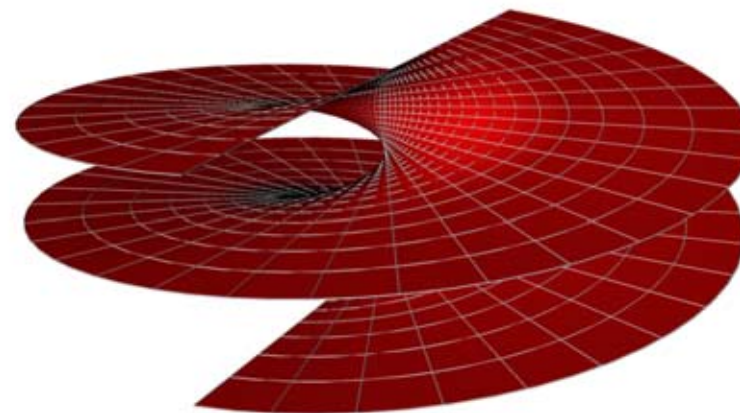
$$X(u, v) = \sinh v \sin u$$

$$Y(u, v) = \sinh v \cos u$$

$$Z(u, v) = u$$

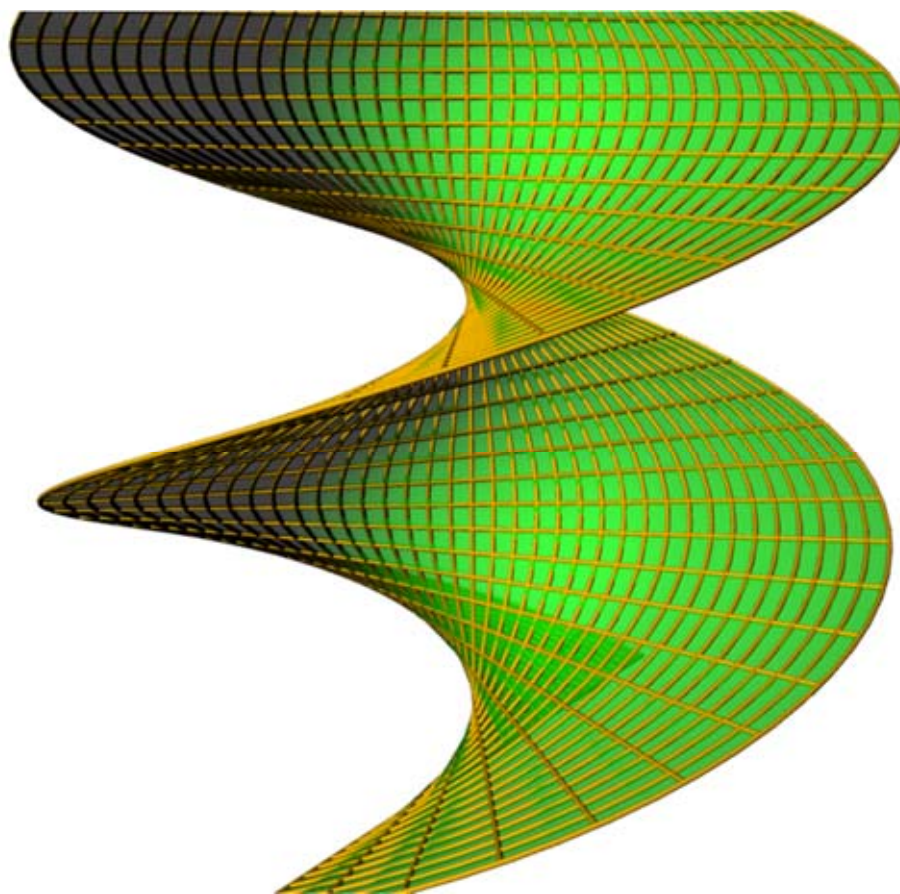
$$-\pi \leq u \leq \pi$$

$$-\pi \leq v \leq \pi$$



POVRŠI U PROSTORU

Izoparametarske linije - reparametrizacija



$$\begin{array}{l|l} -\pi \leq u \leq \pi & X(u, v) = v \sin u \\ -\pi \leq v \leq \pi & X(u, v) = v \cos u \\ & X(u, v) = u \end{array}$$

$$v = c_i \quad -\pi \leq c_i \leq \pi$$

$$X(u, v) = c_i \sin u$$

$$Y(u, v) = c_i \cos u$$

$$Z(u, v) = u$$

$$-\pi \leq u \leq \pi$$

$$u = d_j \quad -\pi \leq d_j \leq \pi$$

$$X(u, v) = v \sin d_j$$

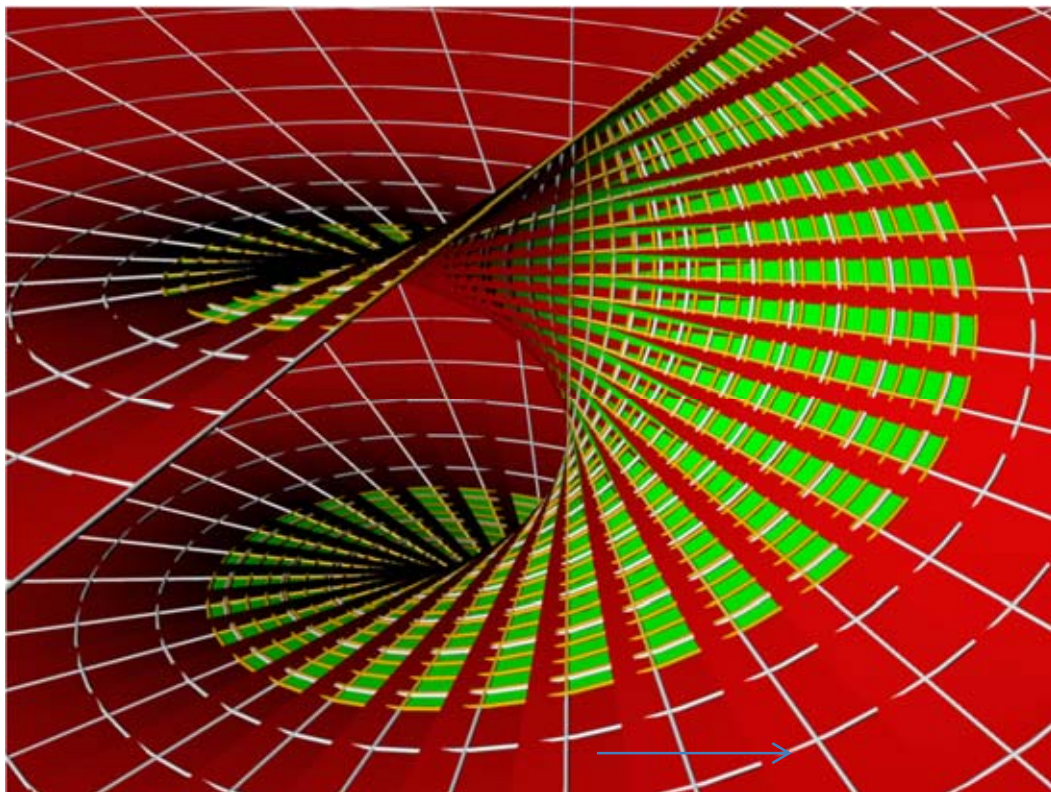
$$Y(u, v) = v \cos d_j$$

$$Z(u, v) = d_j$$

$$-\pi \leq v \leq \pi$$

POVRŠI U PROSTORU

Izoparametarske linije - reparametrizacija



$$u \rightarrow u$$

$$v \rightarrow \sinh v$$

$$X(u, v) = v \sin u$$

$$Y(u, v) = v \cos u$$

$$Z(u, v) = u$$

$$-\pi \leq u \leq \pi$$

$$-\pi \leq v \leq \pi$$

$$X(u, v) = \sinh v \sin u$$

$$Y(u, v) = \sinh v \cos u$$

$$Z(u, v) = u$$

$$-\pi \leq u \leq \pi$$

$$-\pi \leq v \leq \pi$$

POVRŠI U PROSTORU

Izoparametarske linije - reparametrizacija

Helikoid

$$u \rightarrow u$$

$$v \rightarrow u + v$$

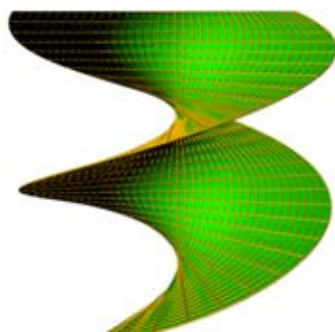
$$X(u, v) = v \sin u$$

$$Y(u, v) = v \cos u$$

$$Z(u, v) = u$$

$$-\pi \leq u \leq \pi$$

$$-\pi \leq v \leq \pi$$



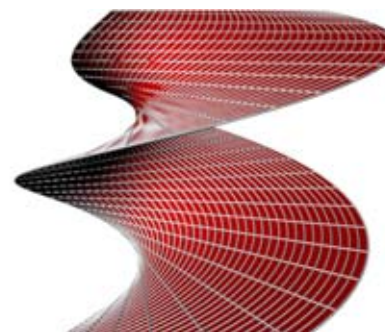
$$X(u, v) = (u + v) \sin u$$

$$Y(u, v) = (u + v) \cos u$$

$$Z(u, v) = u$$

$$-\pi \leq u \leq \pi$$

$$-\pi \leq v \leq \pi$$

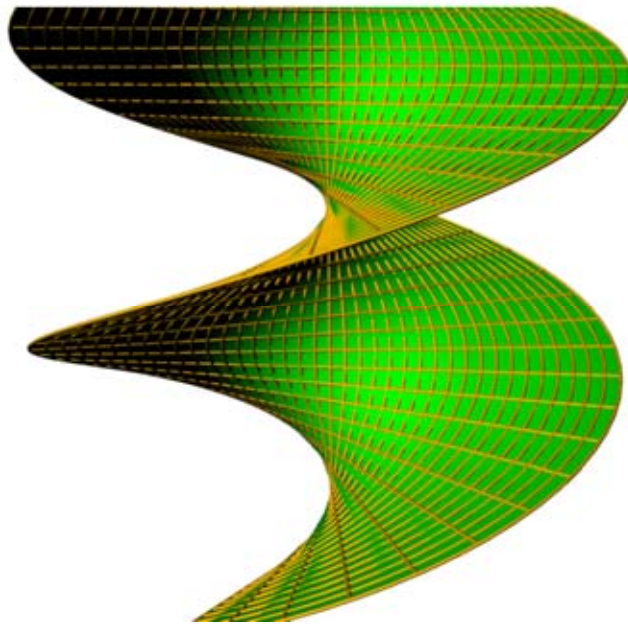


POVRŠI U PROSTORU

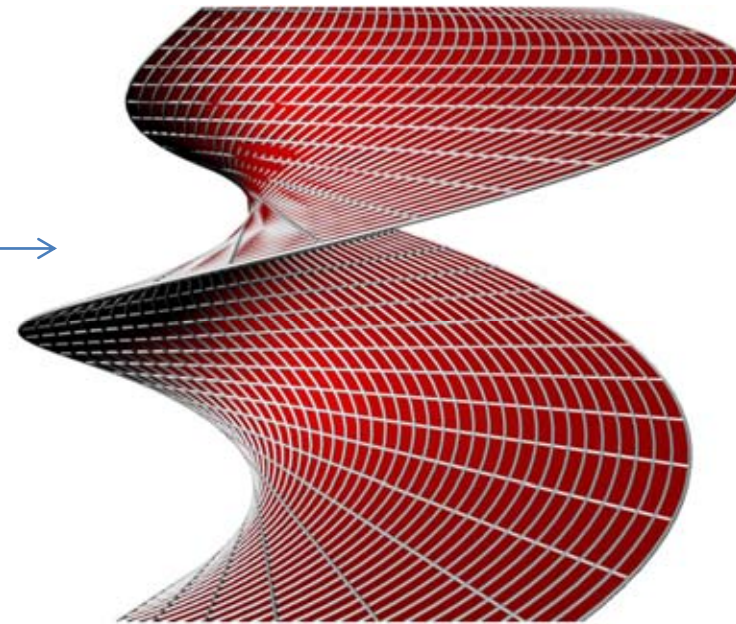
Izoparametarske linije - reparametrizacija

$$u \rightarrow u$$

$$v \rightarrow u + v$$



Helikoid



$$-\pi \leq u \leq \pi$$

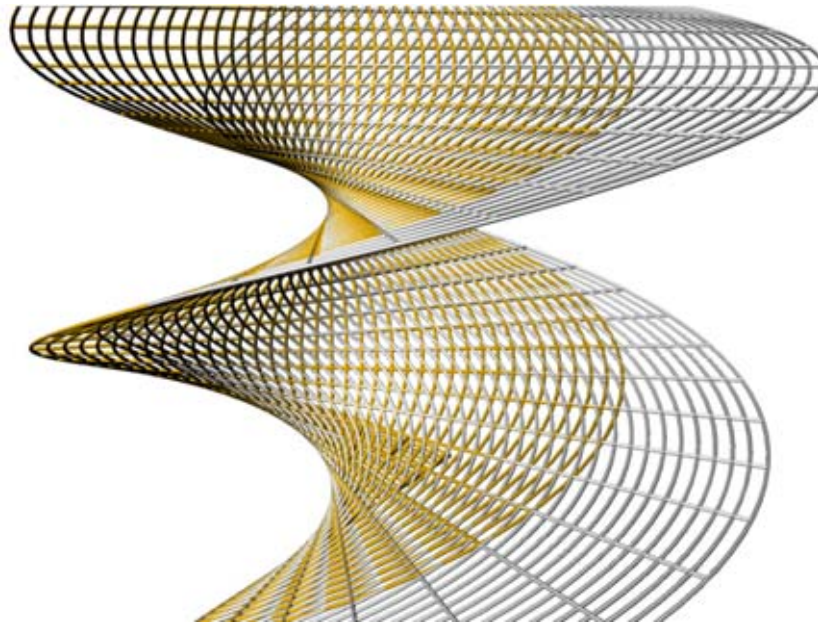
$$-\pi \leq v \leq \pi$$

POVRŠI U PROSTORU

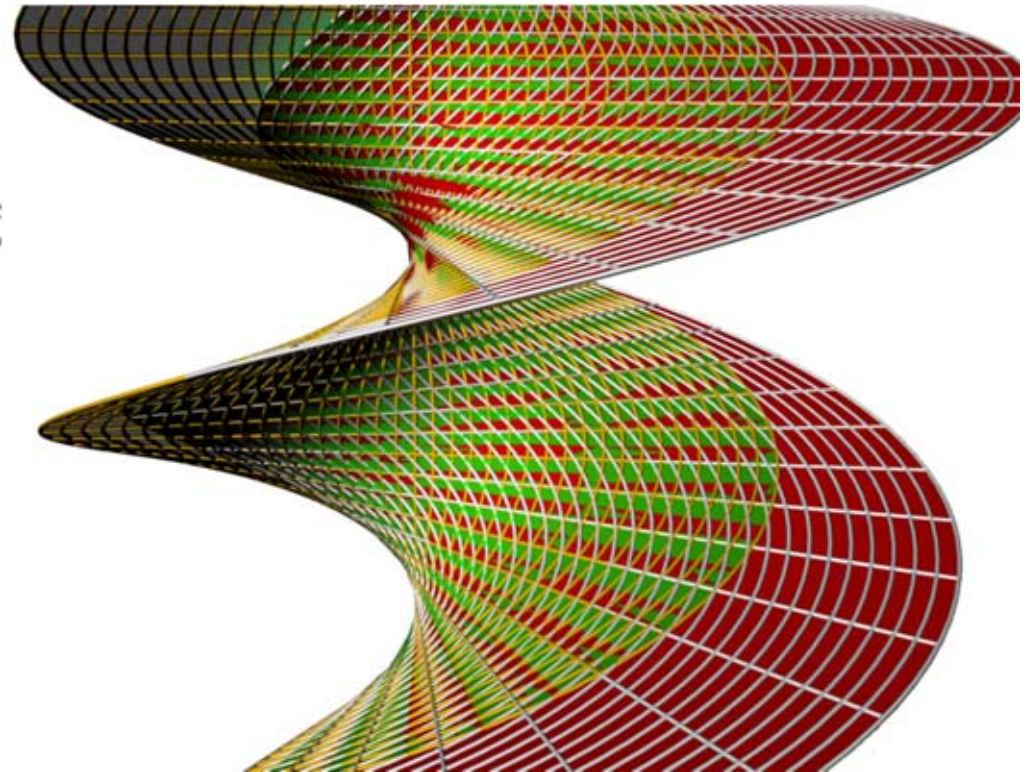
Izoparametarske linije - reparametrizacija

$$u \rightarrow u$$

$$v \rightarrow u + v$$



Helikoid



$$-\pi \leq u \leq \pi$$

$$-\pi \leq v \leq \pi$$

POVRŠI U PROSTORU

Izoparametarske linije - reparametrizacija

$$u \rightarrow u - v$$

$$v \rightarrow u + v$$

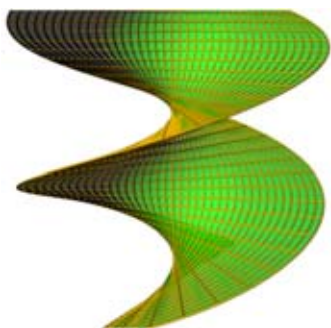
$$X(u, v) = v \sin u$$

$$Y(u, v) = v \cos u$$

$$Z(u, v) = u$$

$$-\pi \leq u \leq \pi$$

$$-\pi \leq v \leq \pi$$



Helikoid

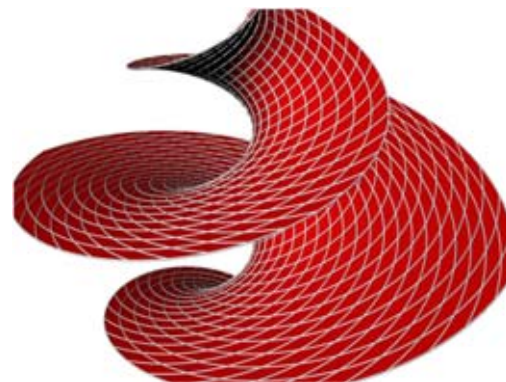
$$X(u, v) = (u + v) \sin(u - v)$$

$$Y(u, v) = (u + v) \cos(u - v)$$

$$Z(u, v) = u - v$$

$$-\pi \leq u \leq \pi$$

$$-\pi \leq v \leq \pi$$

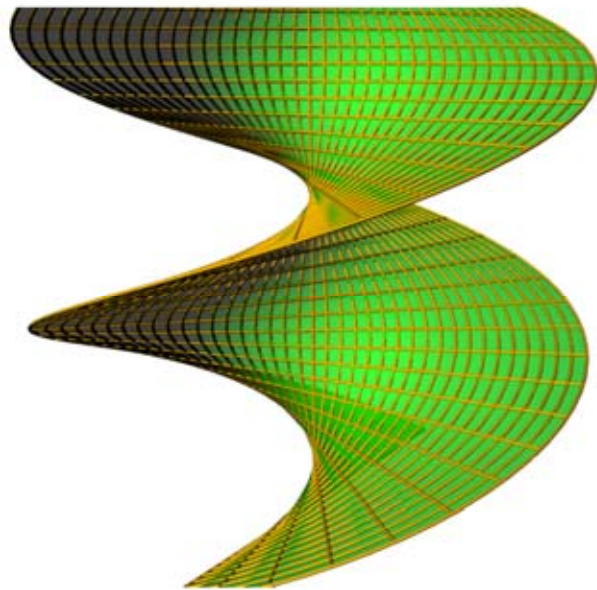


POVRŠI U PROSTORU

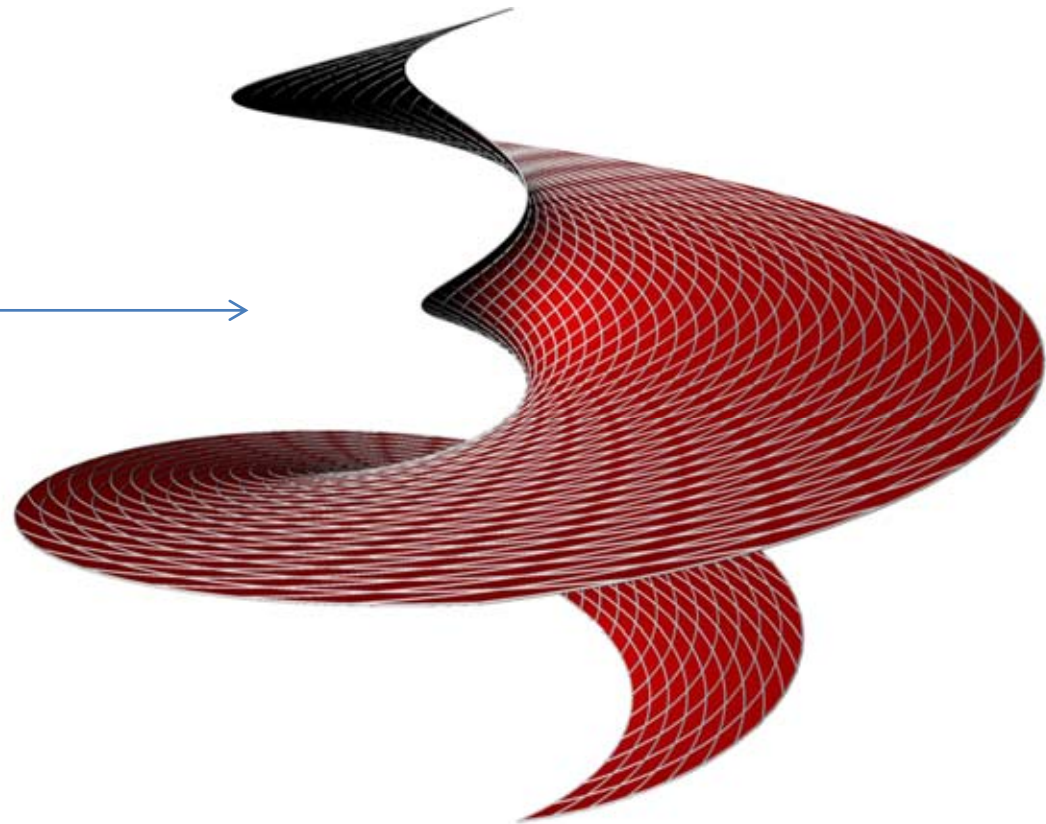
Izoparametarske linije - reparametrizacija

$$u \rightarrow u - v$$

$$v \rightarrow u + v$$



Helikoid



POVRŠI U PROSTORU

Izoparametarske linije - reparametrizacija

Helikoid

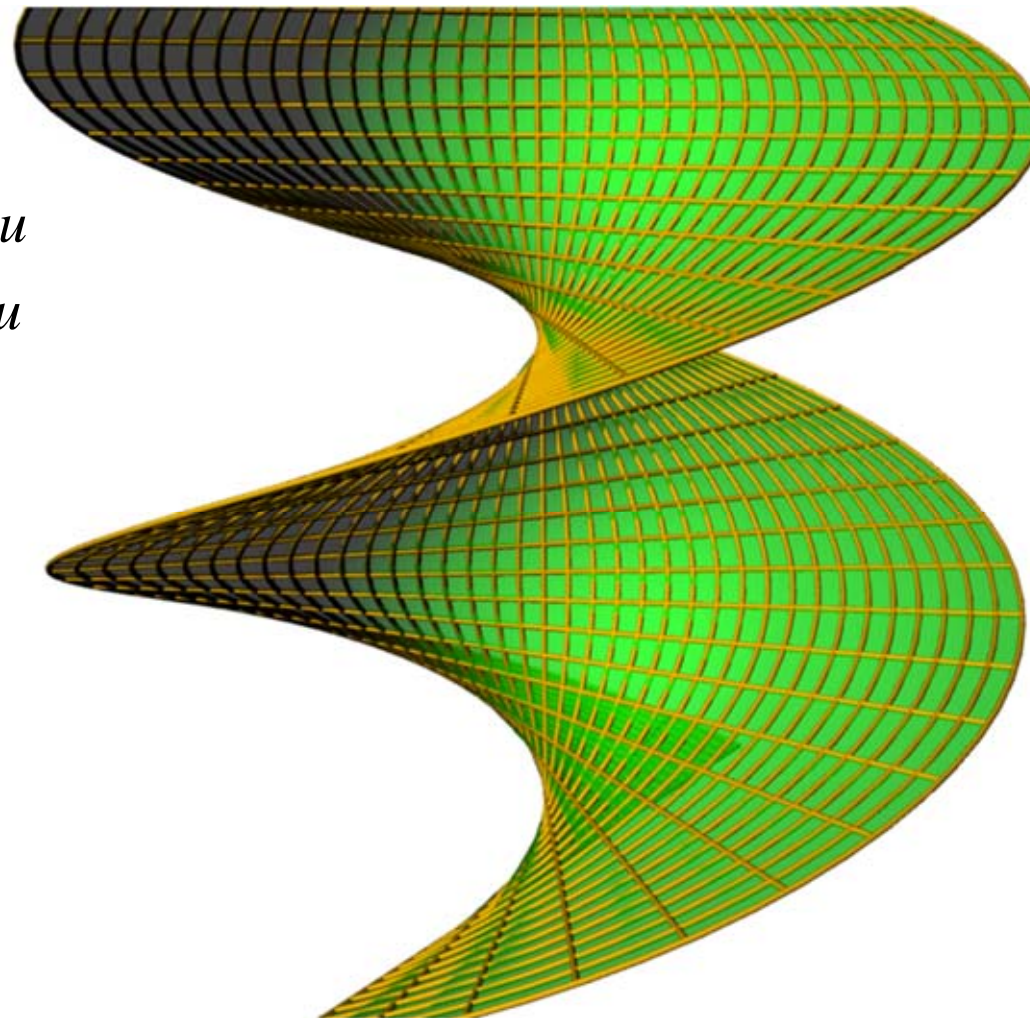
$$X(u, v) = v \sin u$$

$$Y(u, v) = v \cos u$$

$$Z(u, v) = u$$

$$-\pi \leq u \leq \pi$$

$$-\pi \leq v \leq \pi$$



POVRŠI U PROSTORU

Izoparametarske linije - reparametrizacija

Helikoid

$$u \rightarrow u - v$$

$$v \rightarrow u + v$$

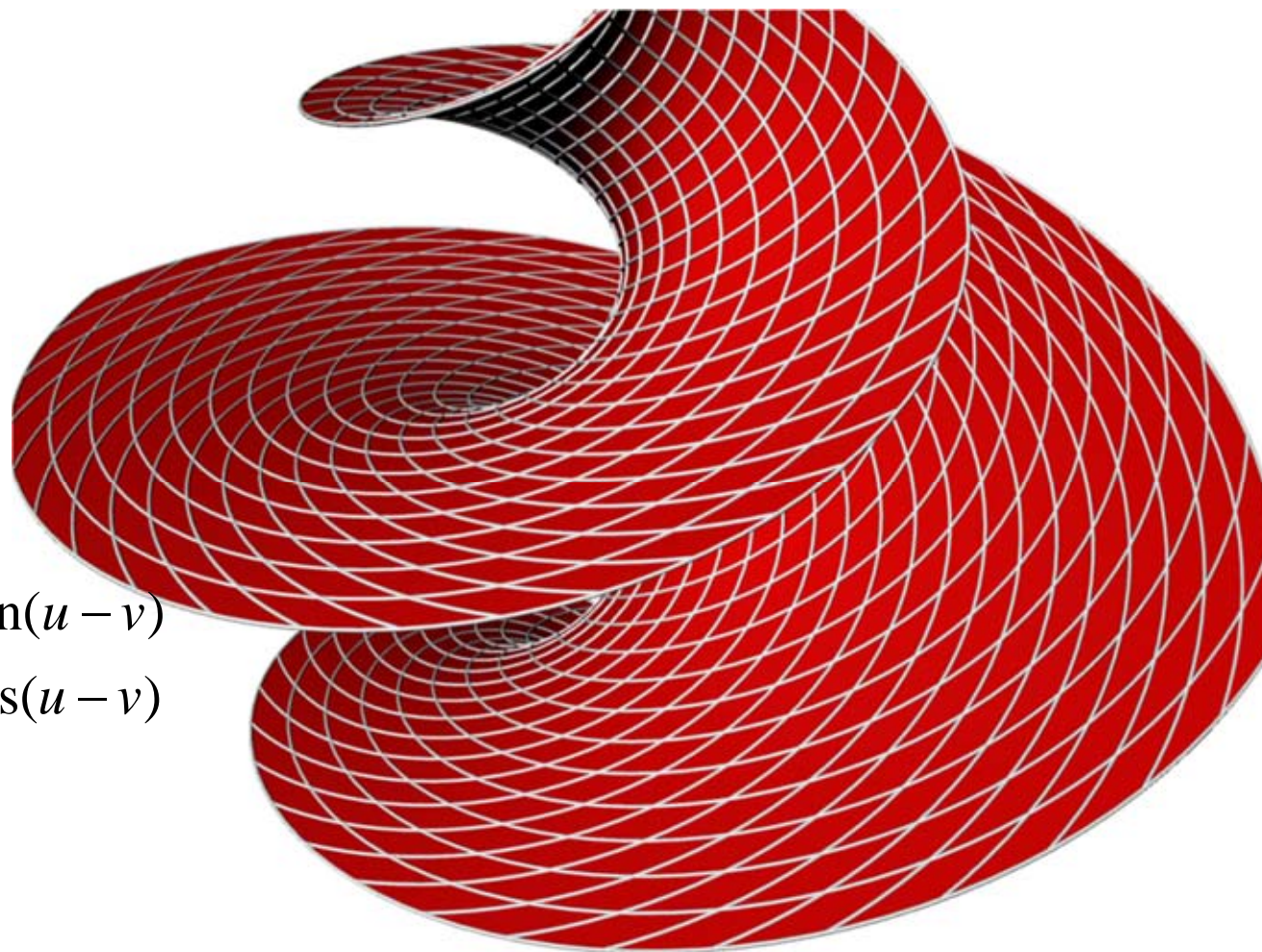
$$X(u, v) = (u + v) \sin(u - v)$$

$$Y(u, v) = (u + v) \cos(u - v)$$

$$Z(u, v) = u - v$$

$$-\pi \leq u \leq \pi$$

$$-\pi \leq v \leq \pi$$



POVRŠI U PROSTORU

Izoparametarske linije - reparametrizacija

Helikoid

$$u \rightarrow u - v$$

$$v \rightarrow u + v$$

Preklopljen sa:

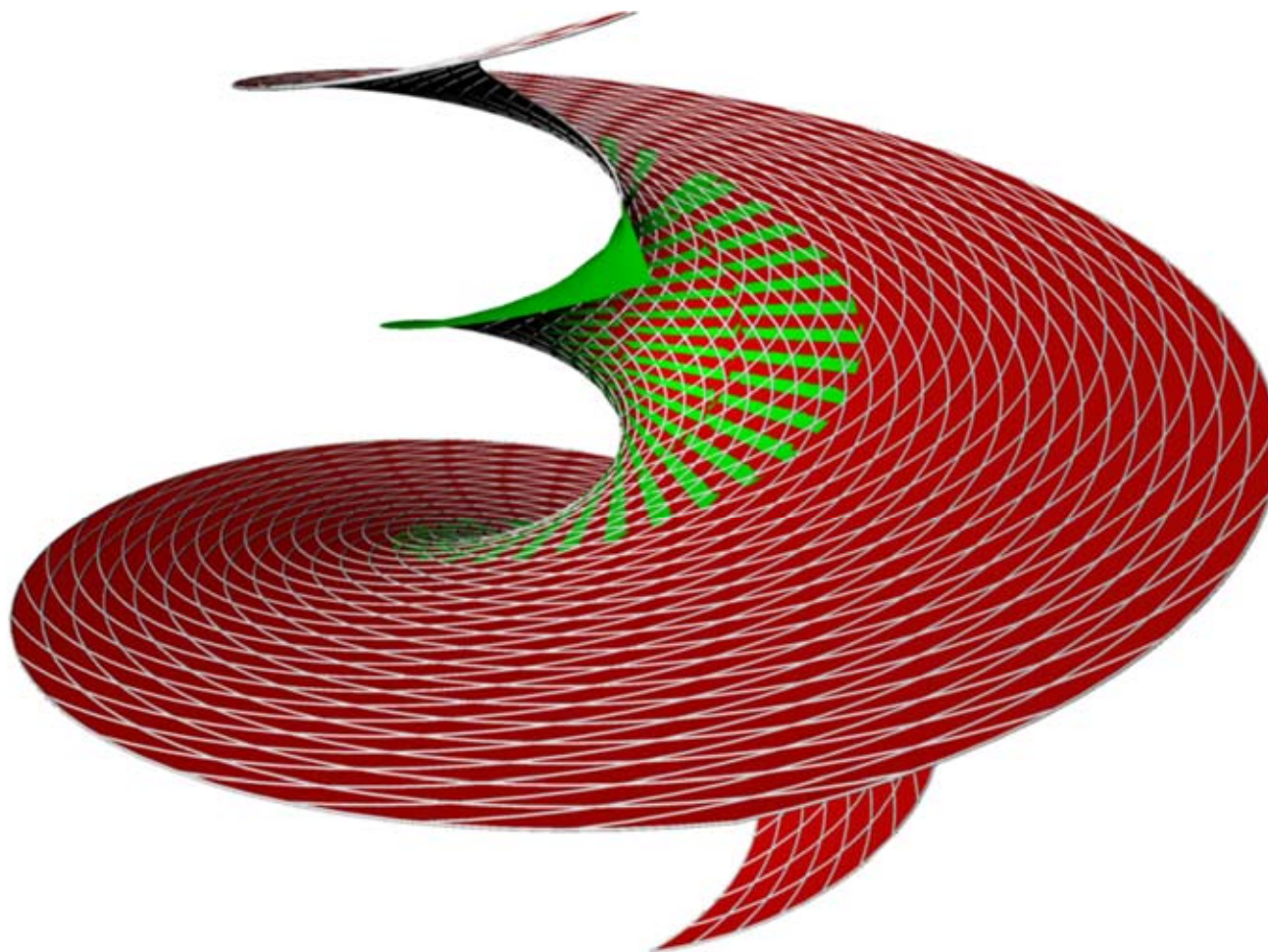
$$X(u, v) = v \sin u$$

$$Y(u, v) = v \cos u$$

$$Z(u, v) = u$$

$$-\pi \leq u \leq \pi$$

$$-\pi \leq v \leq \pi$$



POVRŠI U PROSTORU

Izoparametarske linije - reparametrizacija

Helikoid

$$u \rightarrow u - v$$

$$v \rightarrow u + v$$

Preklopljen sa:

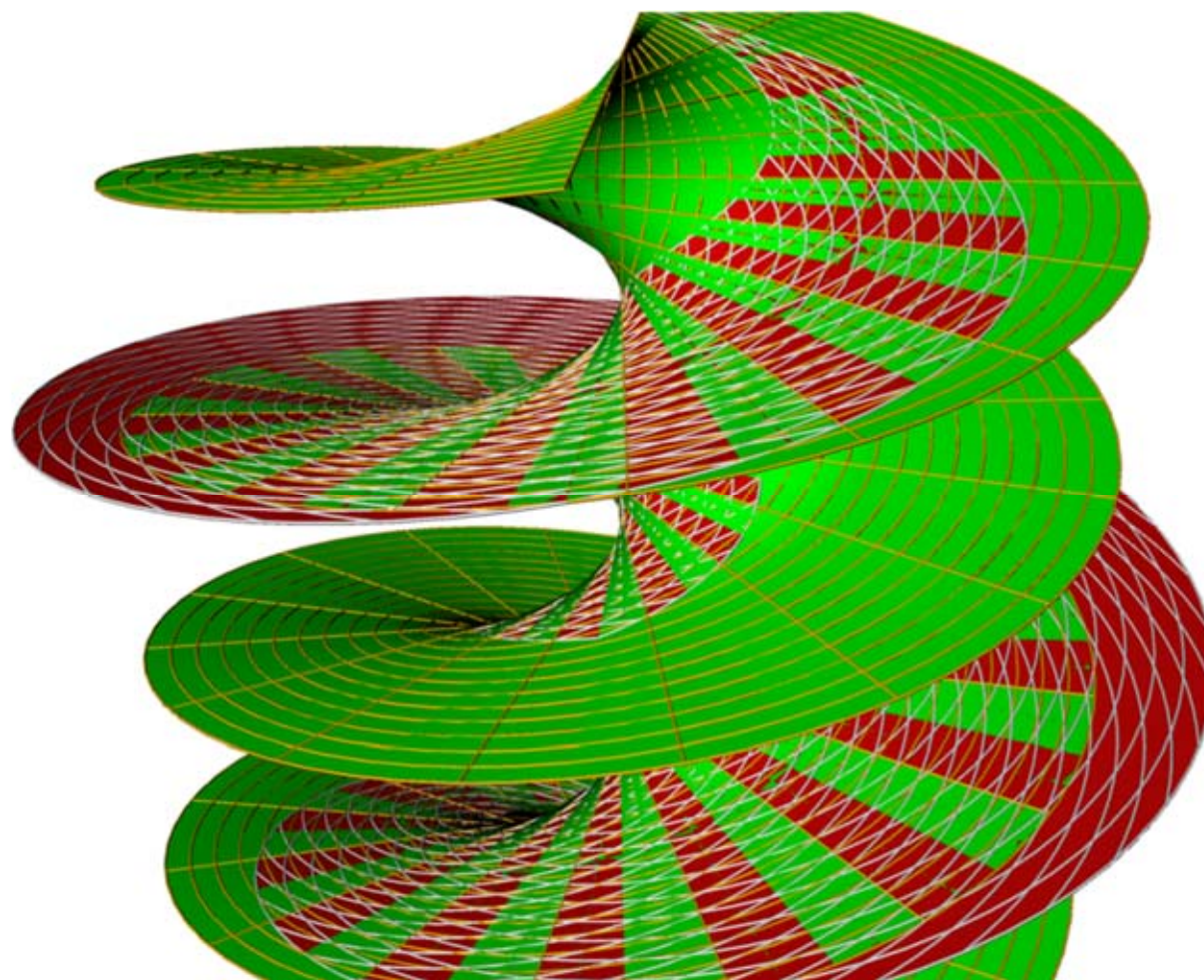
$$X(u, v) = v \sin u$$

$$Y(u, v) = v \cos u$$

$$Z(u, v) = u$$

$$-2\pi \leq u \leq 2\pi$$

$$-2\pi \leq v \leq 2\pi$$



POVRŠI U PROSTORU

Izoparametarske linije - reparametrizacija

Helikoid

$$u \rightarrow u - v$$

$$v \rightarrow u + v$$

Preklopljen sa:

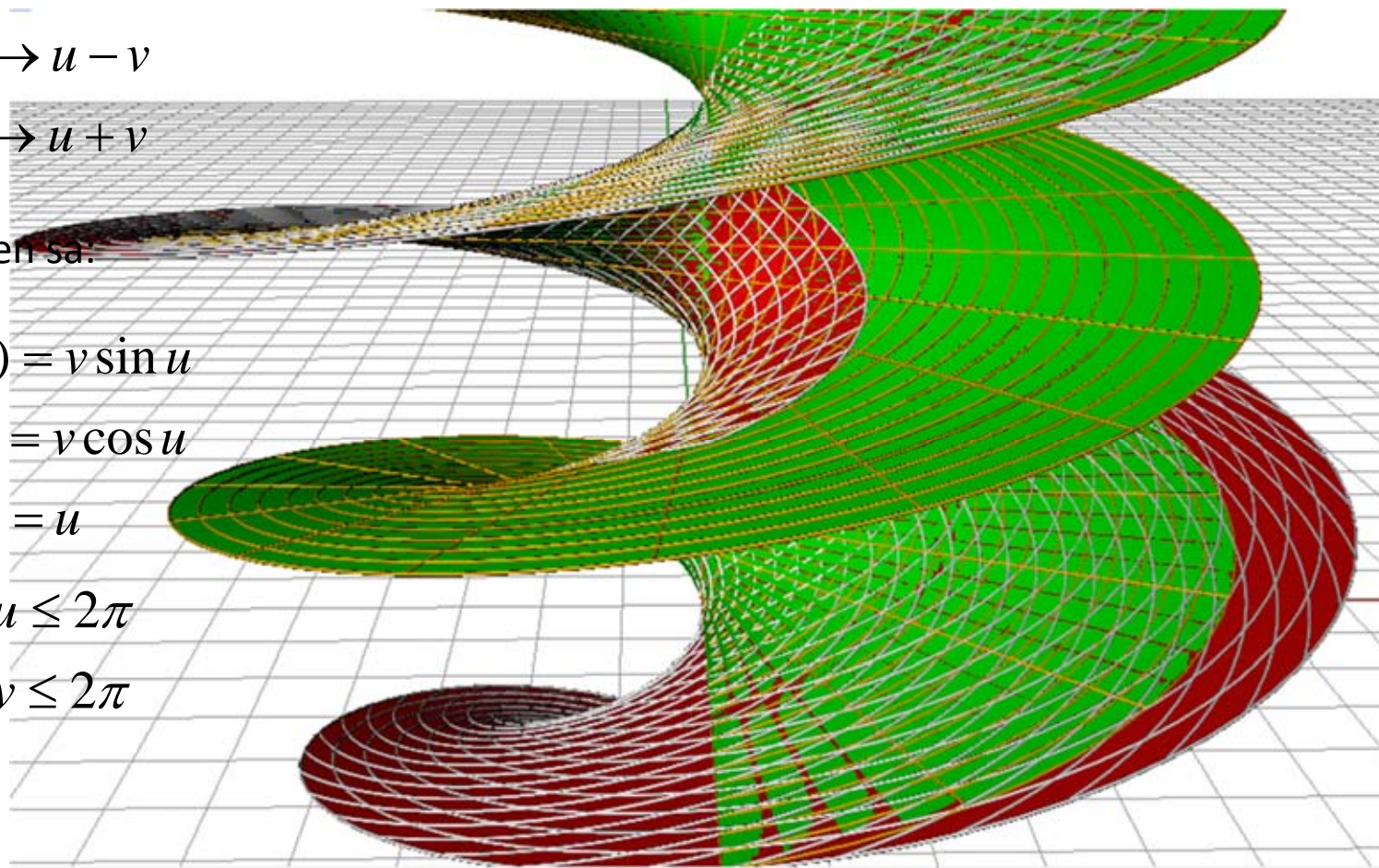
$$X(u, v) = v \sin u$$

$$Y(u, v) = v \cos u$$

$$Z(u, v) = u$$

$$-2\pi \leq u \leq 2\pi$$

$$-2\pi \leq v \leq 2\pi$$



POVRŠI U PROSTORU

Izoparametarske linije - reparametrizacija

Helikoid

$$u \rightarrow u - v$$

$$v \rightarrow u + v$$

Preklopljen sa:

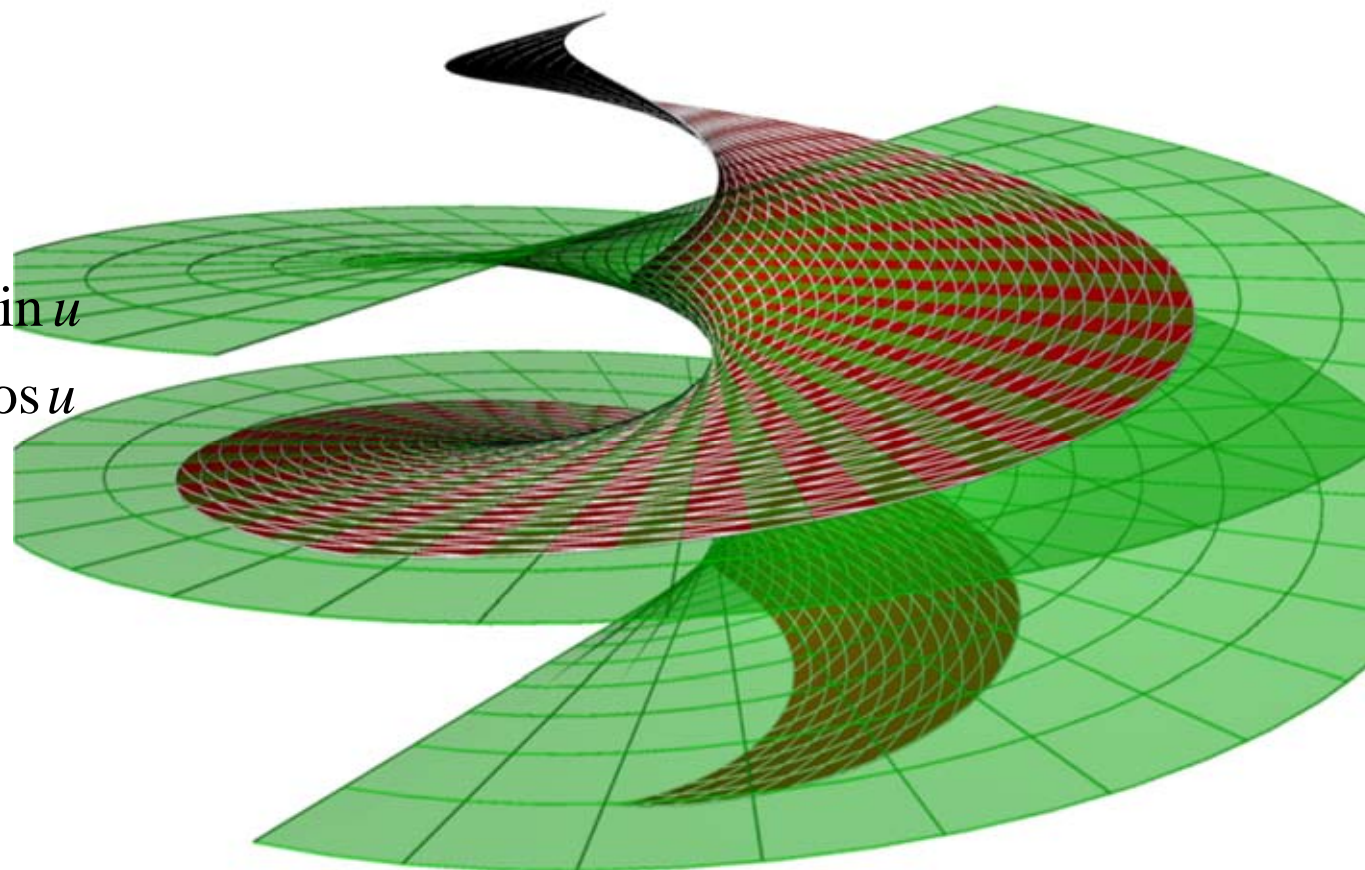
$$X(u, v) = \sinh v \sin u$$

$$Y(u, v) = \sinh v \cos u$$

$$Z(u, v) = u$$

$$-2\pi \leq u \leq 2\pi$$

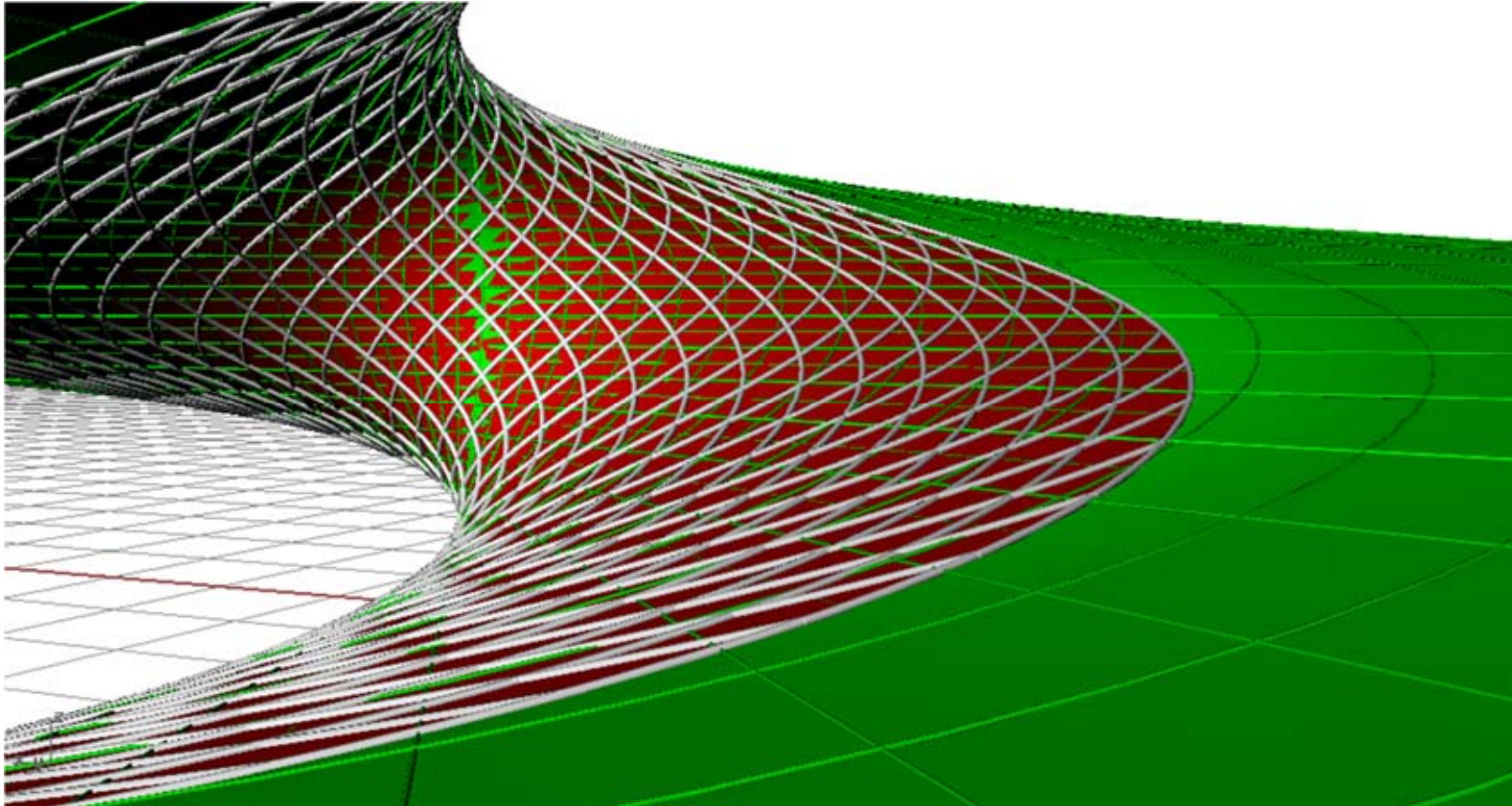
$$-2\pi \leq v \leq 2\pi$$



POVRŠI U PROSTORU

Izoparametarske linije - reparametrizacija

Helikoid



POVRŠI U PROSTORU

Izoparametarske linije - reparametrizacija

$$u \rightarrow u$$

$$v \rightarrow u + v$$

$$x = r \cos u \cos v$$

$$y = r \cos u \sin v$$

$$z = r \sin u$$

$$-\frac{\pi}{2} \leq u \leq \frac{\pi}{2}$$

$$0 \leq v \leq 2\pi$$

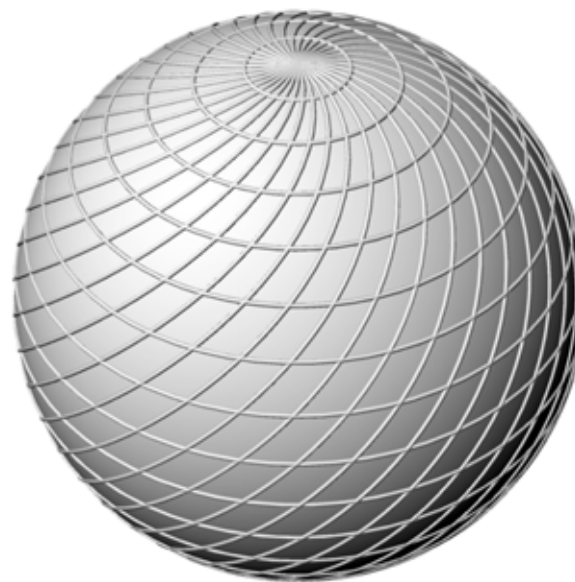
$$x = r \cos u \cos(u + v)$$

$$y = r \cos u \sin(u + v)$$

$$z = r \sin u$$

$$-\frac{\pi}{2} \leq u \leq \frac{\pi}{2}$$

$$0 \leq v \leq 2\pi$$



POVRŠI U PROSTORU

Izoparametarske linije - reparametrizacija

$$u \rightarrow u$$

$$v \rightarrow u + v$$

$$x = r \cos u \cos(u + v)$$

$$y = r \cos u \sin(u + v)$$

$$z = r \sin u$$

$$-\frac{\pi}{2} \leq u \leq \frac{\pi}{2}$$

$$0 \leq v \leq 2\pi$$

$$x = r \cos c_i \cos(c_i + v)$$

$$y = r \cos c_i \sin(c_i + v)$$

$$z = r \sin c_i$$

$$0 \leq v \leq 2\pi$$



$$x = r \cos u \cos(u + d_j)$$

$$y = r \cos u \sin(u + d_j)$$

$$z = r \sin u$$

$$-\frac{\pi}{2} \leq u \leq \frac{\pi}{2}$$

POVRŠI U PROSTORU

$$u \rightarrow u - v$$

Izoparametarske linije - reparametrizacija

$$v \rightarrow u + v$$

$$x = r \cos u \cos v$$

$$x = r \cos(u - v) \cos(u + v)$$

$$y = r \cos u \sin v$$

$$y = r \cos(u - v) \sin(u + v)$$

$$z = r \sin u$$

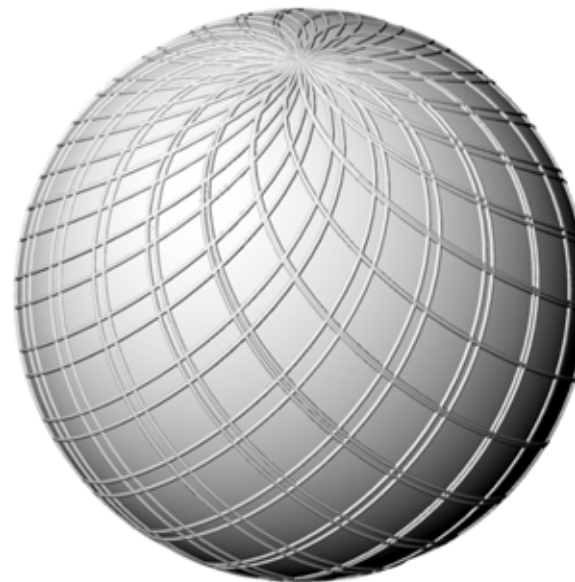
$$z = r \sin(u - v)$$

$$-\frac{\pi}{2} \leq u \leq \frac{\pi}{2}$$

$$-\frac{\pi}{2} \leq u \leq \frac{\pi}{2}$$

$$0 \leq v \leq 2\pi$$

$$0 \leq v \leq 2\pi$$



POVRŠI U PROSTORU

Izoparametarske linije - reparametrizacija

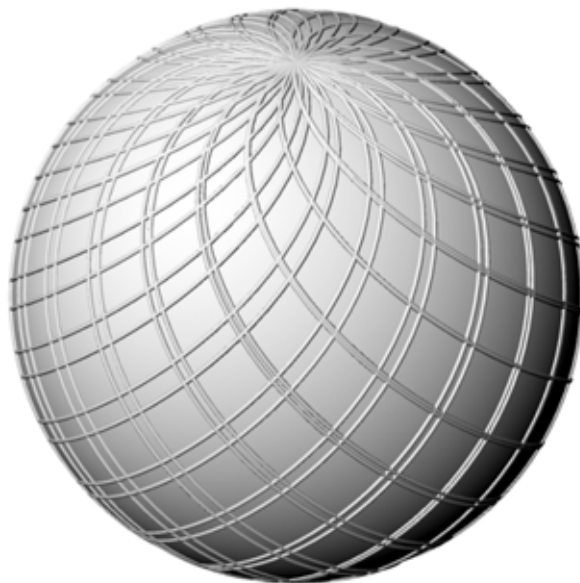
$$x = r \cos(u - v) \cos(u + v)$$

$$y = r \cos(u - v) \sin(u + v)$$

$$z = r \sin(u - v)$$

$$-\frac{\pi}{2} \leq u \leq \frac{\pi}{2}$$

$$0 \leq v \leq 2\pi$$



$$u \rightarrow u - v$$

$$v \rightarrow u + v$$

$$x = r \cos(c_i - v) \cos(c_i + v)$$

$$y = r \cos(c_i - v) \sin(c_i + v)$$

$$z = r \sin(c_i - v)$$

$$0 \leq v \leq 2\pi$$

$$x = r \cos(u - d_i) \cos(u + d_i)$$

$$y = r \cos(u - d_i) \sin(u + d_i)$$

$$z = r \sin(u - d_i)$$

$$-\frac{\pi}{2} \leq u \leq \frac{\pi}{2}$$