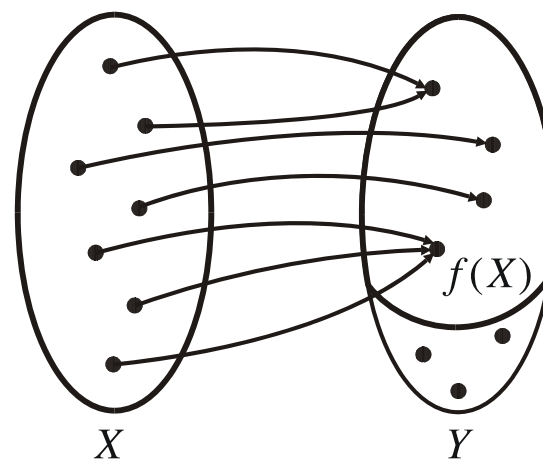


PRESLIKAVANJE (FUNKCIJA)

Neka su dati skupovi X i Y . Ako je svakom elementu $x \in X$ po nekom zakonu f dodeljen (pridružen) tačno jedan određen element $y \in Y$, kaže se da je f **funkcija** ili **preslikavanje** skupa X u skup Y i piše se

$$f: X \rightarrow Y$$



PRESLIKAVANJE (FUNKCIJA)

Funkcija $f: X \rightarrow Y$ se može zadati i opisati pomoću jednakosti

$$y = f(x), x \in X$$

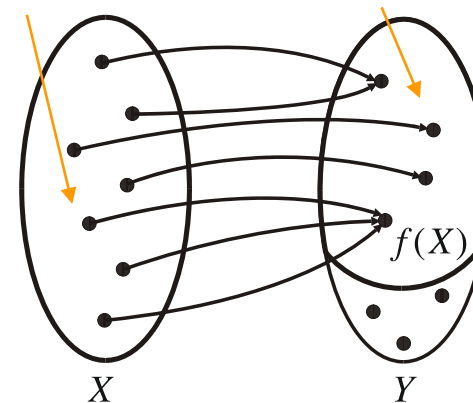
kojom se svakoj pojedinačnoj vrednosti $x \in X$ dodeljuje

tačno jedna određena vrednost $y \in Y$.

Skup vrednosti funkcije $f: X \rightarrow Y$

$$f(X) = \{y \mid y = f(x), x \in X\} \subseteq Y$$

Oblast definisanosti Skup vrednosti funkcije $f: X \rightarrow Y$



PRESLIKAVANJE (FUNKCIJA)

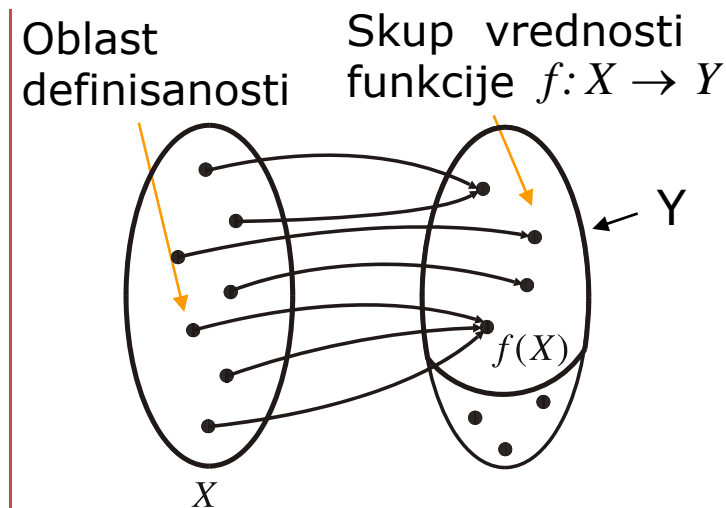
Skup vrednosti funkcije $f : X \rightarrow Y$

$$f(X) = \{y \mid y = f(x), x \in X\} \subseteq Y$$

Ako je

$$f(X) = Y$$

kaže se da funkcija f vrši
preslikavanje skupa X na skup Y

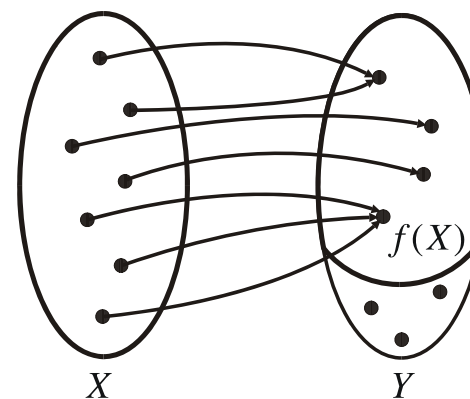


PRESLIKAVANJE (FUNKCIJA)

Jednoznačnost preslikavanja

$$x_1 = x_2 \Rightarrow f(x_1) = f(x_2)$$

$$f(x_1) \neq f(x_2) \Rightarrow x_1 \neq x_2$$



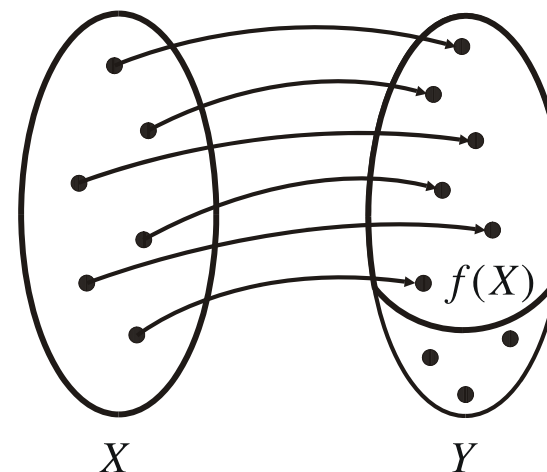
PRESLIKAVANJE (FUNKCIJA)

Ako važi obrnuta implikacija

$$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$$

odnosno

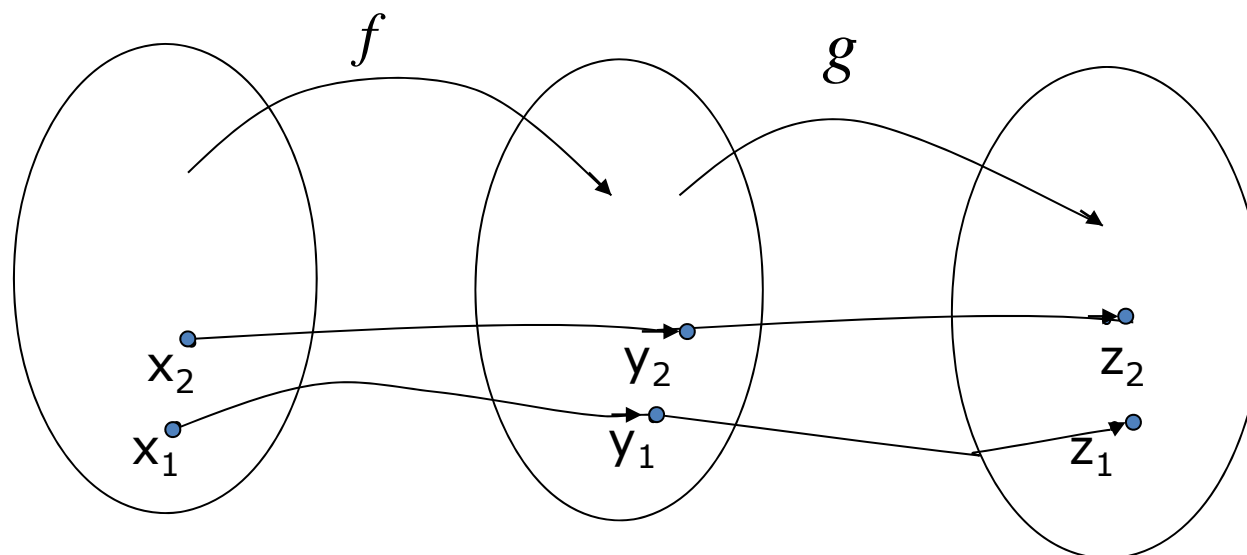
$$x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$$



preslikavanje je obostrano jednoznačno ili 1-1 preslikavanje i predstavlja uzajamnu biunivoku korespondenciju između skupova X i $f(X)$.

PRESLIKAVANJE (FUNKCIJA)

$$f: X \rightarrow Y \quad g: Y \rightarrow Z$$



Kompozicija preslikavanja

$$f \circ g(x) = g(f(x)) \quad f \circ g: X \rightarrow Z$$

PRESLIKAVANJE (FUNKCIJA)

$$I: X \rightarrow X \quad I = I(x) = x \quad \text{Identično preslikavanje}$$

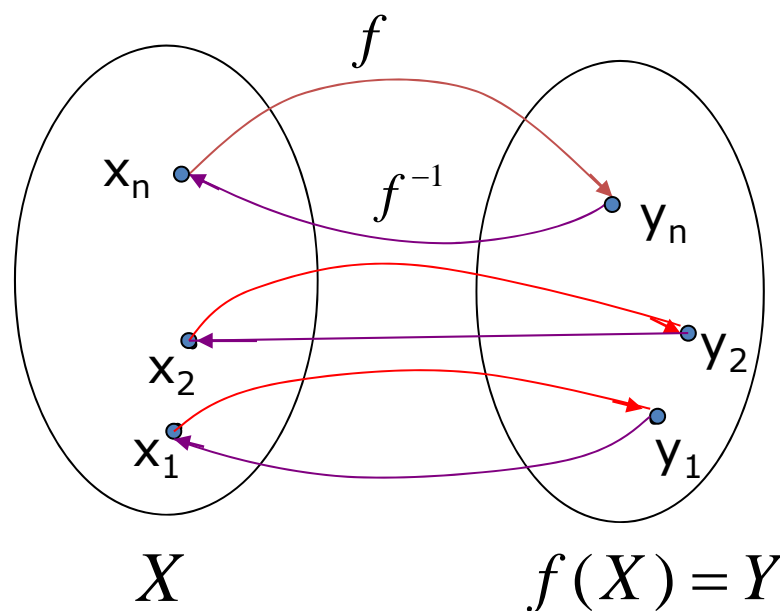
$$f: X \rightarrow Y \quad f(X) = Y$$

Ako postoji funkcija $f^{-1}: Y \rightarrow X$ takva da je

$$f \circ f^{-1} = f^{-1} \circ f = I$$

funkcija f^{-1} je inverzna za funkciju f .

PRESLIKAVANJE (FUNKCIJA)



f - obostrano jednoznačno
prelikavanje i

$$f(X) = Y$$

Inverzno preslikavanje

$$f \circ f^{-1}(x) = f^{-1}(f(x)) = x$$

LINIJE (KRIVE) U RAVNI

$$f : R \rightarrow R$$

$$y = f(x) \quad x \in R$$

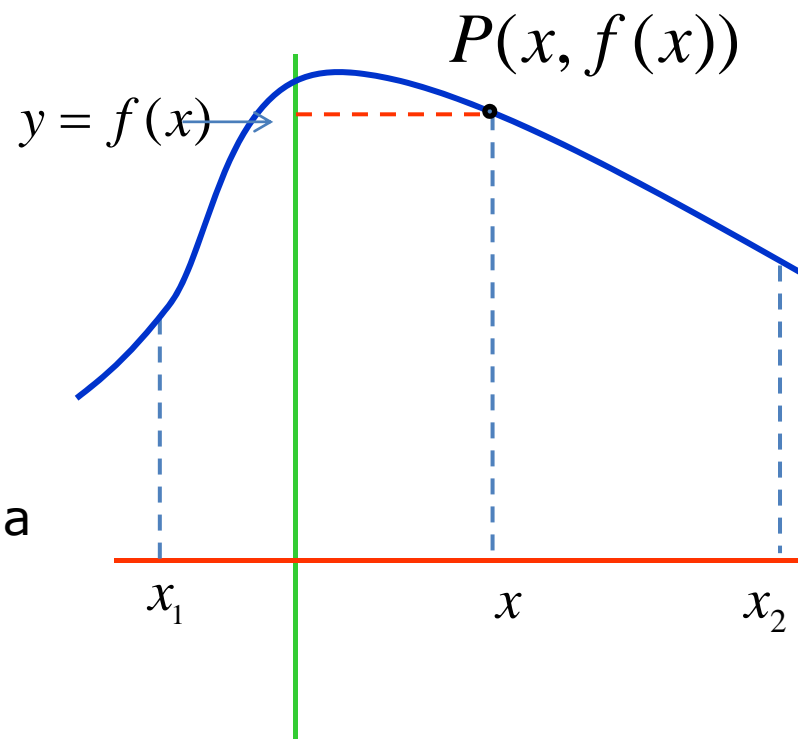
$y = f(x)$ eksplicitno zadana funkcija

Skup tačaka

$$\{(x, f(x)), x \in R\}$$

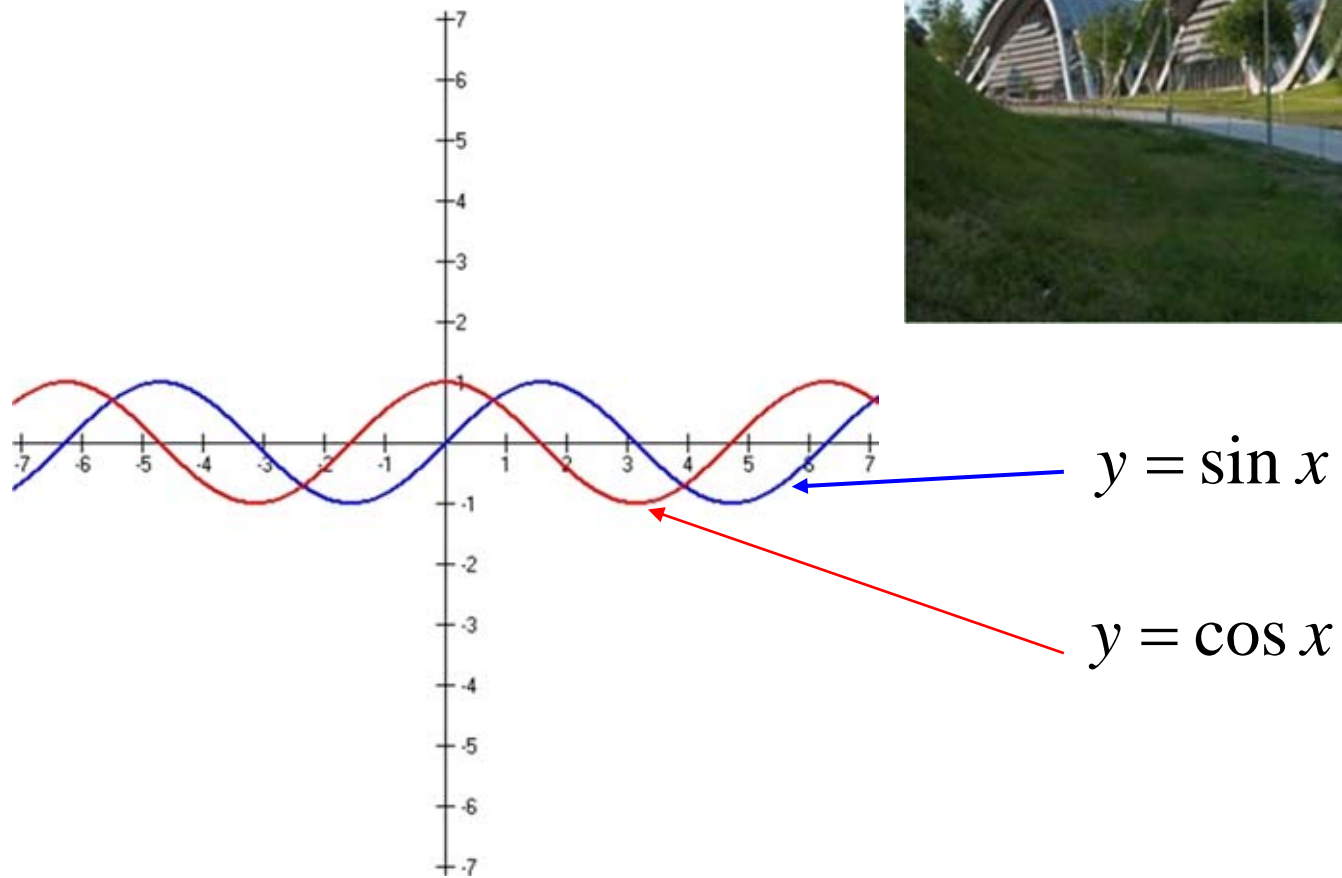
je grafik funkcije $y = f(x)$ i predstavlja liniju (ili krivu) u ravni.

Jednačina $y = f(x)$ je jednačina krive.



LINIJE (KRIVE) U RAVNI

Primeri



LINIJE (KRIVE) U RAVNI

Implicitno zadana funkcija

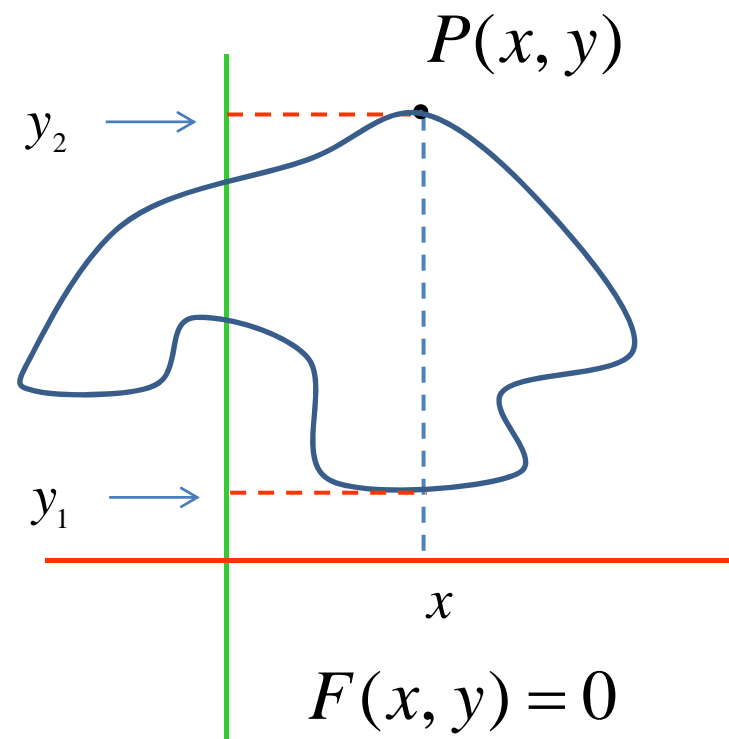
$$F(x, y) = 0 \quad x, y \in \mathbb{R}$$

Skup tačaka

$$\{(x, y), F(x, y) = 0\}$$

je grafik funkcije f i predstavlja liniju (ili krivu) u ravni.

Jednačina $F(x, y) = 0$ je jednačina krive.



LINIJE (KRIVE) U RAVNI

Krive drugog reda

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0 \quad A^2 + B^2 + C^2 \neq 0$$

Svodjenjem na kanonski oblik, odnosno transformacijom koordinatnog sistema (translacijom i rotacijom) dobija se jedna od sledećih krivih:

Krug: $x^2 + y^2 = r^2$

Elipsa:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Hiperbola: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$

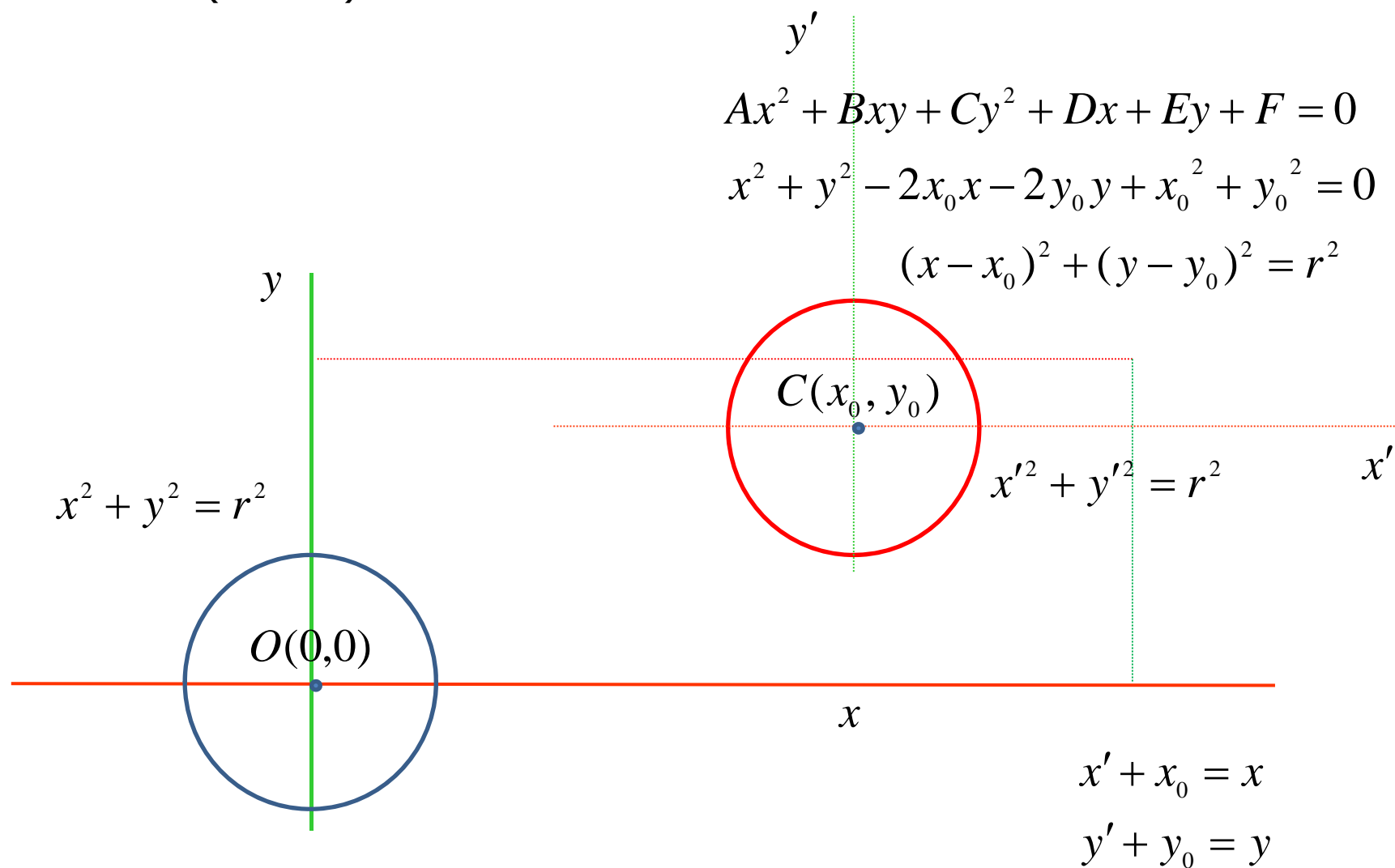
Parabola: $x^2 = 2py$ $y^2 = 2px$

Dve prave koje se seku ili su paralelne

Jedna tačka ili prazan skup.

LINIJE (KRIVE) U RAVNI

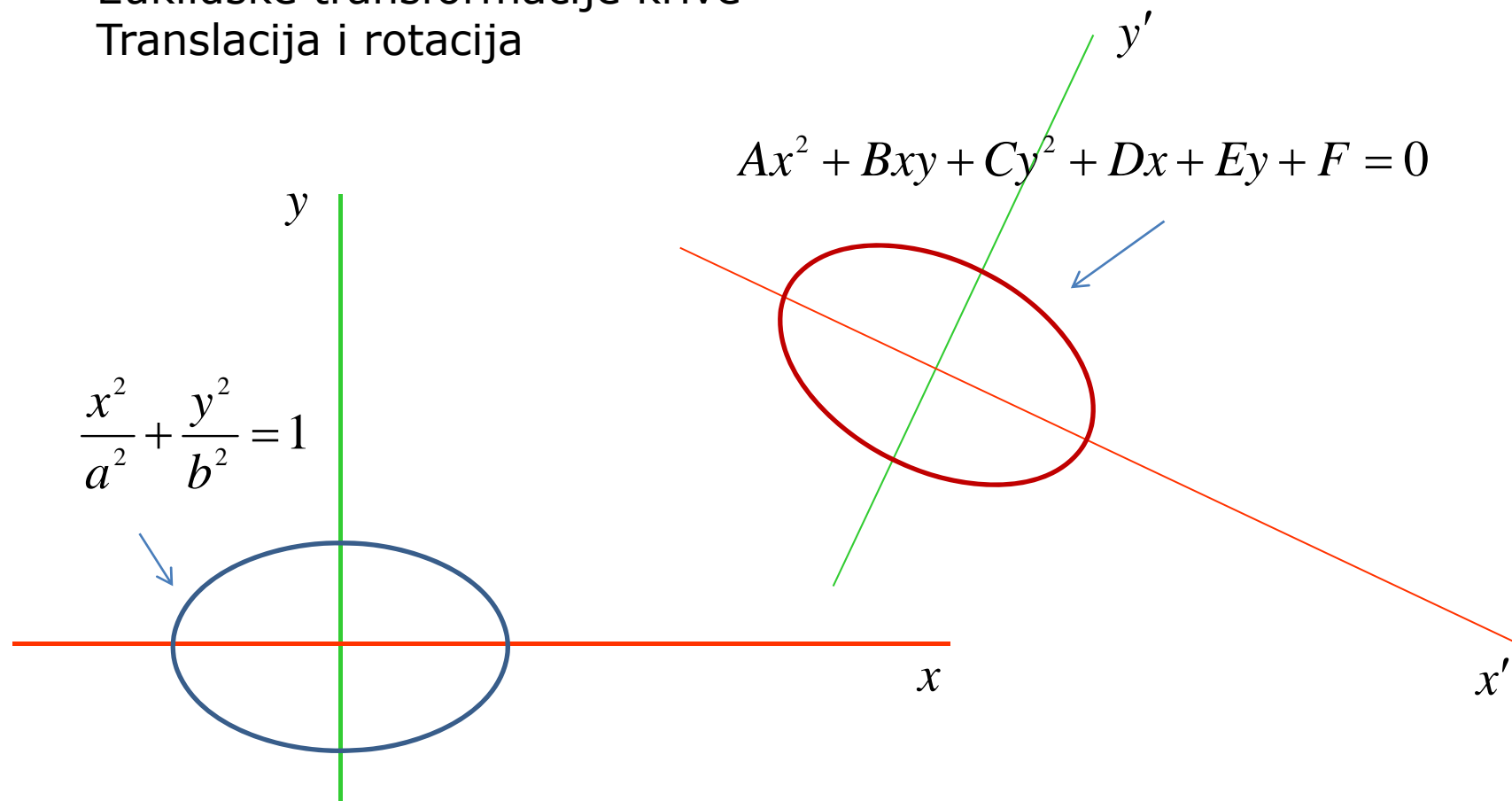
KRUG



LINIJE (KRIVE) U RAVNI

ELIPSA

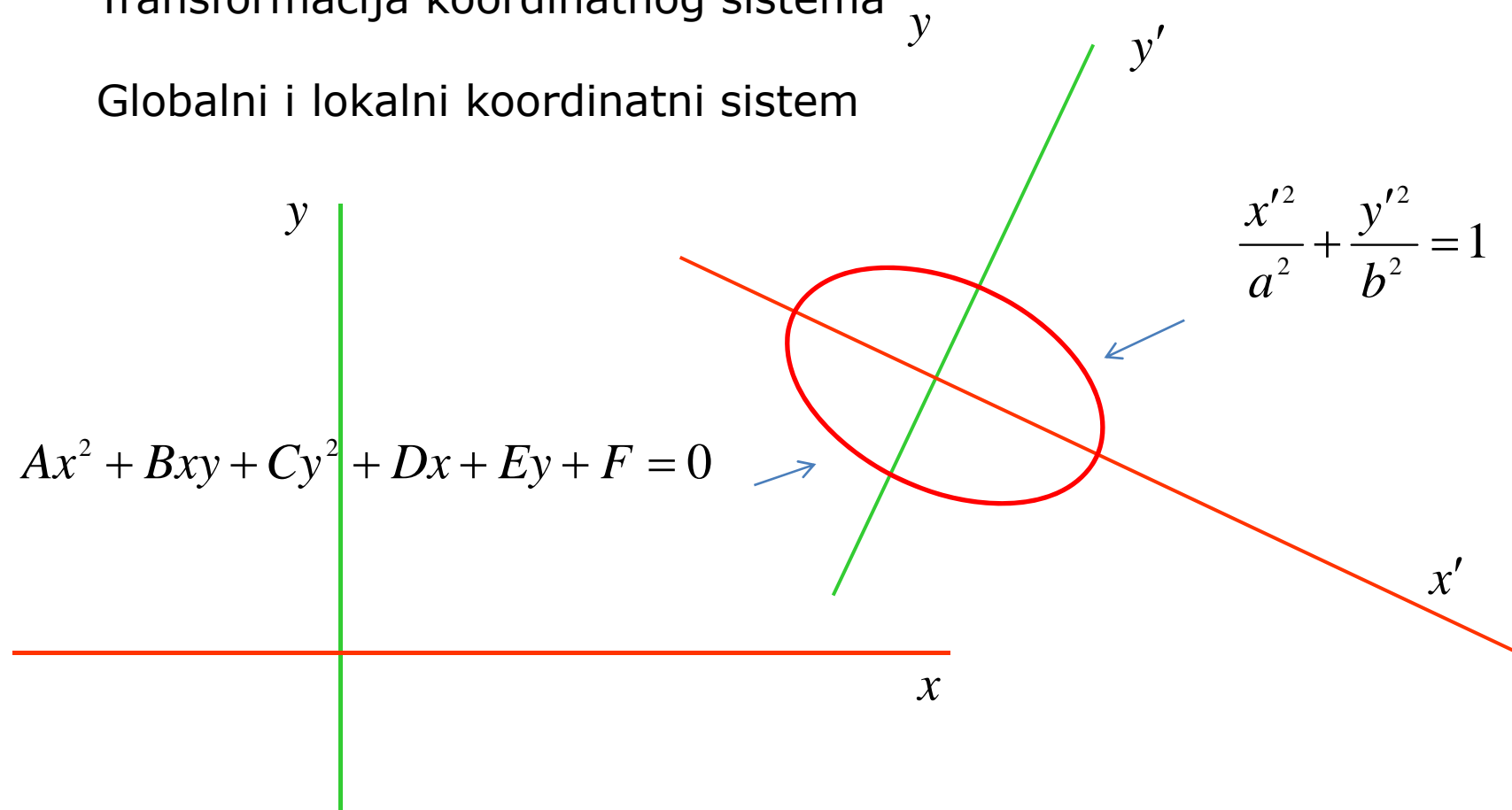
Euklidske transformacije krive
Translacija i rotacija



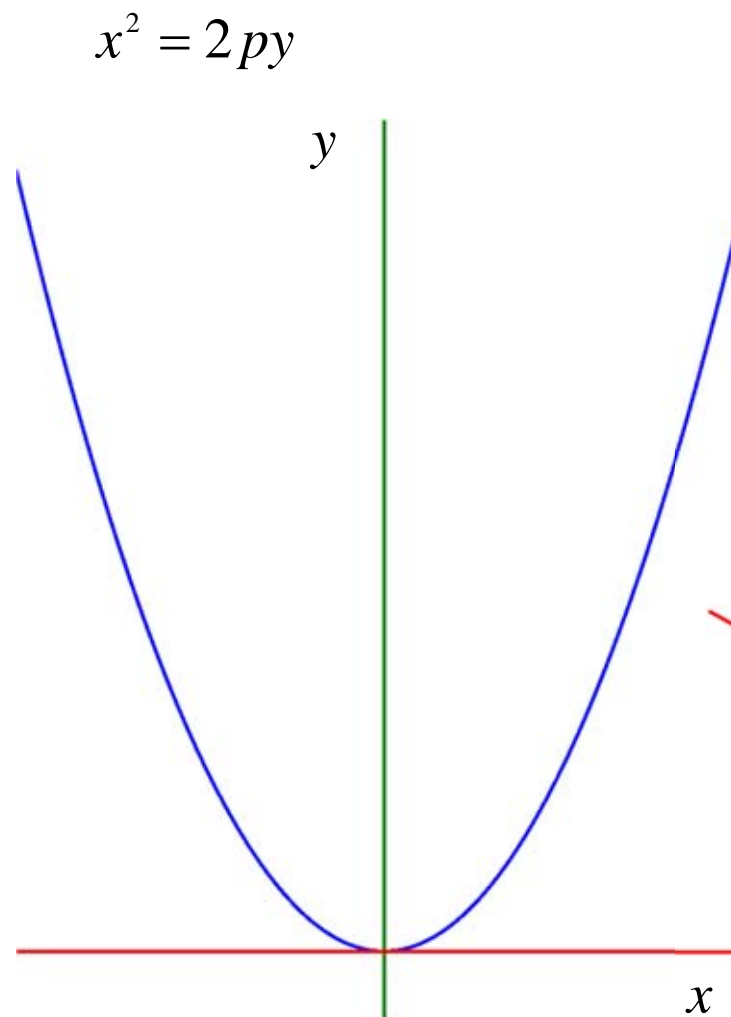
LINIJE (KRIVE) U RAVNI

Transformacija koordinatnog sistema

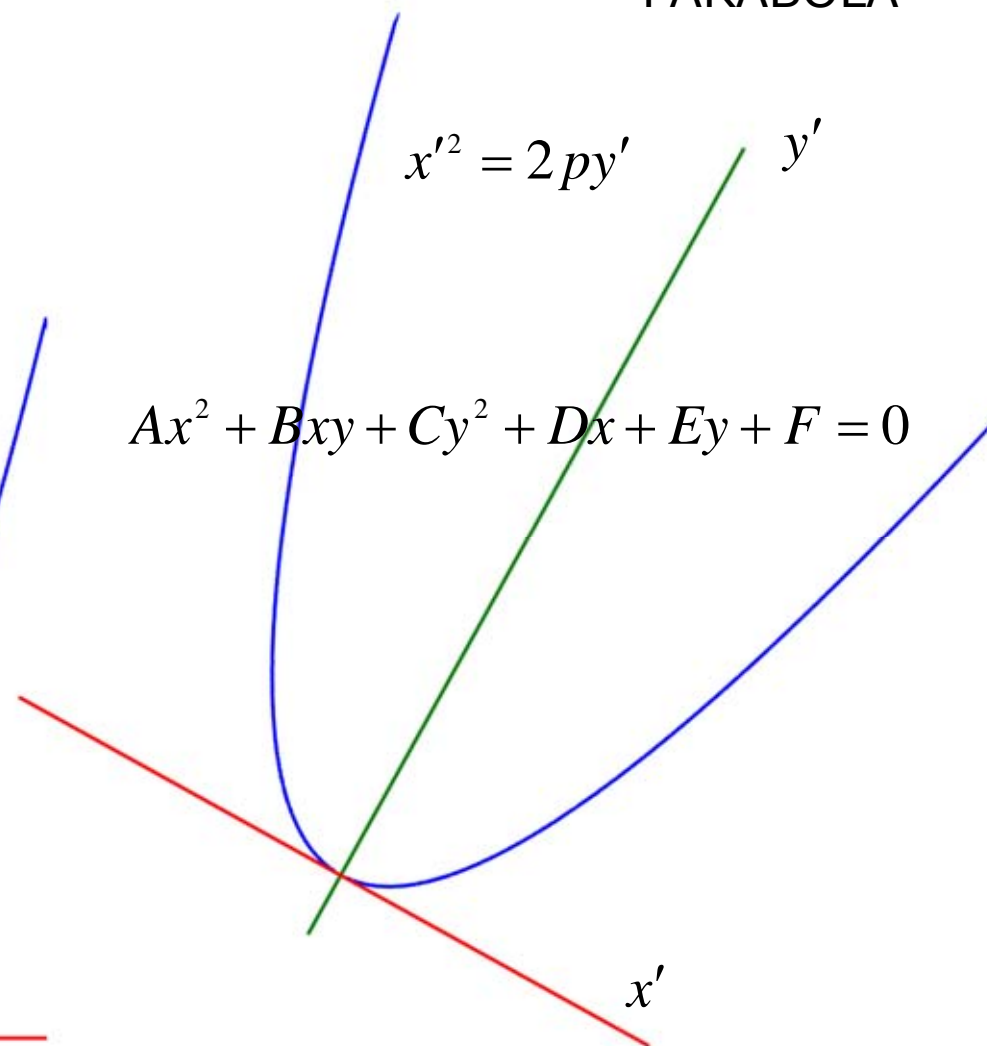
Globalni i lokalni koordinatni sistem



LINIJE (KRIVE) U RAVNI

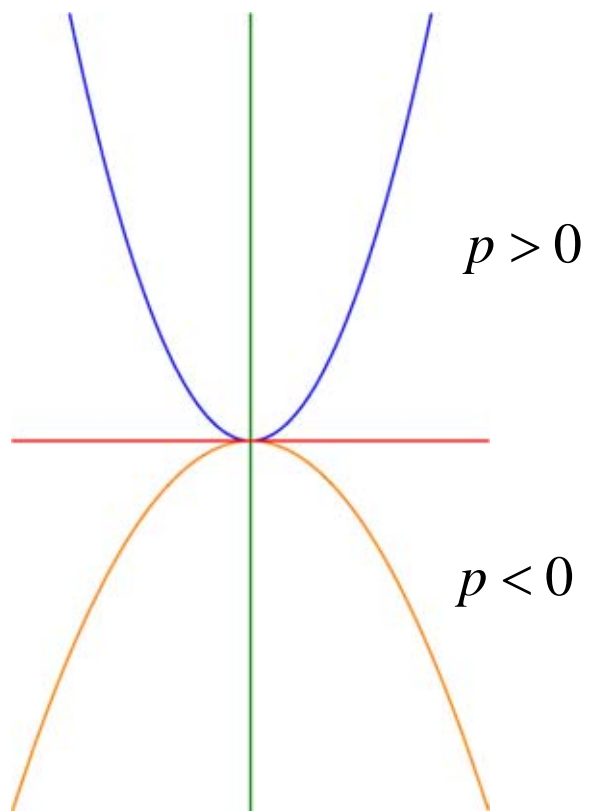


PARABOLA



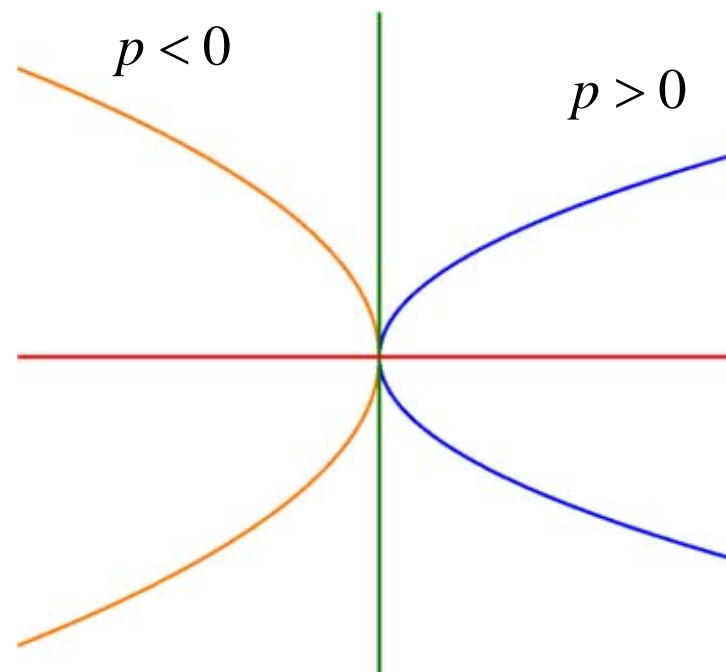
LINIJE (KRIVE) U RAVNI

$$x^2 = 2py$$



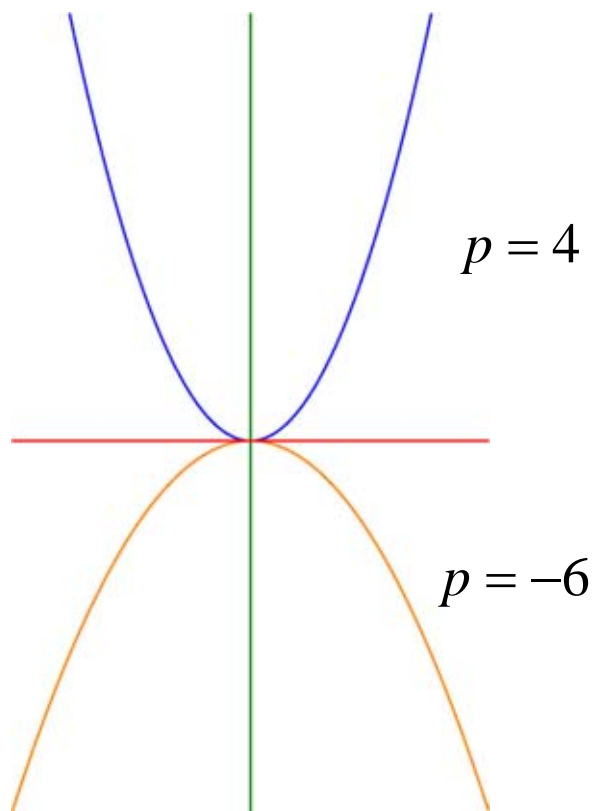
PARABOLA

$$y^2 = 2px$$



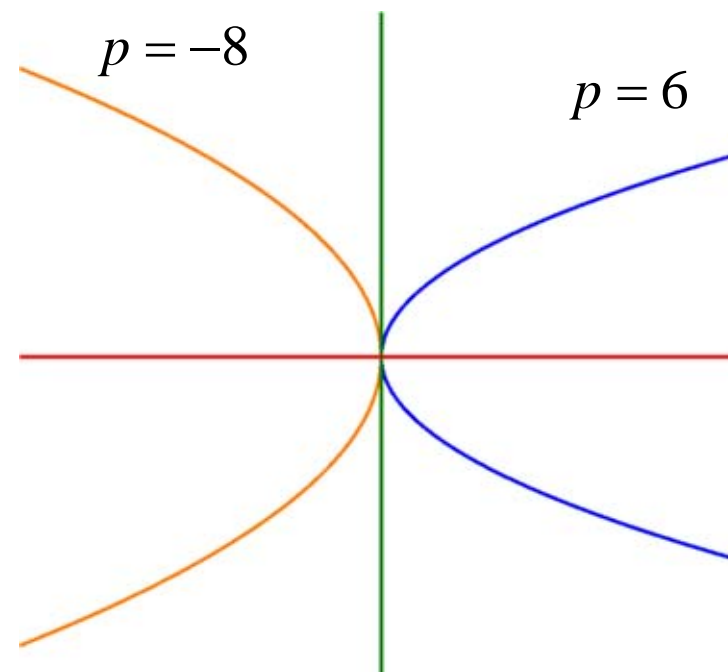
LINIJE (KRIVE) U RAVNI

$$x^2 = 2py$$



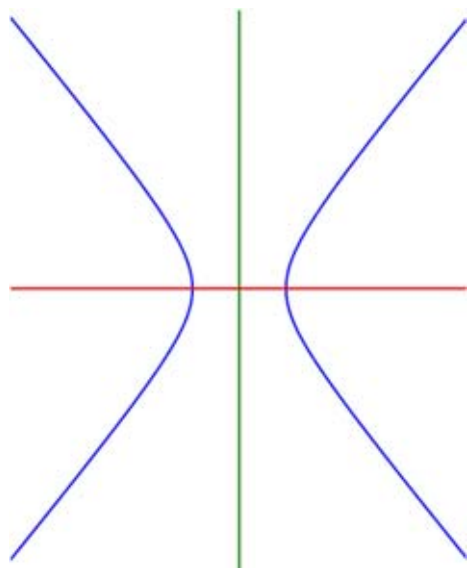
PARABOLA

$$y^2 = 2px$$



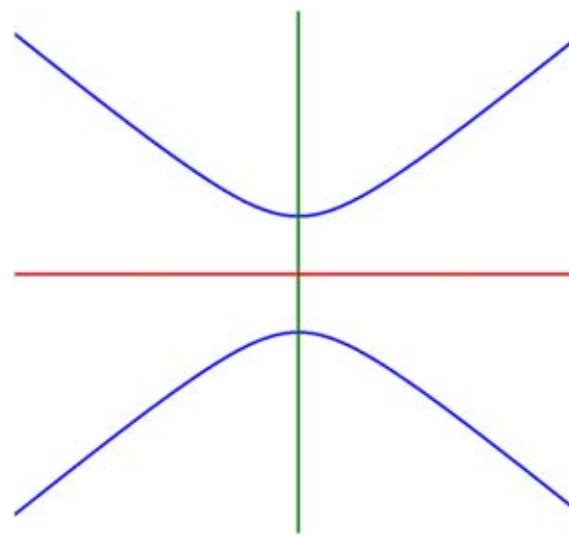
LINIJE (KRIVE) U RAVNI

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



HIPERBOLA

$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$$



LINIJE (KRIVE) U RAVNI

$$I = [a, b] \subset \mathbb{R} \quad I = (a, b) \subset \mathbb{R}$$

$$I = (-\infty, +\infty) = \mathbb{R}$$

$$I = (a, +\infty) \subset \mathbb{R}$$

$$I = (-\infty, a) \subset \mathbb{R}$$

Preslikavanje

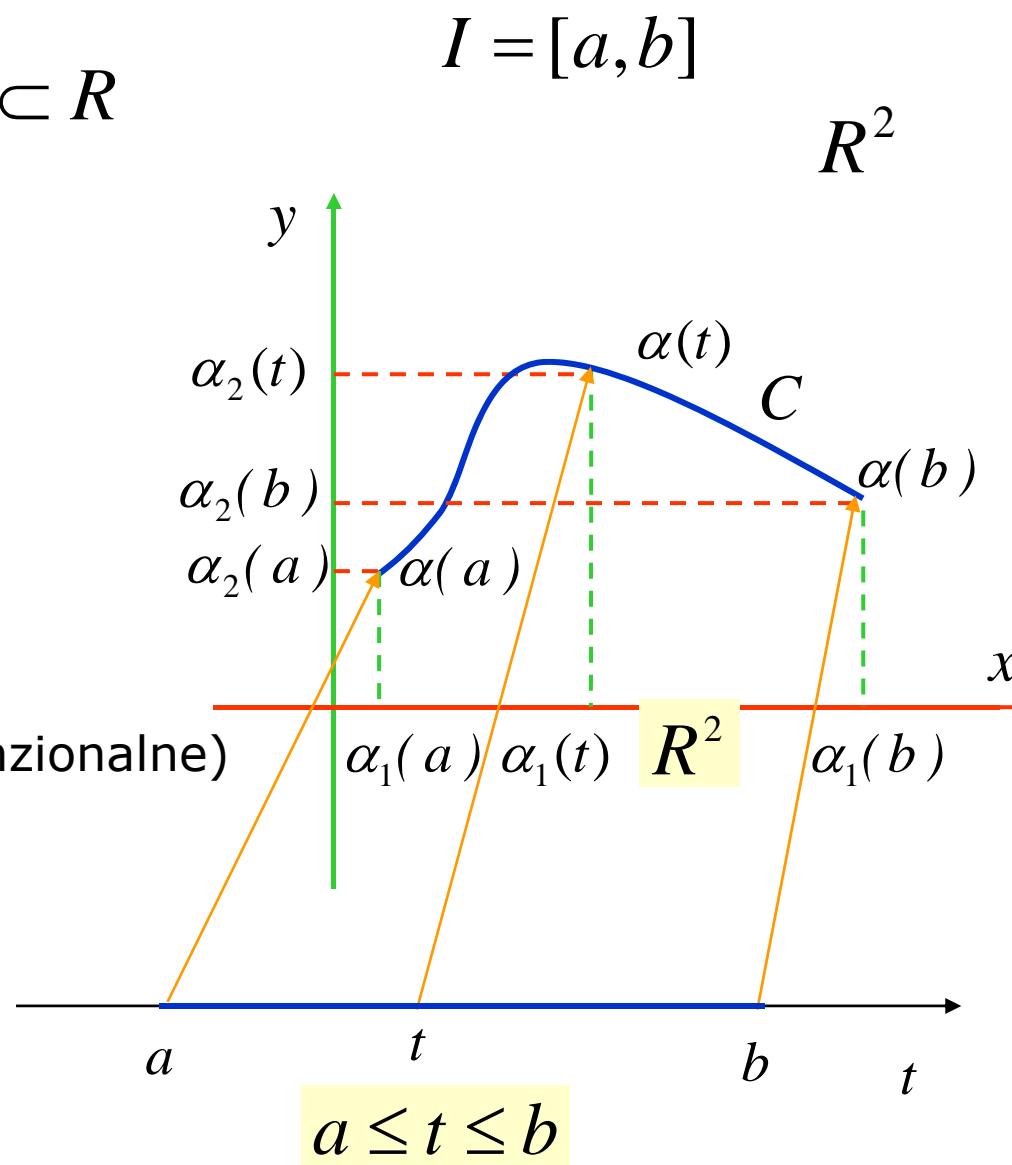
$$\alpha : I \rightarrow \mathbb{R}^2, \quad t \in I$$

nekog intervala I (jednodimensionalne)

realne prave \mathbb{R} u \mathbb{R}^2

je linija (ili kriva) u ravni

\mathbb{R} – parametarska prava



LINIJE (KRIVE) U RAVNI

$$\alpha : I \rightarrow \mathbb{R}^2$$

$$\alpha(t) = (\alpha_1(t), \alpha_2(t))$$

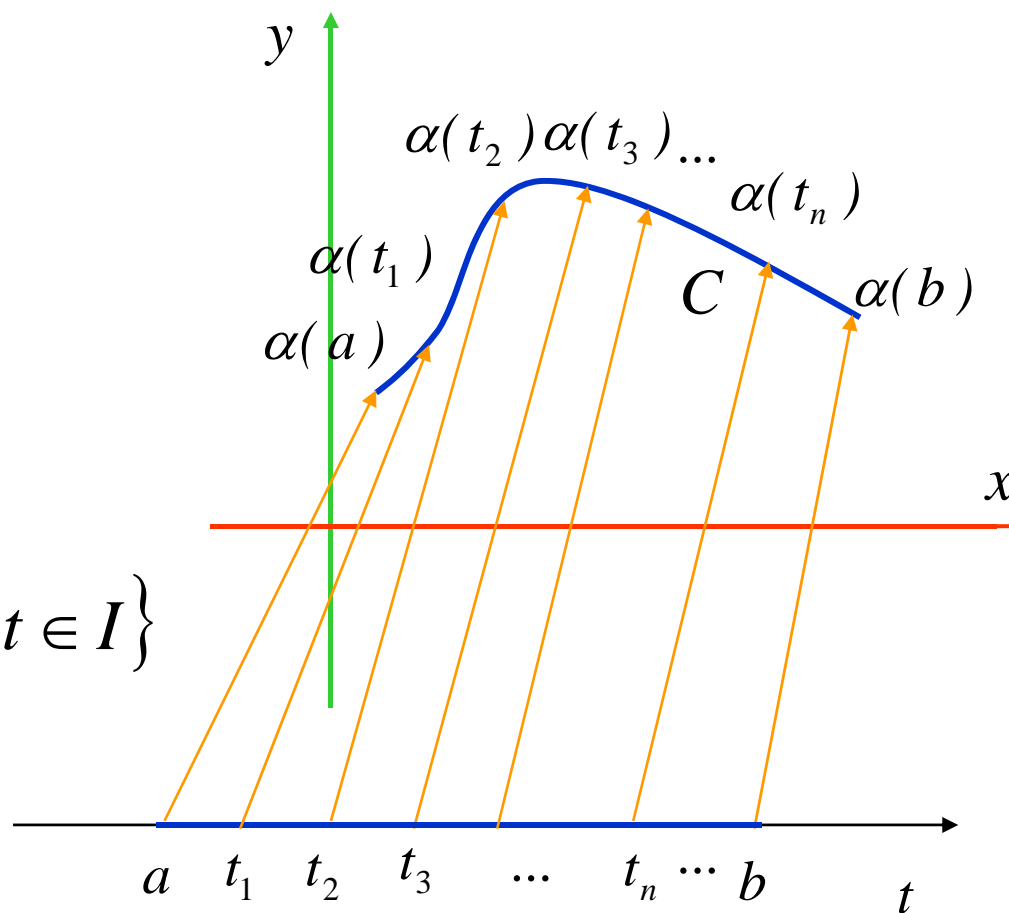
$$t \in I$$

Skup tačaka

$$\{\alpha(t) \mid t \in I\} = \{(\alpha_1(t), \alpha_2(t)) \mid t \in I\}$$

je linija (ili kriva) u ravni.

$$I = [a, b] \subset \mathbb{R}$$



LINIJE (KRIVE) U RAVNI

$$\alpha : I \rightarrow \mathbb{R}^2$$

$$\alpha(t) = (\alpha_1(t), \alpha_2(t))$$

$$a \leq t \leq b$$

Jednačine

$$x = \alpha_1(t)$$

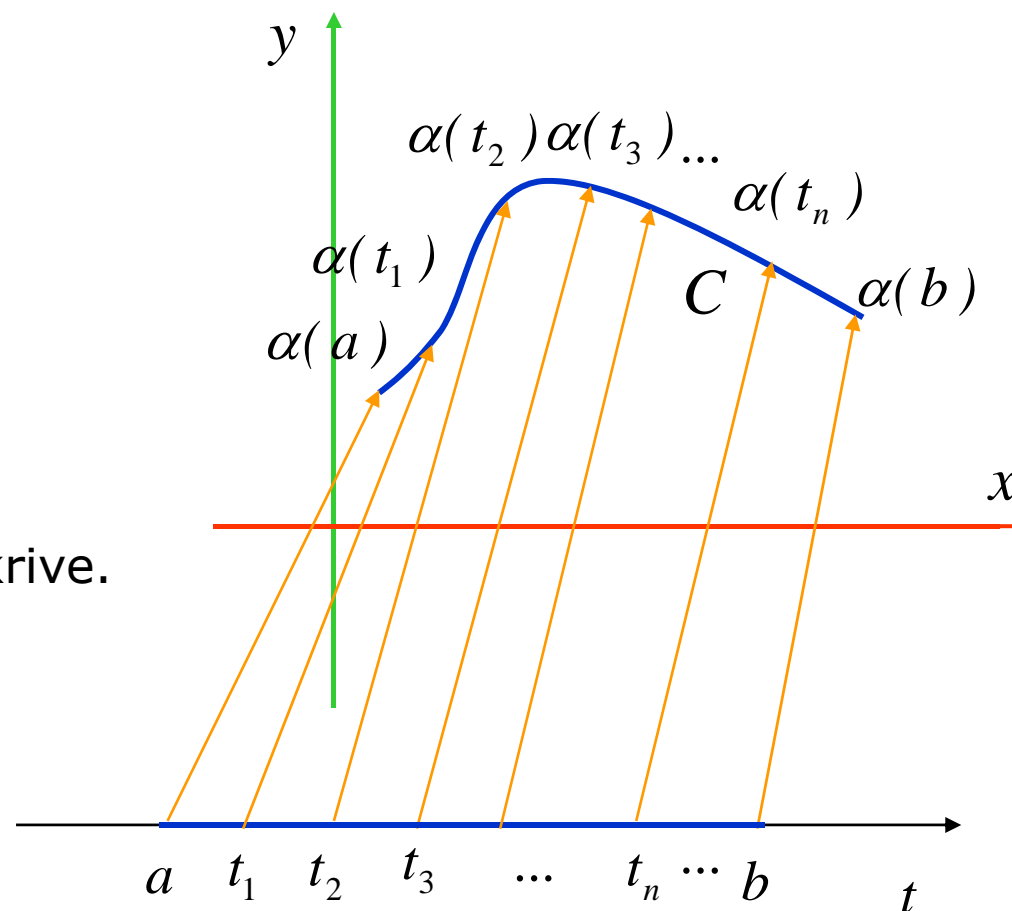
$$y = \alpha_2(t)$$

$$a < t < b$$

su parametarske jednačine te krive.

Za krivu α se kaže da je zadata u parametarskom obliku.

$$I = [a, b] \subset \mathbb{R}$$



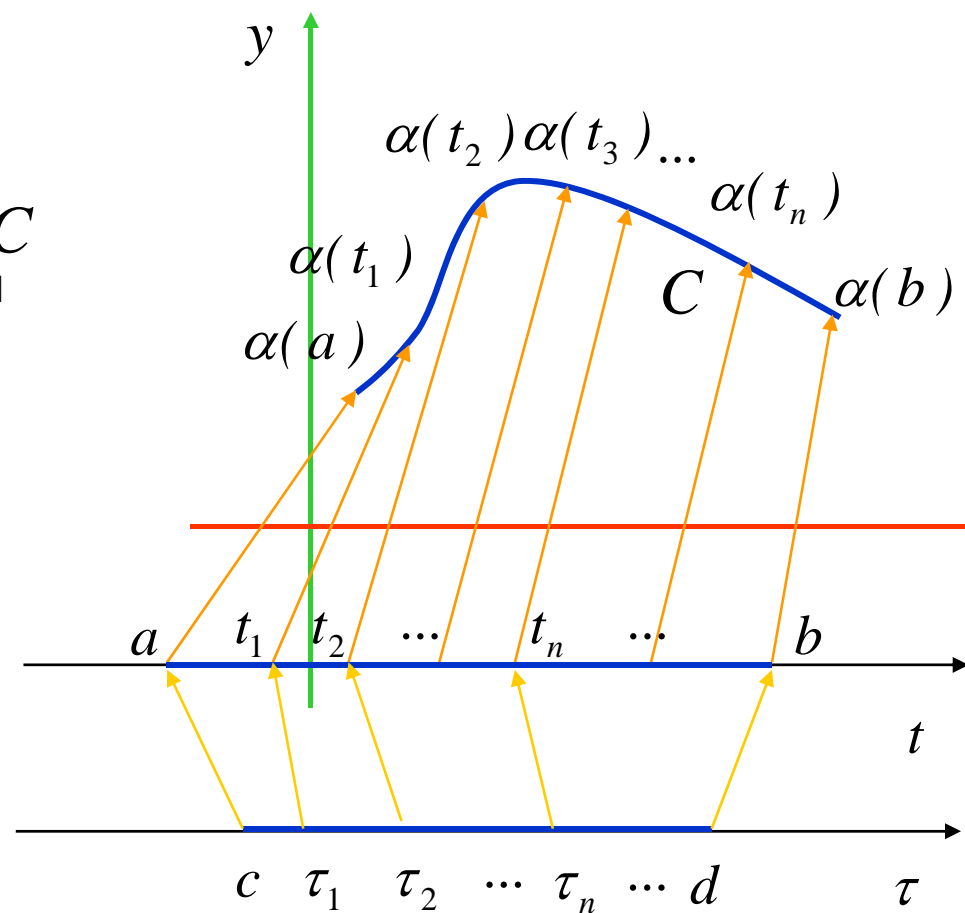
LINIJE (KRIVE) U RAVNI

Matematički prikaz krive linije C
u ravni u parametarskom obliku

$$x = \alpha_1(t) \quad a < t < b$$

$$y = \alpha_2(t)$$

nije jednoznačan.



LINIJE (KRIVE) U RAVNI

$$t = \varphi(\tau)$$

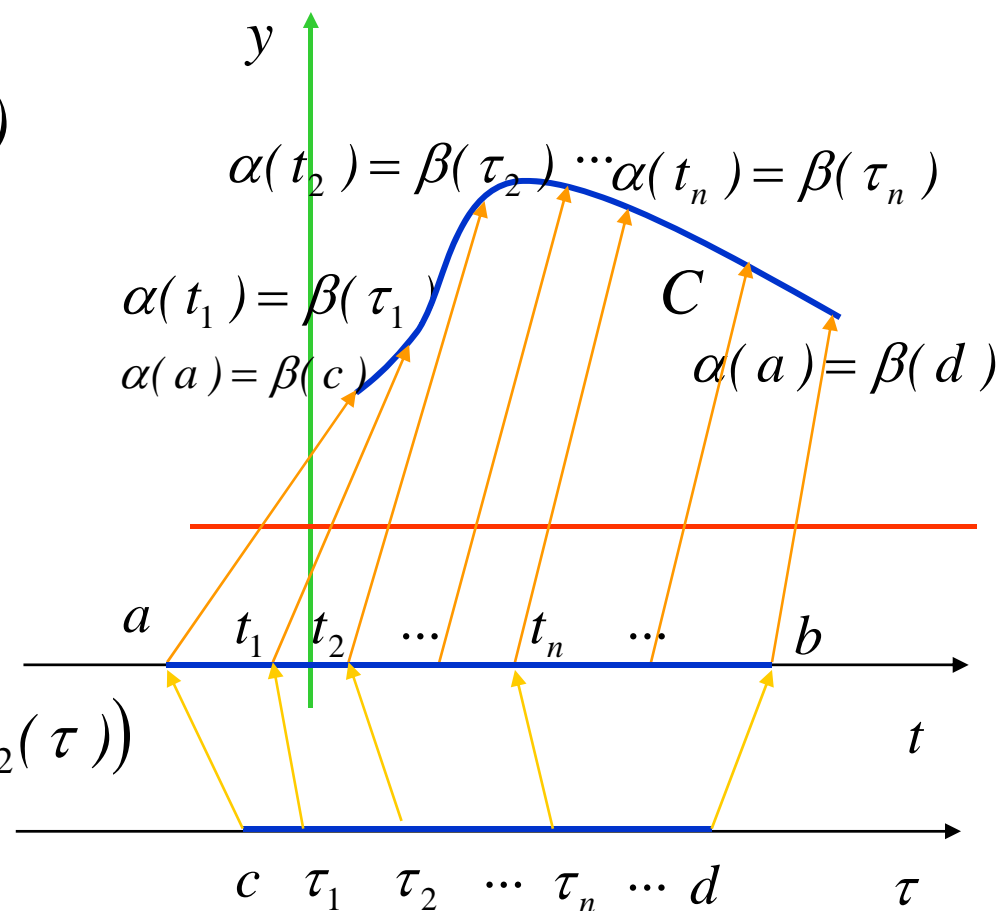
$$\alpha(t) = \beta(\tau)$$

$$\begin{aligned} x &= \alpha_1(t) \\ y &= \alpha_2(t) \quad a < t < b \quad t = \varphi(\tau) \end{aligned}$$

$$\begin{aligned} x &= \alpha_1(\varphi(\tau)) = \beta_1(\tau) \\ y &= \alpha_2(\varphi(\tau)) = \beta_2(\tau) \quad c < \tau < d \end{aligned}$$

$$C : \alpha(t) = (\alpha_1(t), \alpha_2(t))$$

$$C : C(\varphi(\tau)) = \beta(\tau) = (\beta_1(\tau), \beta_2(\tau))$$



LINIJE (KRIVE) U RAVNI



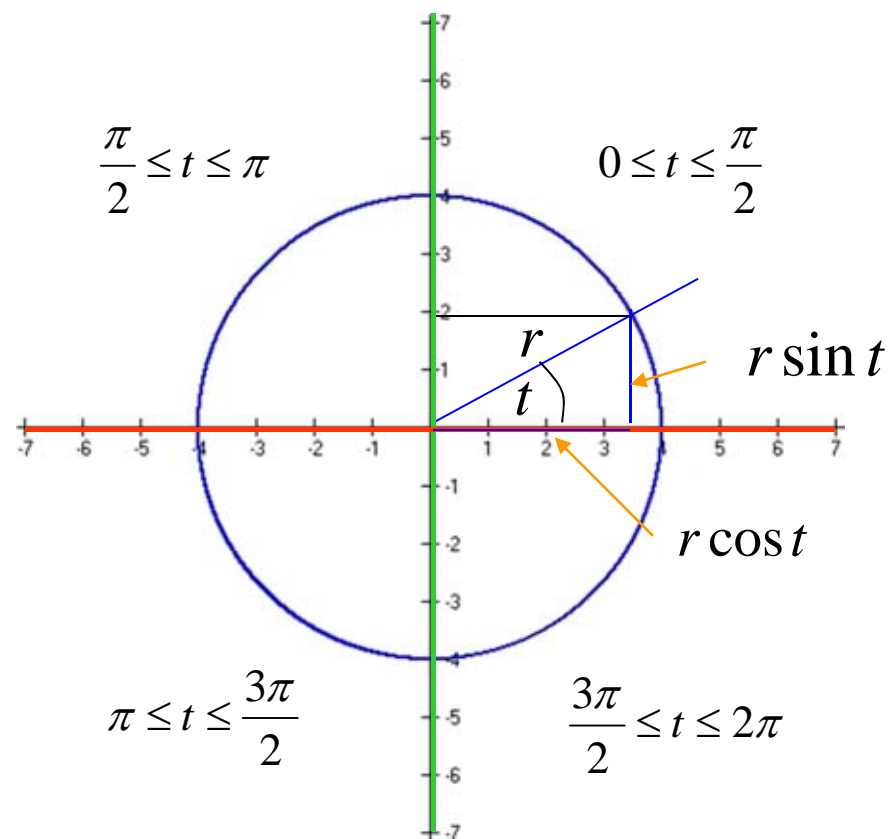
Parametarske jednačine kruga

$$C(0,0)$$

$$x = r \cos t$$

$$y = r \sin t$$

$$0 \leq t \leq 2\pi$$



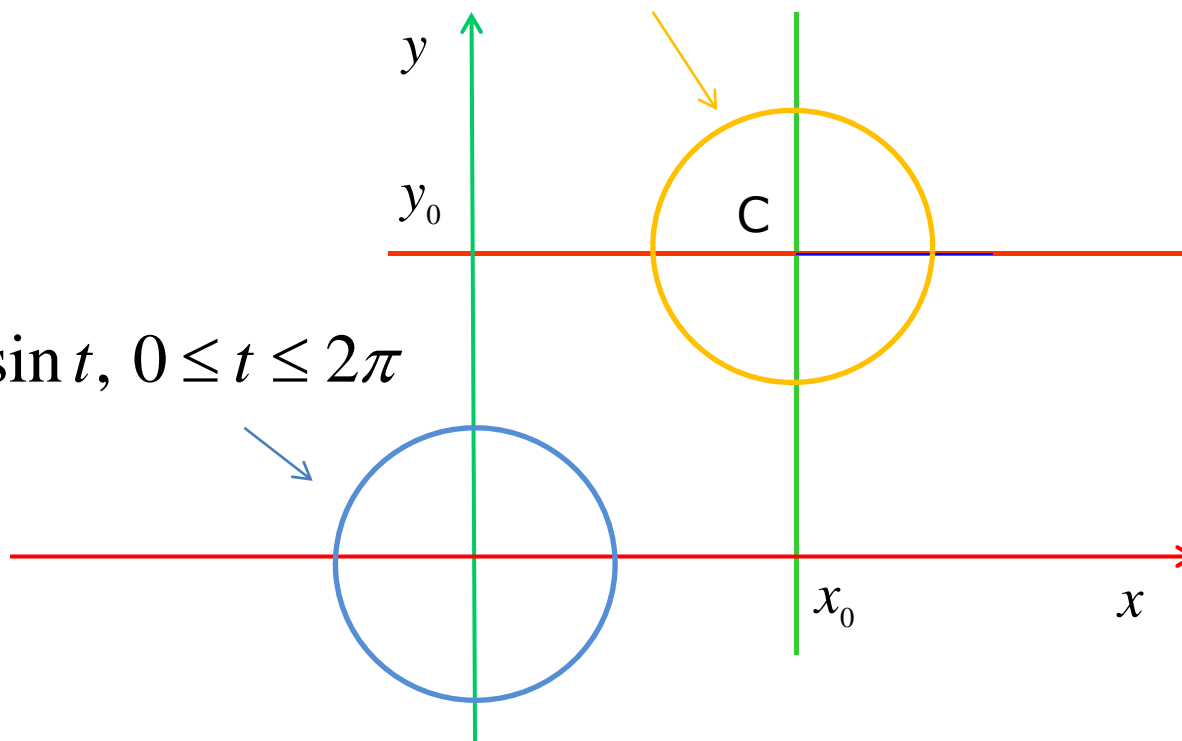
LINIJE (KRIVE) U RAVNI

$$C(x_0, y_0)$$

Parametarske jednačine kruga

$$x = x_0 + r \cos t, \quad y = y_0 + r \sin t, \quad 0 \leq t \leq 2\pi$$

$$x = r \cos t, \quad y = r \sin t, \quad 0 \leq t \leq 2\pi$$



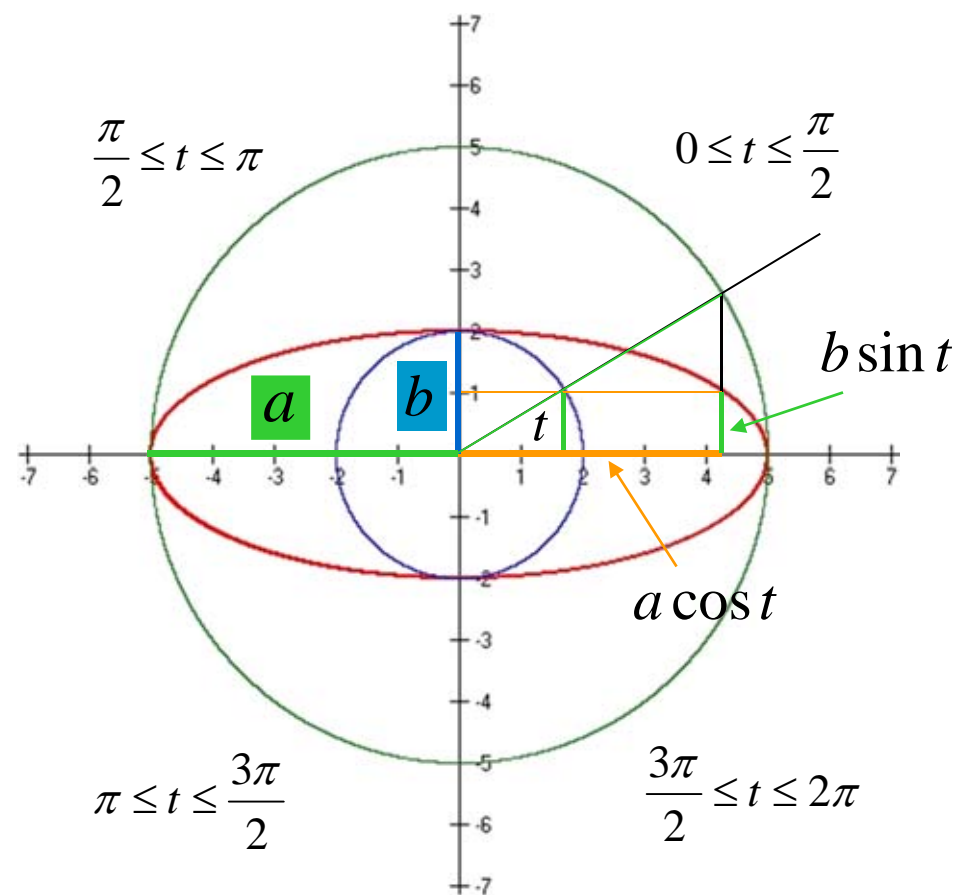
LINIJE (KRIVE) U RAVNI

Parametarske jednačine elipse

$$x = a \cos t$$

$$y = b \sin t$$

$$0 \leq t \leq 2\pi$$

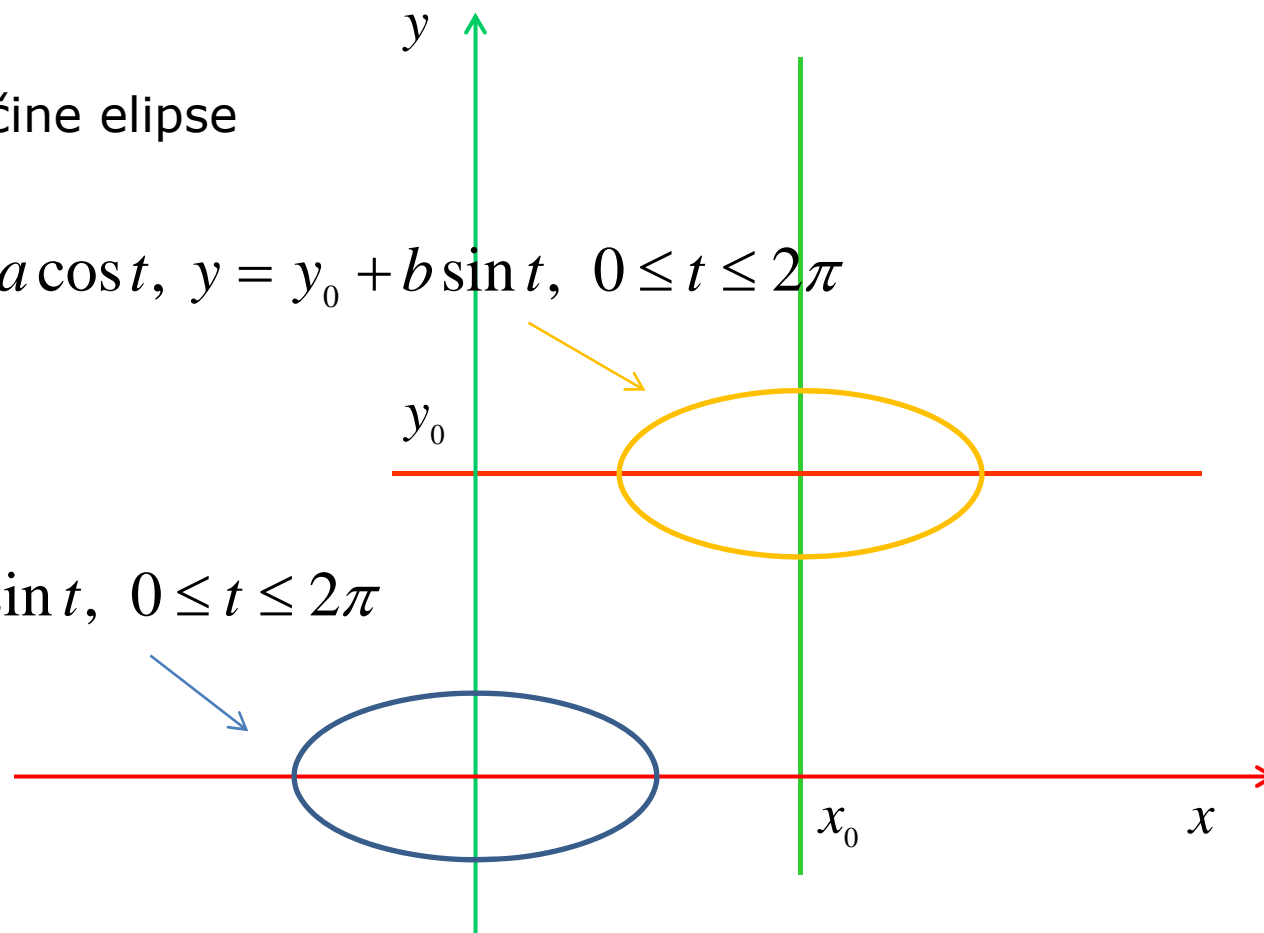


LINIJE (KRIVE) U RAVNI

Parametarske jednačine elipse

$$x = x_0 + a \cos t, \quad y = y_0 + b \sin t, \quad 0 \leq t \leq 2\pi$$

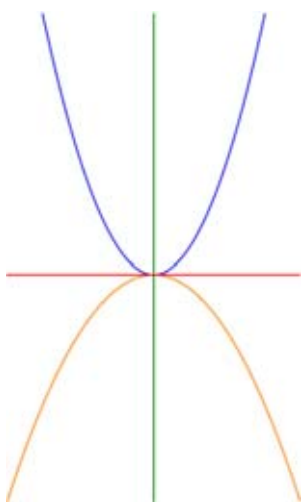
$$x = a \cos t, \quad y = b \sin t, \quad 0 \leq t \leq 2\pi$$



LINIJE (KRIVE) U RAVNI

Parametarske jednačine parabole

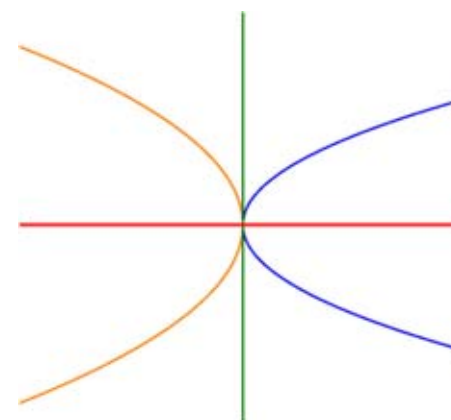
$$x^2 = 2py$$



$$x = t$$

$$y = \frac{1}{2p}t^2$$

$$y^2 = 2px$$



$$x = \frac{1}{2p}t^2$$

$$y = t$$

LINIJE (KRIVE) U RAVNI



Parametarske jednačine hiperbole

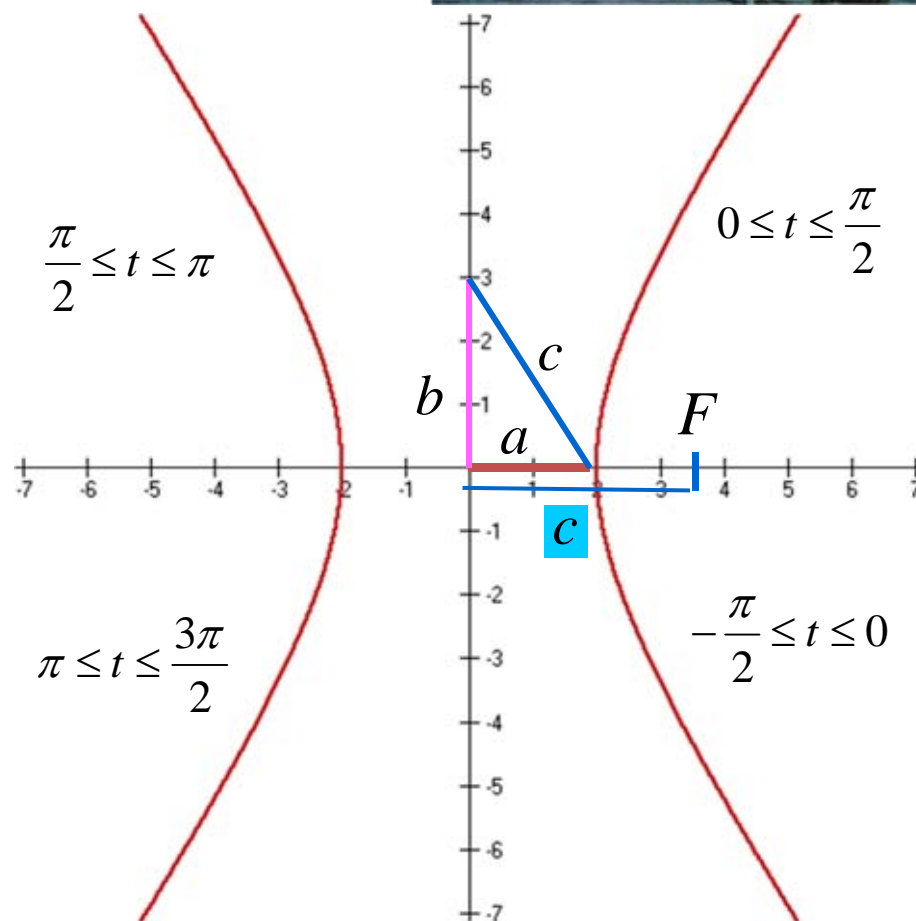
$$x = a \sec t$$

$$y = b \tan t$$

$$-\frac{\pi}{2} \leq t \leq \frac{3\pi}{2}$$

$$\frac{\pi}{2} \leq t \leq \pi$$

$$0 \leq t \leq \frac{\pi}{2}$$



LINIJE (KRIVE) U RAVNI

Parametarske jednačine hiperbole

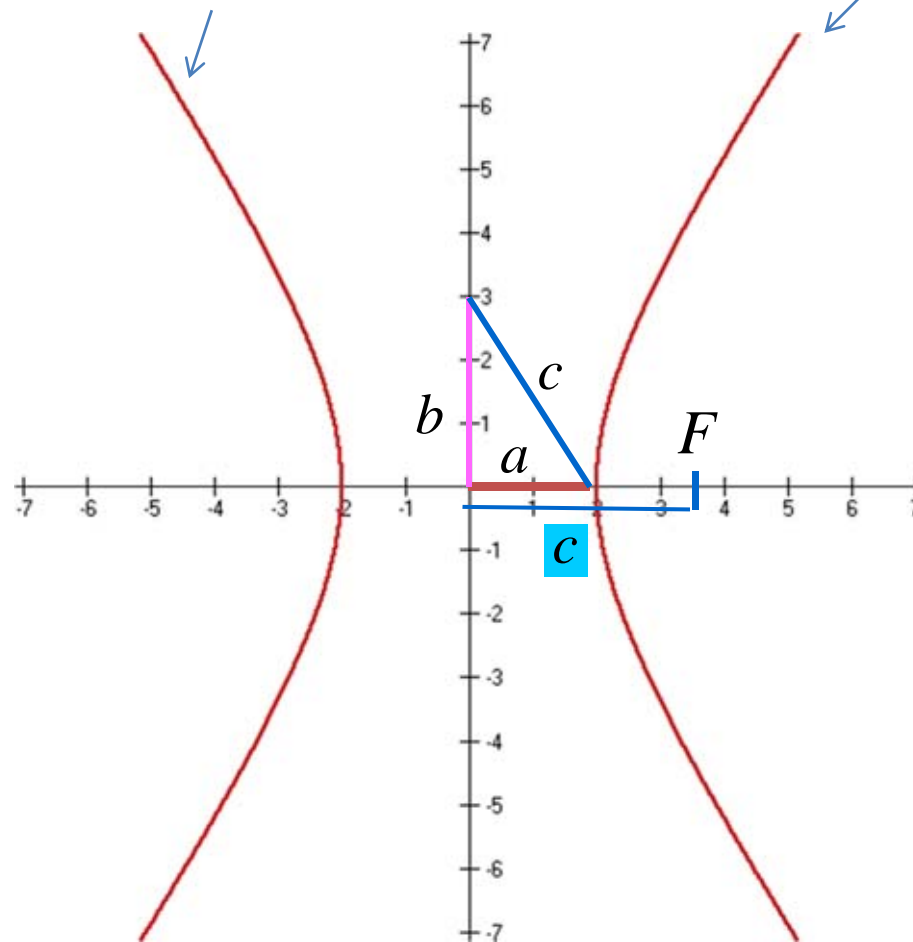
$$\begin{aligned}x &= a \cosh t & t \in \mathbb{R} \\y &= b \sinh t\end{aligned}$$

$$\sinh x = \frac{e^x - e^{-x}}{2} = \frac{e^{2x} - 1}{2e^x}$$

$$\cosh x = \frac{e^x + e^{-x}}{2} = \frac{e^{2x} + 1}{2e^x}$$

$$\begin{aligned}x &= -a \cosh t \\y &= b \sinh t \\t &\in \mathbb{R}\end{aligned}$$

$$\begin{aligned}x &= a \cosh t \\y &= b \sinh t \\t &\in \mathbb{R}\end{aligned}$$



LINIJE (KRIVE) U RAVNI

$$x = a \sec t, \quad y = b \tan t$$

$$x = \sec t, \quad y = \tan t$$

$$x = 2 \sec t, \quad y = 3 \tan t$$

$$x = 3 \sec t, \quad y = \tan t$$

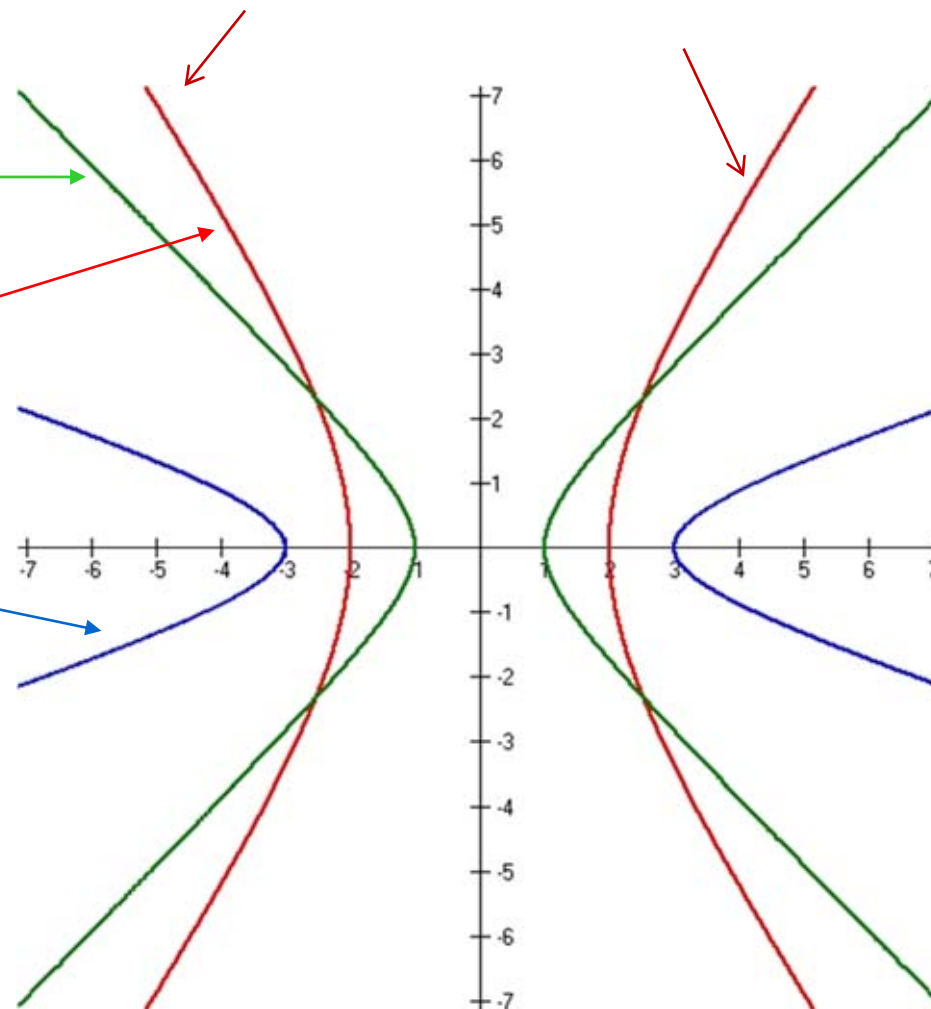
$$-\frac{\pi}{2} \leq t \leq \frac{3\pi}{2}$$

$$x = -2 \cosh t$$

$$y = 3 \sinh t$$

$$x = 2 \cosh t$$

$$y = 3 \sinh t$$



LINIJE (KRIVE) U RAVNI

Astroida - parametarske jednačine

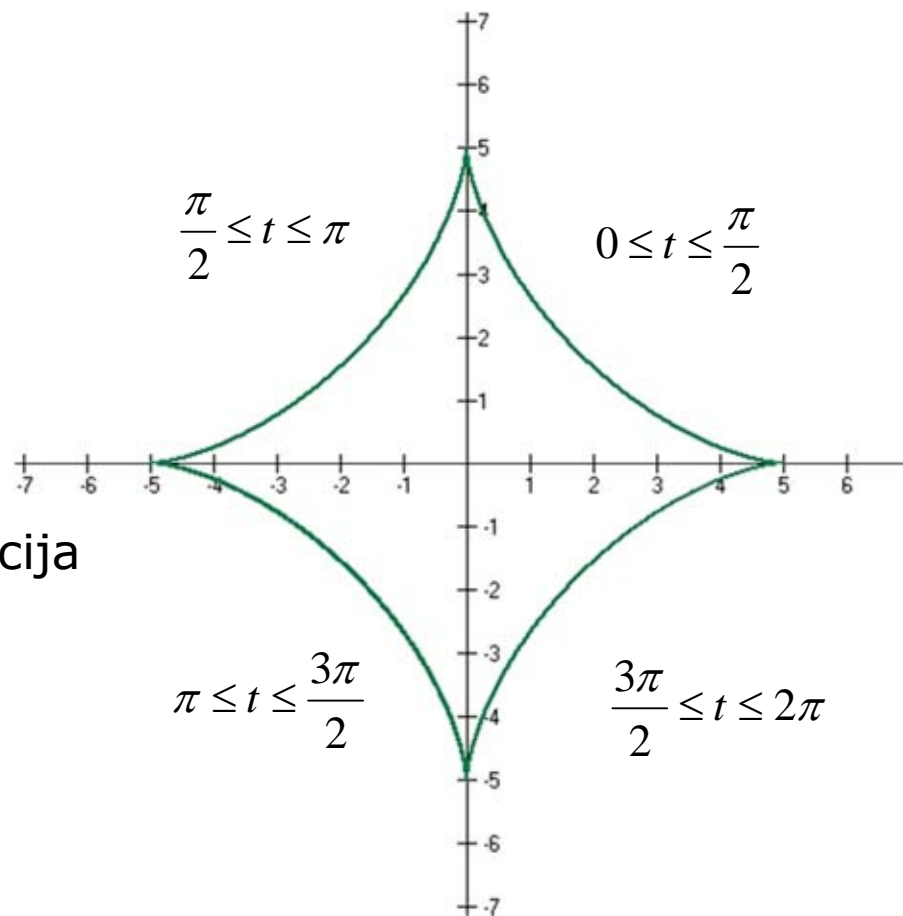
$$x = a \cos^3 t$$

$$y = a \sin^3 t$$

$$0 \leq t \leq 2\pi$$

Astroida - implicitno zadata funkcija

$$x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^2$$



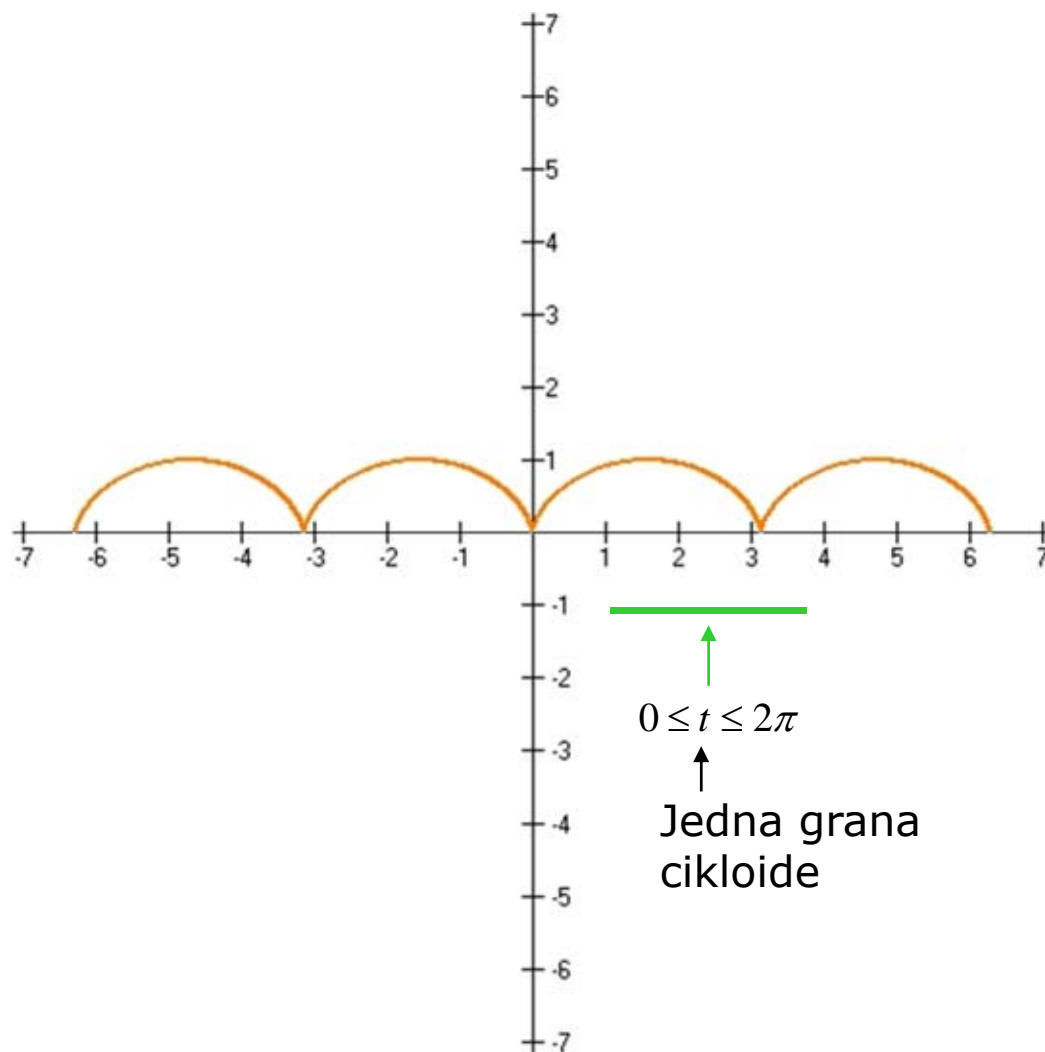
LINIJE (KRIVE) U RAVNI

Cikloida

$$x = a(t - \sin t)$$

$$y = a(1 - \cos t)$$

$$t \in R$$



LINIJE (KRIVE) U RAVNI

$$y = t, x = t^2, -0.5 \leq t \leq 2.5$$

$$y = t^2, x = t, -2 \leq t \leq 2$$

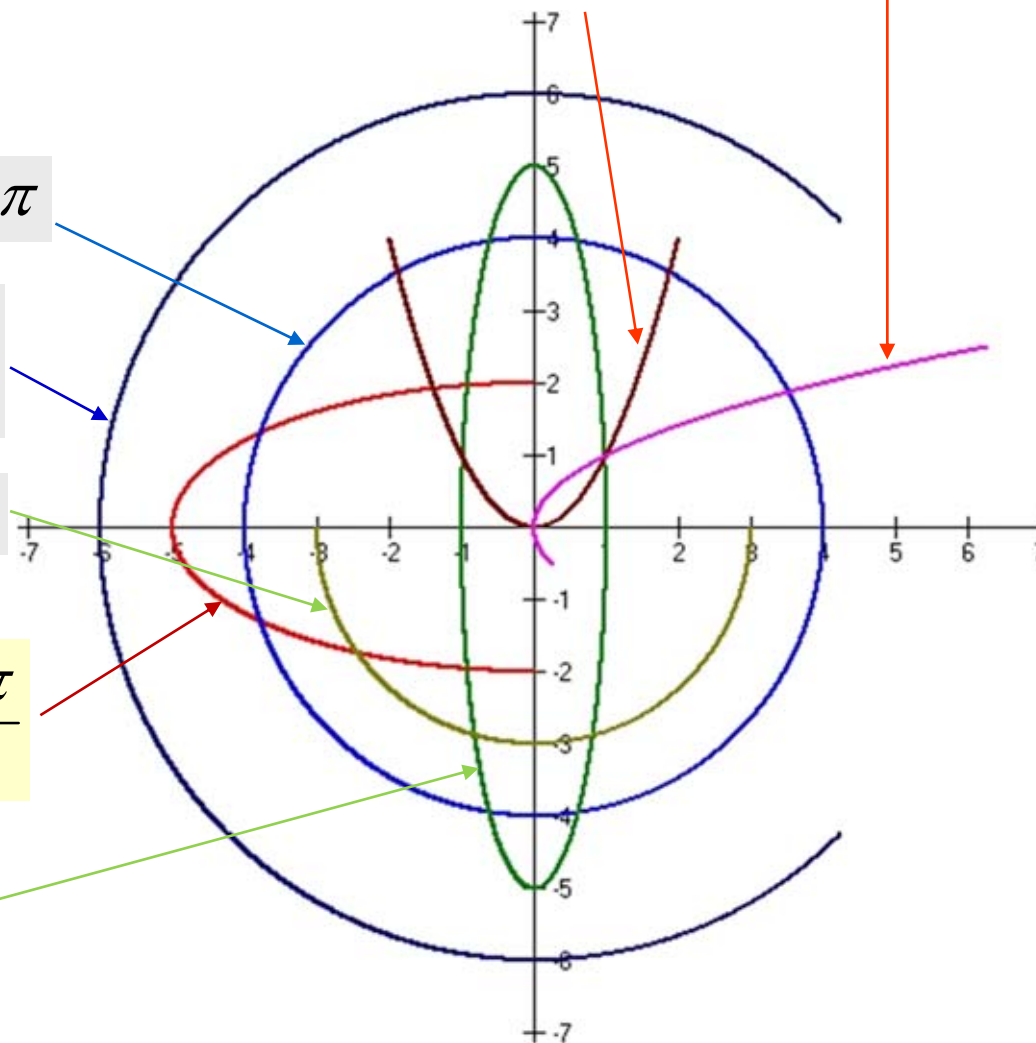
$$x = 4 \cos t, y = 4 \sin t, 0 \leq t \leq 2\pi$$

$$x = 6 \cos t, y = 6 \sin t, \frac{\pi}{4} \leq t \leq \frac{7\pi}{4}$$

$$x = 3 \cos t, y = 3 \sin t, \pi \leq t \leq 2\pi$$

$$x = 5 \cos t, y = 2 \sin t, \frac{\pi}{2} \leq t \leq \frac{3\pi}{2}$$

$$x = \cos t, y = 5 \sin t, 0 \leq t \leq 2\pi$$



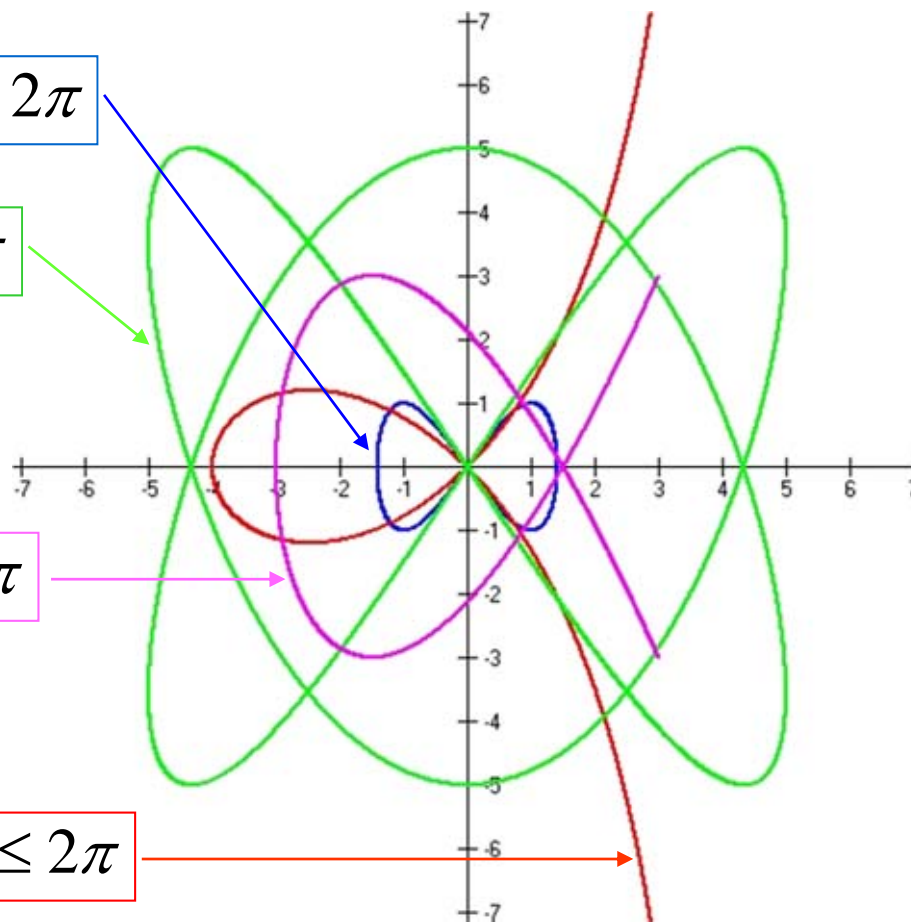
LINIJE (KRIVE) U RAVNI

$$x = \sin t + \cos t, \quad y = \cos 2t, \quad 0 \leq t \leq 2\pi$$

$$x = 5 \sin 2t, \quad y = 5 \sin 3t, \quad 0 \leq t \leq 2\pi$$

$$x = 3 \cos 2t, \quad y = 3 \cos 3t, \quad 0 \leq t \leq 2\pi$$

$$x = 4 \sin t, \quad y = \operatorname{tg} t(1 + \sin t), \quad 0 \leq t \leq 2\pi$$



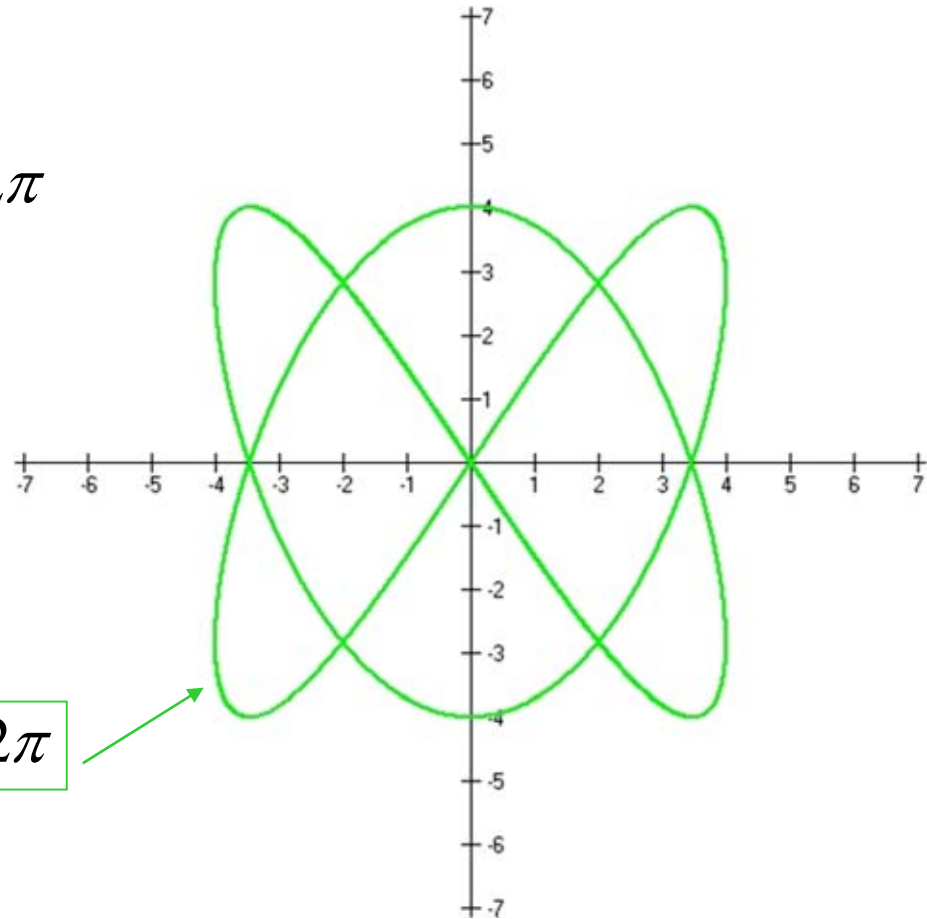
LINIJE (KRIVE) U RAVNI

$$x = a \sin 2t, \quad y = b \sin 3t, \quad 0 \leq t \leq 2\pi$$

$$a = b$$

$$a = b = 4$$

$$x = 4 \sin 2t, \quad y = 4 \sin 3t, \quad 0 \leq t \leq 2\pi$$



LINIJE (KRIVE) U RAVNI

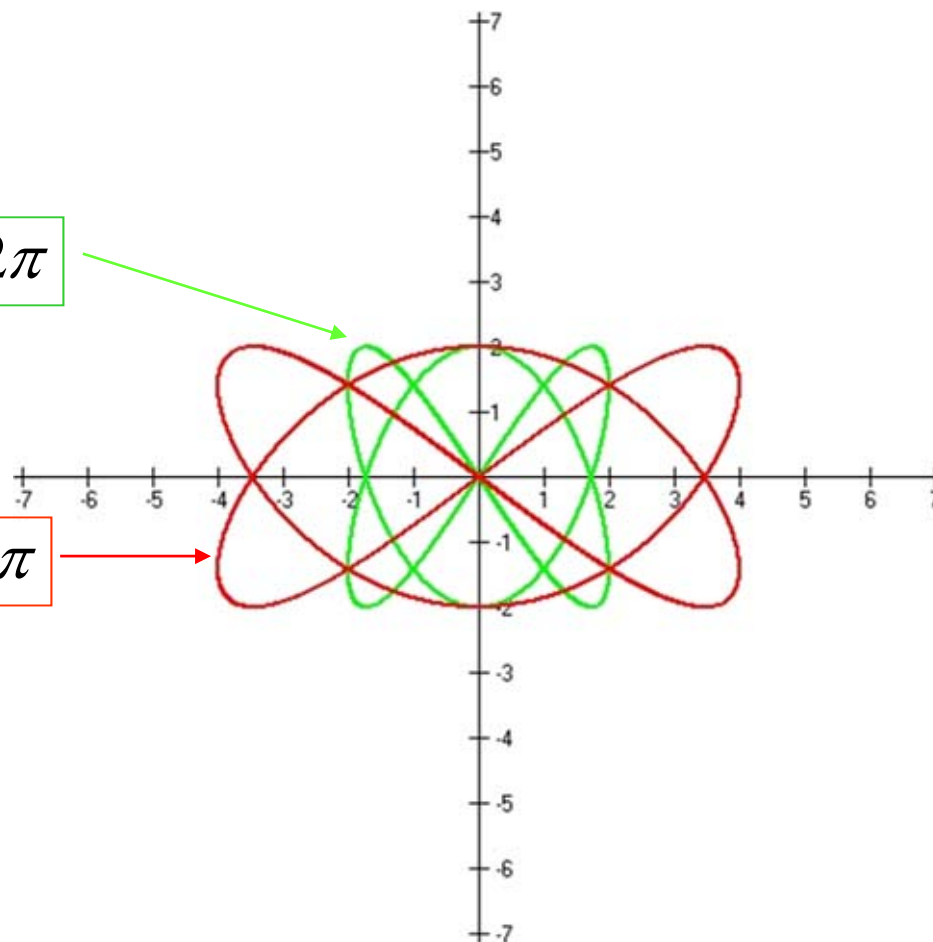
$$x = a \sin 2t, \quad y = b \sin 3t, \quad 0 \leq t \leq 2\pi$$

$$a = b = 2$$

$$x = 2 \sin 2t, \quad y = 2 \sin 3t, \quad 0 \leq t \leq 2\pi$$

$$a \neq b \quad a = 4, \quad b = 2$$

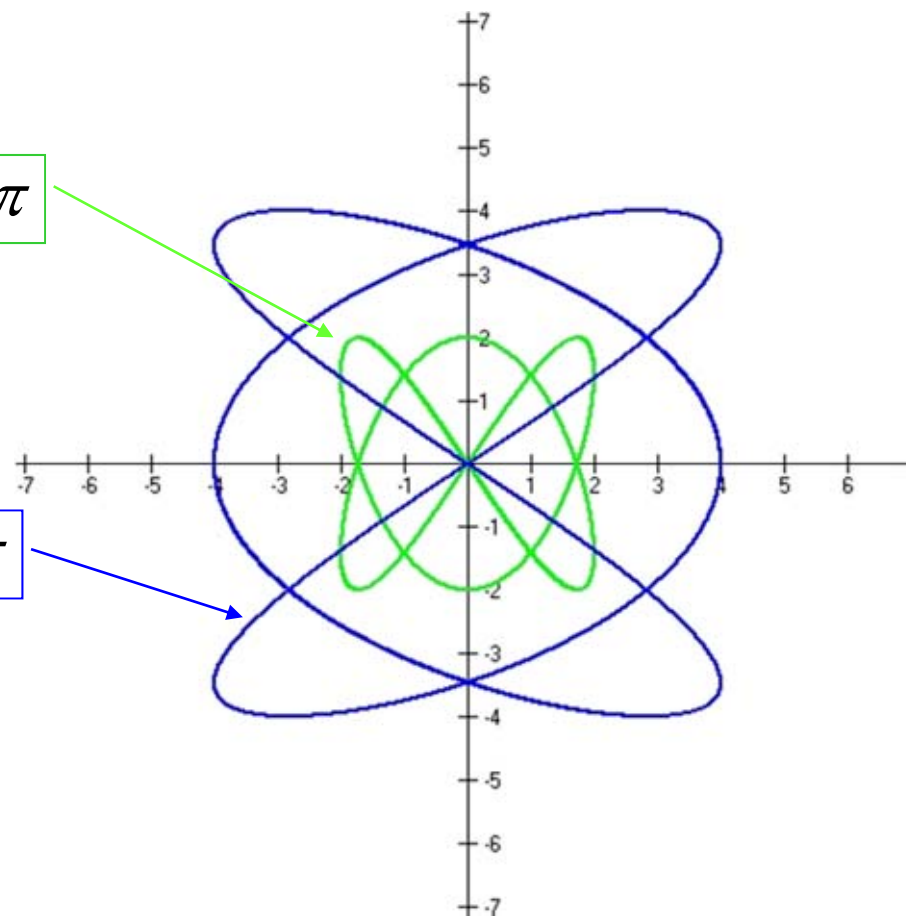
$$x = 4 \sin 2t, \quad y = 2 \sin 3t, \quad 0 \leq t \leq 2\pi$$



LINIJE (KRIVE) U RAVNI

$$x = 2 \sin 2t, \quad y = 2 \sin 3t, \quad 0 \leq t \leq 2\pi$$

$$x = 4 \sin 3t, \quad y = 4 \sin 2t, \quad 0 \leq t \leq 2\pi$$



LINIJE (KRIVE) U RAVNI

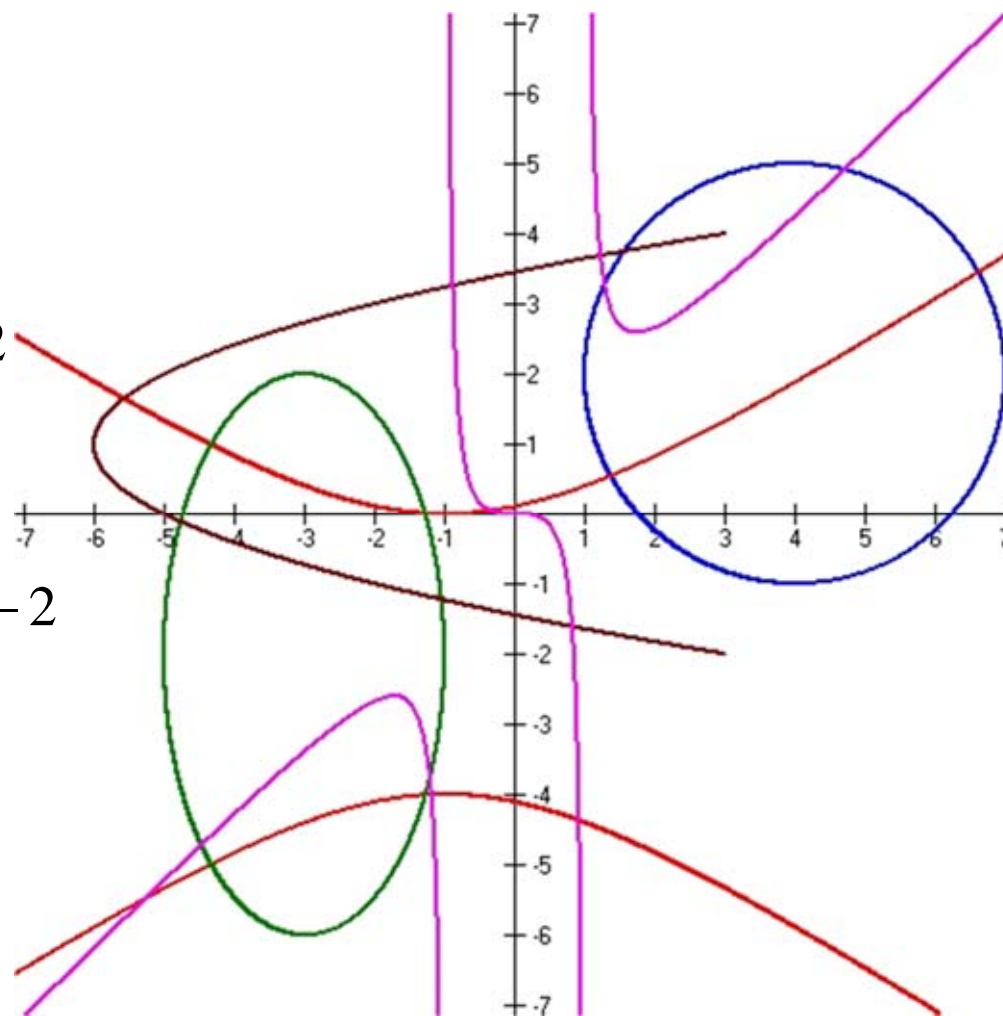
krug $x = 3 \cos t + 4, y = 3 \sin t + 2$

elipsa $x = 2 \cos t - 3, y = 4 \sin t - 2$

hiperbola $x = 3 \tan t - 1, y = 2 \sec t - 2$

parabola $x = t^2 - 6, y = t + 1$

funkcija $y = \frac{x^3}{x^2 - 1}$

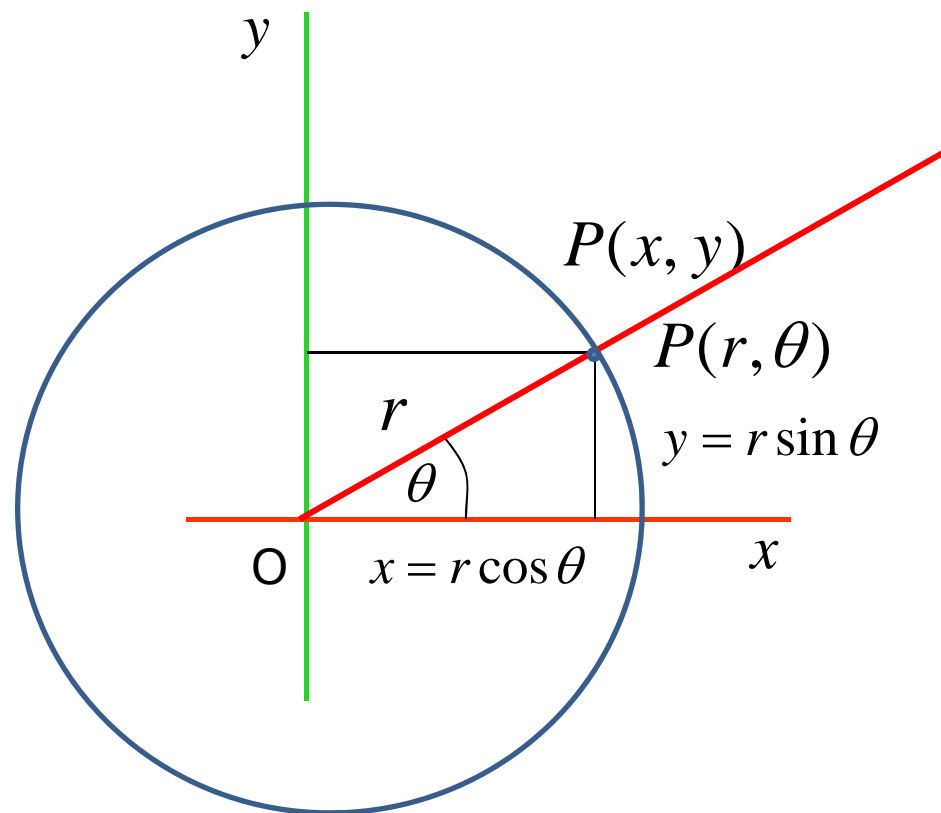


LINIJE (KRIVE) U RAVNI

Polarni koordinatni sistem

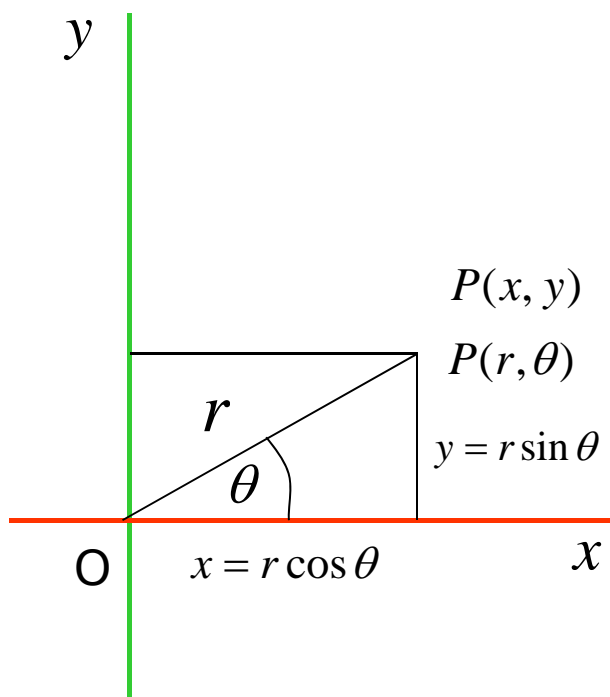
$$x = r \cos \theta$$

$$y = r \sin \theta$$



LINIJE (KRIVE) U RAVNI

Polarni koordinatni sistem



Odnos

Dekartovih i polarnih koordinata

$$x = r \cos \theta$$

$$y = r \sin \theta$$

Kriva u polarnim koordinatama

$$r = \varphi(\theta)$$

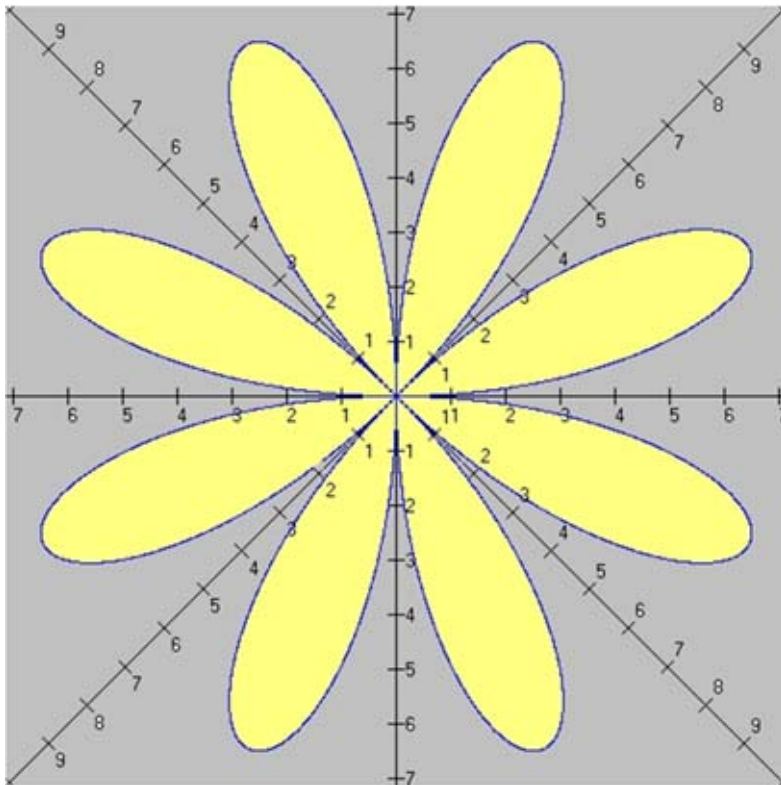
Prelazak na parametarski oblik

$$x = r \cos \theta = \varphi(\theta) \cos \theta$$

$$y = r \sin \theta = \varphi(\theta) \sin \theta$$

LINIJE (KRIVE) U RAVNI

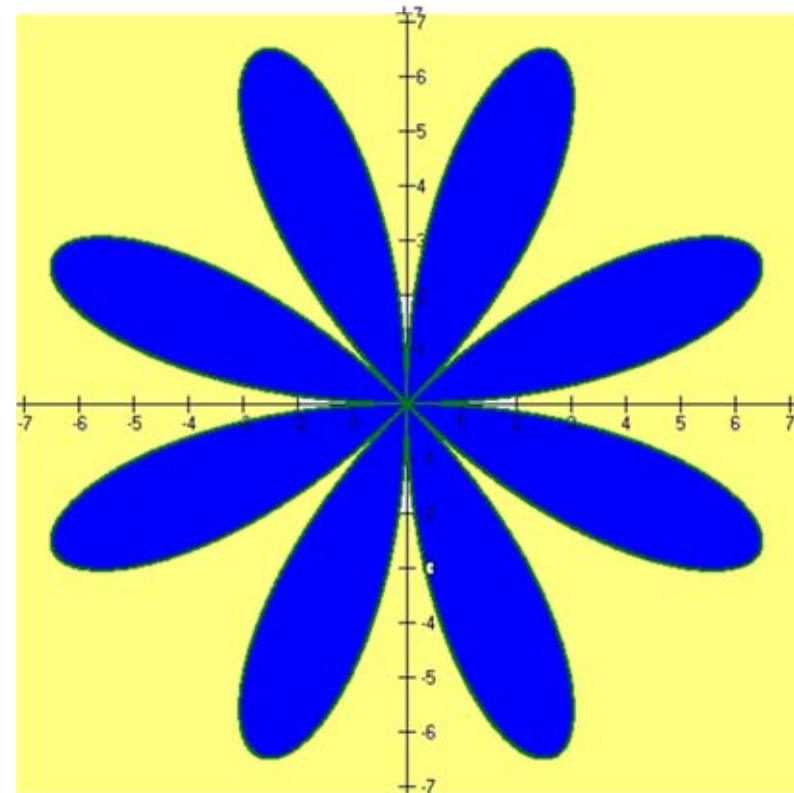
$$r = 7 \sin 4\theta, 0 \leq \theta \leq 2\pi$$



$$x = 7 \sin 4\theta \cos \theta$$

$$y = 7 \sin 4\theta \sin \theta$$

$$0 \leq \theta \leq 2\pi$$



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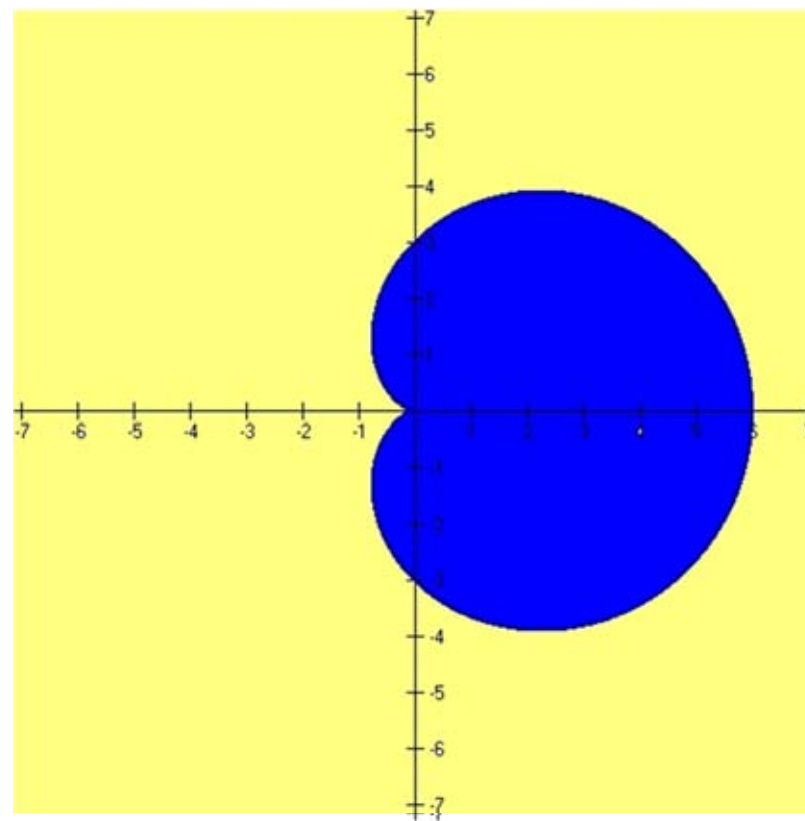
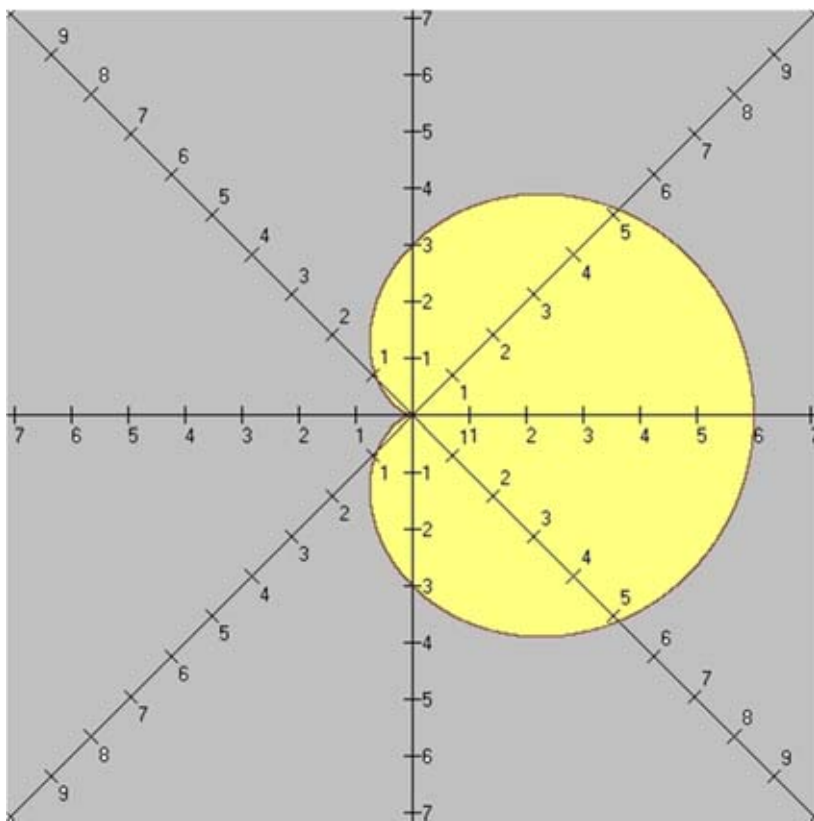
kardioida

$$r = 3(1 + \cos \theta), 0 \leq \theta \leq 2\pi$$

$$x = 3(1 + \cos t) \cos t$$

$$y = 3(1 + \cos t) \sin t$$

$$0 \leq t \leq 2\pi$$



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Arhimedova spirala

$$\underline{r = a\theta}$$

$$x = at \cos t$$

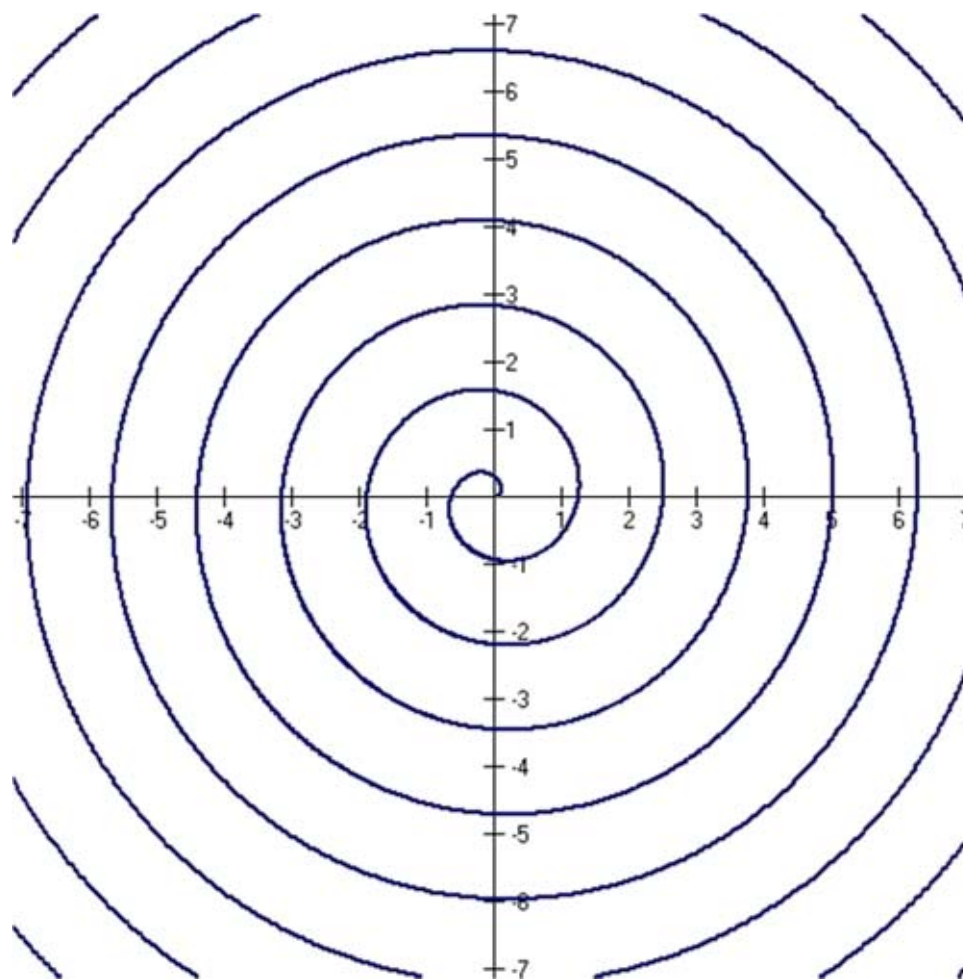
$$y = at \sin t$$

Grafički prikaz

$$x = 0.2t \cos t$$

$$y = 0.2t \sin t$$

$$0 \leq t \leq 20\pi$$



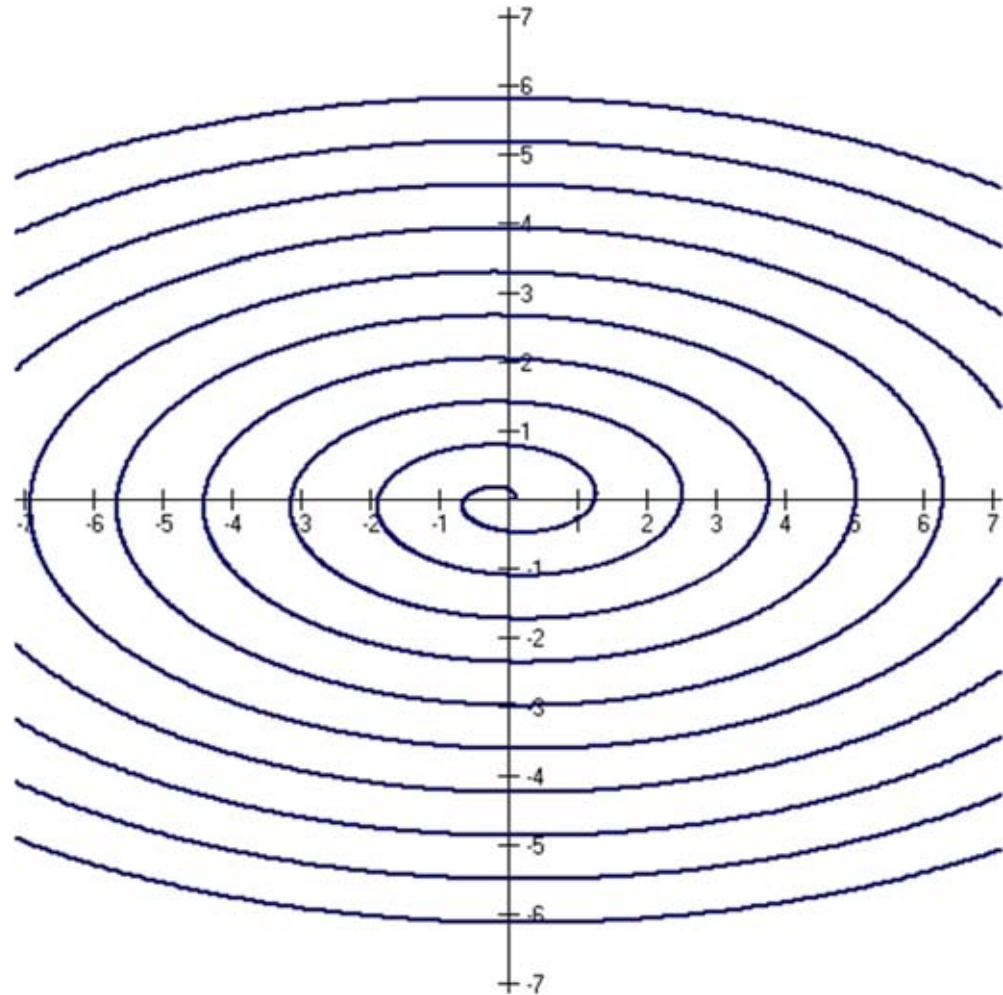
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Modifikovana
Arhimedova spirala

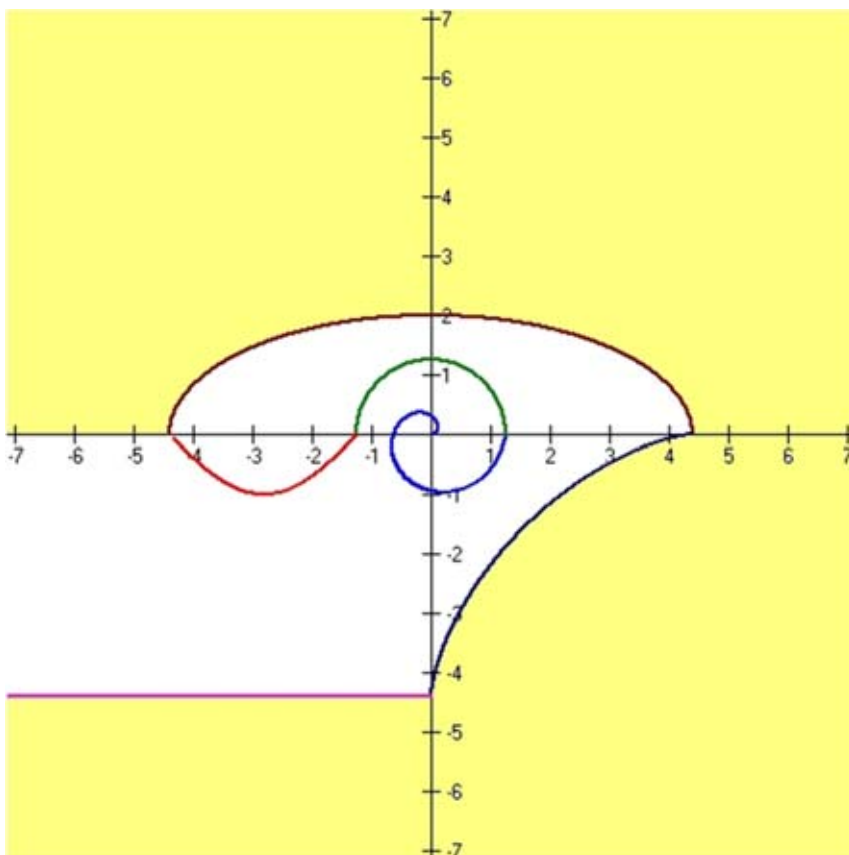
$$x = 0.2t \cos t$$

$$y = 0.1t \sin t$$

$$0 \leq t \leq 20\pi$$



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$$\begin{array}{l|l} x = 0.2t \cos t & x = 0.4\pi \cos t \\ y = 0.2t \sin t & y = 0.4\pi \sin t \\ 0 \leq t \leq 2\pi & 0 \leq t \leq \pi \end{array}$$

$$y = \sin(x + 0.4\pi), \quad -1.4\pi \leq x \leq -0.4\pi$$

$$\begin{array}{l|l} x = 1.4\pi \cos t & x = 1.4\pi \cos^3 t \\ y = 2 \sin t & y = 1.4\pi \sin^3 t \\ 0 \leq t \leq \pi & \frac{3\pi}{2} \leq t \leq 2\pi \end{array}$$

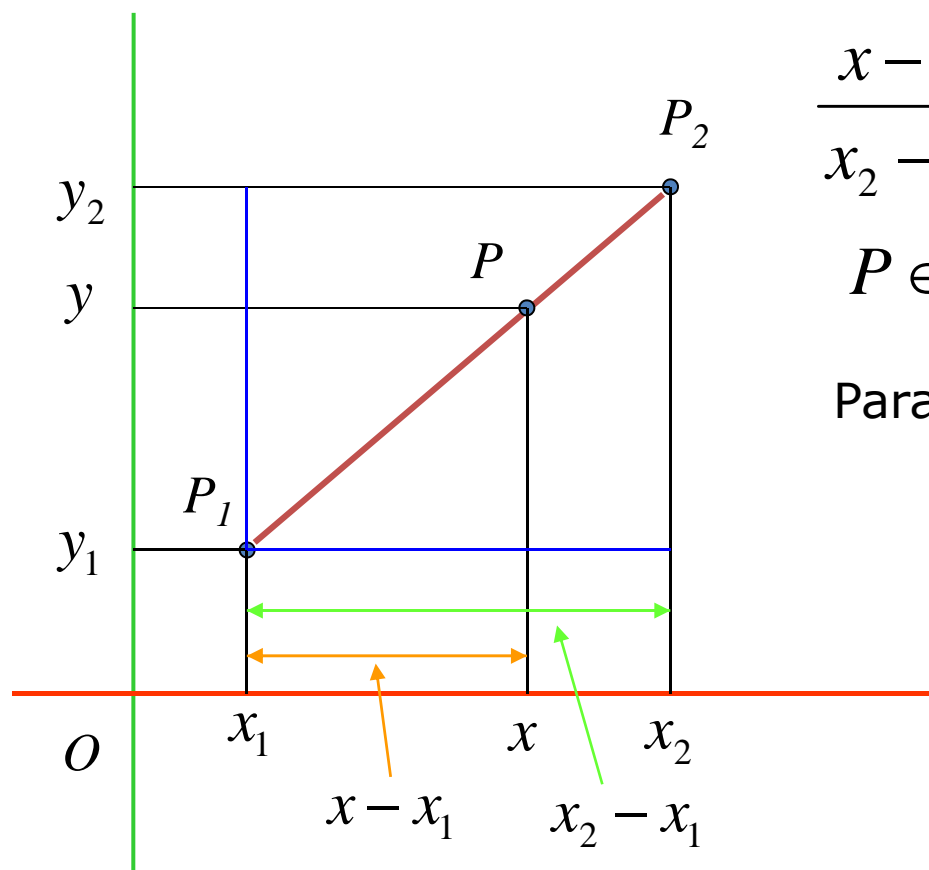
$$y = -1.4\pi, \quad -8 \leq x \leq 0$$

LINIJE U RAVNI

prava linija

$$P_1(x_1, y_1)$$

$$P_2(x_2, y_2)$$



$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{d(P_1, P)}{d(P_1, P_2)} = t$$

$$P \in P_1P_2 \Leftrightarrow 0 \leq t \leq 1$$

Parametarske jednačine duži P_1P_2

$$x = x_1 + t(x_2 - x_1)$$

$$y = y_1 + t(y_2 - y_1)$$

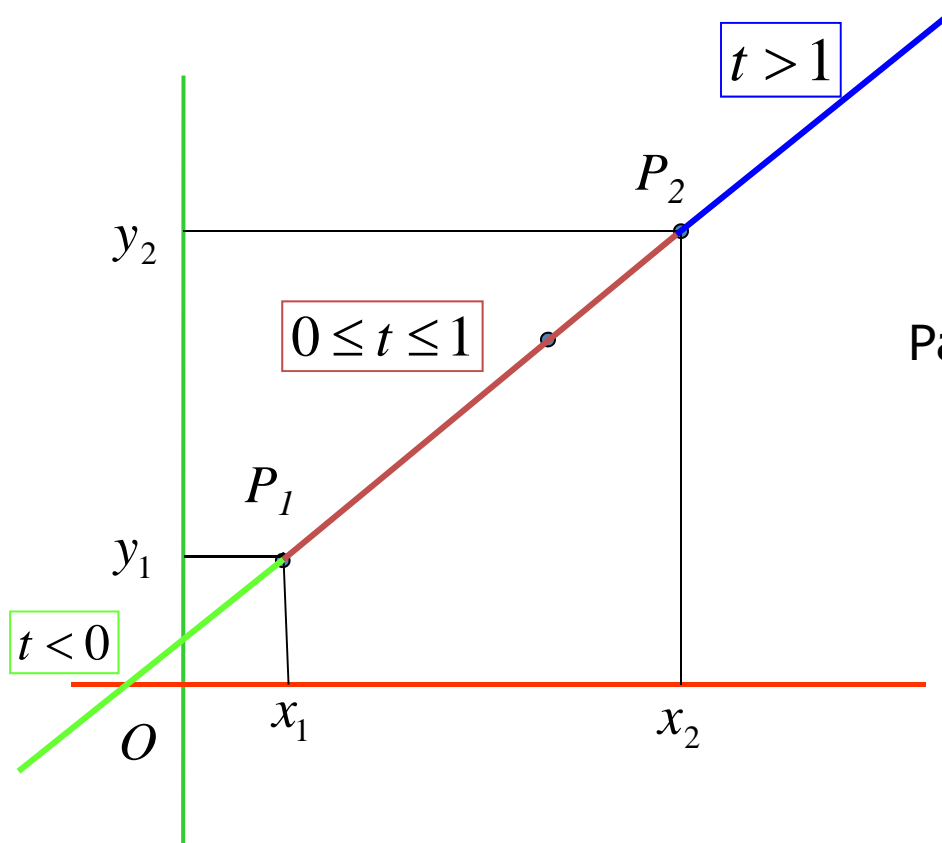
$$0 \leq t \leq 1$$

LINIJE U RAVNI

prava linija

$$P_1(x_1, y_1)$$

$$P_2(x_2, y_2)$$



$$\frac{d(P_1, P)}{d(P_1, P_2)} = t$$

Parametarske jednačine prave P_1P_2

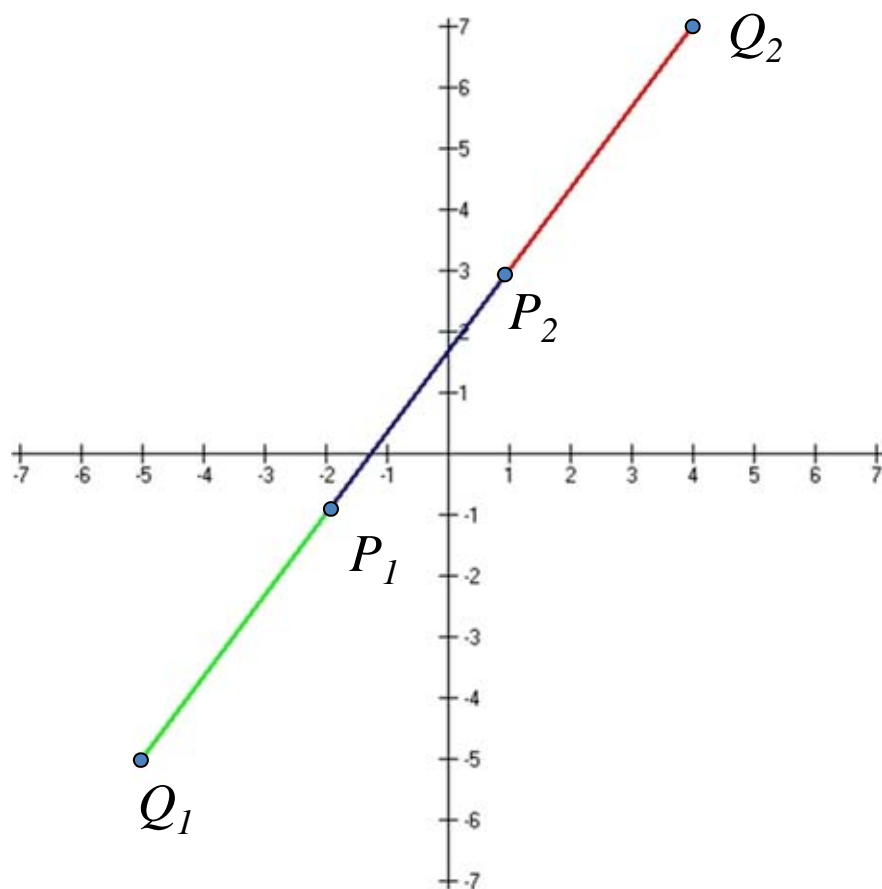
$$x = x_1 + t(x_2 - x_1)$$

$$y = y_1 + t(y_2 - y_1)$$

$$t \in \mathbb{R}$$

LINIJE U RAVNI

prava linija



$$x = -2 + 3t$$

$$y = -1 + 4t$$

$$P_1P_2 \text{ — } 0 \leq t \leq 1$$

$$P_2Q_2 \text{ — } 1 \leq t \leq 2$$

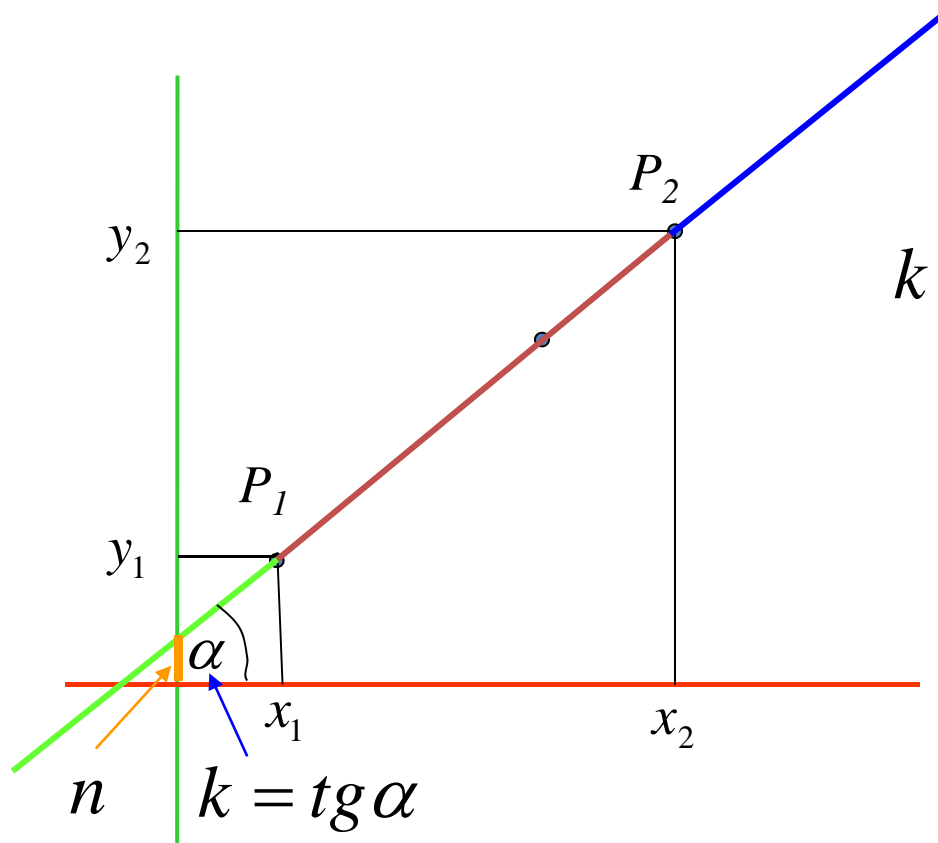
$$Q_1P_1 \text{ — } -1 \leq t \leq 0$$

$$P_1Q_2 \text{ — } 0 \leq t \leq 2$$

$$Q_1P_2 \text{ — } -1 \leq t \leq 1$$

LINIJE U RAVNI

prava linija



$$P_1(x_1, y_1)$$

$$P_2(x_2, y_2)$$

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}$$

$$k = \frac{y_2 - y_1}{x_2 - x_1} \quad n = y_1 - \frac{y_2 - y_1}{x_2 - x_1} x_1$$

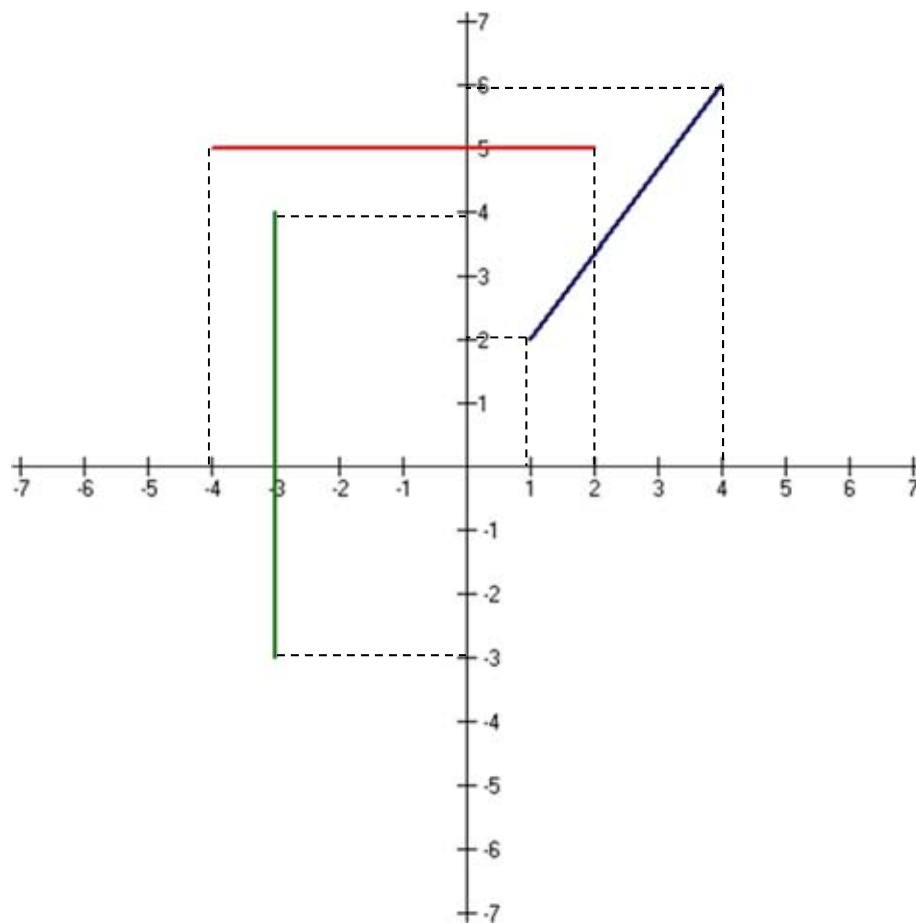
$$y = kx + n$$

$x \in \mathbb{R}$ — prava

$x_1 \leq x \leq x_2$ — duž P_1P_2

LINIJE U RAVNI

prava linija



$$x = x_1 + t(x_2 - x_1)$$

$$y = y_1 + t(y_2 - y_1)$$

$$0 \leq t \leq 1$$

— (1,2), (4,6) $x = 1 + 3t$
 $y = 2 + 4t$
 $0 \leq t \leq 1$

— (-4,5), (2,5) $x = -4 + 6t$
 $y = 5$
 $0 \leq t \leq 1$

— (-3,-3), (-3,4) $x = -3$
 $y = -3 + 7t$
 $0 \leq t \leq 1$