MATRICE

TRANSPONOVANA MATRICA

Transponovana matrica matrice $A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$

je matrica $A^T = \begin{bmatrix} a_{11} & a_{21} & \dots & a_{m1} \\ a_{12} & a_{22} & \dots & a_{m2} \\ \dots & \dots & \dots & \dots \\ a_{1n} & a_{2n} & \dots & a_{mn} \end{bmatrix}$

koja se dobija zamenom mesta vrsta i kolona date matrice A.

TRANSPONOVANA MATRICA - primer

Odrediti transponovane matrice matrica

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -5 & -6 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 4 \\ 2 & -5 \\ 3 & -6 \end{bmatrix} \qquad B^T = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad C^T = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

TRANSPONOVANA MATRICA

Osobine transponovane matrice:

$$(A+B)^T = A^T + B^T$$

$$\left(A^T\right)^T = A$$

$$(kA)^T = k\,A^T \qquad k \in R$$

$$(AB)^T = B^T A^T$$

INVERZNA MATRICA

Inverzna matrica kvadratne matrice *A* je matrica A^{-1}

takva da je

$$AA^{-1} = A^{-1}A = I$$

gde je *I* jedinična matrica.

Ukoliko postoji inverzna matrica, ona je jedinstvena.

INVERZNA MATRICA

Matrice

$$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} \quad \text{i} \quad \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$$

su uzajamno inverzne zato što je:

$$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

INVERZNA MATRICA

Osobine inverzne matrice:

$$\left(A^{-1}\right)^{-1} = A$$

$$\left(A^{T}\right)^{-1} = \left(A^{-1}\right)^{T}$$

$$\left(AB\right)^{-1} = B^{-1}A^{-1}$$

MATRICE

Determinanta matrice *A*

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}$$

$$\det A = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}$$

MATRICE - DETERMINANTE

minori M_{ij}

$(-1)^{i+j}$

Parna i neparna mesta

$$\begin{vmatrix} a_{11} & a_{12} & \dots & a_{1j} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2j} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{i1} & a_{i2} & \dots & a_{ij} & \dots & a_{in} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nj} & \dots & a_{nn} \end{vmatrix}$$

$$\begin{bmatrix} + & - & + & - & \dots \\ - & + & - & + & \dots \\ + & - & + & - & \\ \dots & \dots & \dots & \dots & \dots \end{bmatrix}$$

kofaktori $A_{ij} = (-1)^{i+j} M_{ij}$

MATRICE - DETERMINANTE

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix} \qquad \det A = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}$$

Determinanta matrice *A* je jednaka zbiru proizvoda svih elemenata neke vrste ili kolone i odgovarajućih kofaktora:

$$det(A) = \sum_{j=1}^{n} a_{ij}A_{ij} = a_{i1}A_{i1} + a_{i2}A_{i2} + \ldots + a_{in}A_{in}$$

$$det(A) = \sum_{i=1}^{n} a_{ij}A_{ij} = a_{1j}A_{1j} + a_{2j}A_{2j} + \ldots + a_{nj}A_{nj}$$

MATRICE – DETERMINANTE - primer

$$D = \begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix} = 2 \cdot 4 - 3 \cdot 1 = 5$$

$$D = \begin{vmatrix} -2 & 5 \\ 1 & 2 \end{vmatrix} = (-2) \cdot 2 - 5 \cdot 1 = -9$$

MATRICE – DETERMINANTE - primer

$$D=\begin{vmatrix}1 & -2 & 5\\ 1 & 2 & 3\\ 3 & 1 & 2\end{vmatrix}=\begin{vmatrix}2 & 3\\ 1 & 2\end{vmatrix}-(-2)\begin{vmatrix}1 & 3\\ 3 & 2\end{vmatrix}+5\begin{vmatrix}1 & 2\\ 3 & 1\end{vmatrix}$$

$$D=\left(2\cdot 2-3\cdot 1\right)+2\left(1\cdot 2-3\cdot 3\right)+5\left(1\cdot 1-2\cdot 3\right)=-38$$

MATRICE – ADJUNGOVANA MATRICA

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}$$

Transponovana matrica matrice odgovarajućih kofaktora

$$adj\,A = \begin{bmatrix} A_{11} & A_{21} & \dots & A_{n1} \\ A_{12} & A_{22} & \dots & A_{n2} \\ \dots & \dots & \dots & \dots \\ A_{1n} & A_{2n} & \dots & A_{nn} \end{bmatrix}$$

zove se ***adjungovana matrica*** matrice A.

MATRICE – ADJUNGOVANA MATRICA

Matrica $adj\,A$ se može formirati na dva načina:

1. Elementi matrice *A* se menjaju odgovarajućim kofaktorima i zatim se dobijena matrica transponuje.

2. U transponovanoj matrici A^T matrice *A* elementi se menjaju odgovarajućim kofaktorima.

MATRICE – ADJUNGOVANA MATRICA - primer

$$A = \begin{bmatrix} 2 & 3 & -4 \\ 0 & -4 & 2 \\ 1 & -1 & 5 \end{bmatrix}$$

$$A_{11} = +\begin{vmatrix} -4 & 2 \\ -1 & 5 \end{vmatrix} = -18 \qquad A_{12} = -\begin{vmatrix} 0 & 2 \\ 1 & 5 \end{vmatrix} = 2 \qquad A_{13} = +\begin{vmatrix} 0 & -4 \\ 1 & -1 \end{vmatrix} = 4$$

$$A_{21} = -\begin{vmatrix} 3 & -4 \\ -1 & 5 \end{vmatrix} = -11 \qquad A_{22} = +\begin{vmatrix} 2 & -4 \\ 1 & 5 \end{vmatrix} = 14 \qquad A_{23} = -\begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix} = 5$$

$$A_{31} = +\begin{vmatrix} 3 & -4 \\ -4 & 2 \end{vmatrix} = -10 \qquad A_{32} = -\begin{vmatrix} 2 & -4 \\ 0 & 2 \end{vmatrix} = -4 \qquad A_{33} = +\begin{vmatrix} 2 & 3 \\ 0 & -4 \end{vmatrix} = -8$$

$$adj\, A = \begin{bmatrix} -18 & -11 & -10 \\ 2 & 14 & -4 \\ 4 & 5 & -8 \end{bmatrix}$$

MATRICE – ADJUNGOVANA MATRICA - primer

$$A=\begin{bmatrix} 2 & 3 & -4 \\ 0 & -4 & 2 \\ 1 & -1 & 5 \end{bmatrix}$$

$$A^T=\begin{bmatrix} 2 & 0 & 1 \\ 3 & -4 & -1 \\ -4 & 2 & 5 \end{bmatrix}$$

$$A_{11}^T=+\begin{vmatrix} -4 & 2 \\ -1 & 5 \end{vmatrix}=-18 \qquad A_{12}^T=-\begin{vmatrix} 3 & -4 \\ -1 & 5 \end{vmatrix}=-11 \qquad A_{13}^T=+\begin{vmatrix} 3 & -4 \\ -4 & 2 \end{vmatrix}=-10$$

$$A_{21}^T=-\begin{vmatrix} 0 & 2 \\ 1 & 5 \end{vmatrix}=2 \qquad A_{22}^T=+\begin{vmatrix} 2 & -4 \\ 1 & 5 \end{vmatrix}=14 \qquad A_{23}^T=-\begin{vmatrix} 2 & -4 \\ 0 & 2 \end{vmatrix}=-4$$

$$A_{31}^T=+\begin{vmatrix} 0 & -4 \\ 1 & -1 \end{vmatrix}=4 \qquad A_{32}^T=-\begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix}=5 \qquad A_{33}^T=+\begin{vmatrix} 2 & 3 \\ 0 & -4 \end{vmatrix}=-8$$

$$adj\,A=\begin{bmatrix} -18 & -11 & -10 \\ 2 & 14 & -4 \\ 4 & 5 & -8 \end{bmatrix}$$

MATRICE – ADJUNGOVANA MATRICA - primer

$$A = \begin{bmatrix} 2 & 3 \\ 0 & -4 \end{bmatrix}$$

$$A_{11} = -4 \qquad A_{12} = 0$$

$$A_{21} = -3 \qquad A_{22} = 2$$

$$adj A = \begin{bmatrix} -4 & -3 \\ 0 & 2 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 2 & 0 \\ 3 & -4 \end{bmatrix}$$

$$A^T_{11} = -4 \qquad A^T_{12} = -3$$

$$A^T_{21} = 0 \qquad A^T_{22} = 2$$

$$adj A = \begin{bmatrix} -4 & -3 \\ 0 & 2 \end{bmatrix}$$

INVERZNA MATRICA

Za proizvoljnu kvadratnu matricu A važi matrična jednakost

$$A \cdot (adj\, A) = (adj\, A) \cdot A = det(A) \cdot I$$

gde je *I* jedinična matrica.

Za datu kvadratnu matricu *A* postoji inverzna matrica A^{-1}

ako i samo ako je $det(A) \neq 0$ (regularna matrica).

Tada je

$$A^{-1} = \frac{1}{\det(A)}(adj A)$$

INVERZNA MATRICA - primer

$$A = \begin{bmatrix} 2 & 3 & -4 \\ 0 & -4 & 2 \\ 1 & -1 & 5 \end{bmatrix} \qquad adj\, A = \begin{bmatrix} -18 & -11 & -10 \\ 2 & 14 & -4 \\ 4 & 5 & -8 \end{bmatrix}$$

$$\det(A) = \begin{vmatrix} 2 & 3 & -4 \\ 0 & -4 & 2 \\ 1 & -1 & 5 \end{vmatrix} = 2\cdot(-18) + 0\cdot(-11) + 1\cdot(-10) = -46$$

$$A^{-1} = \frac{1}{det(A)}\, adj\, A = \frac{1}{-46}\begin{bmatrix} -18 & -11 & -10 \\ 2 & 14 & -4 \\ 4 & 5 & -8 \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} \frac{9}{23} & \frac{11}{46} & \frac{5}{23} \\ -\frac{1}{23} & -\frac{7}{23} & \frac{2}{23} \\ -\frac{2}{23} & -\frac{5}{46} & \frac{4}{23} \end{bmatrix}$$

INVERZNA MATRICA - primer

$$A = \begin{bmatrix} 2 & 3 \\ 0 & -4 \end{bmatrix}$$

$$A_{11} = -4 \qquad A_{12} = 0$$

$$A_{21} = -3 \qquad A_{22} = 2$$

$$adj A = \begin{bmatrix} -4 & -3 \\ 0 & 2 \end{bmatrix} \quad \det A = \begin{vmatrix} 2 & 3 \\ 0 & -4 \end{vmatrix} = -8 \neq 0$$

$$A^{-1} = \frac{1}{-8} adj A = -\frac{1}{8}\begin{bmatrix} -4 & -3 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{3}{8} \\ 0 & -\frac{1}{4} \end{bmatrix}$$

MATRICE I SISTEMI JEDNAČINA

$$a_{11}\ x_1 + a_{12}\ x_2 + \ldots + a_{1n}\ x_n = b_1$$

$$a_{21}\ x_1 + a_{22}\ x_2 + \ldots + a_{2n}\ x_n = b_2$$

. . . .

$$a_{n1}\ x_1 + a_{n2}\ x_2 + \ldots + a_{nn}\ x_n = b_n$$

$$\begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \ldots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \ldots \\ b_n \end{bmatrix}$$

$$AX = B$$

Matrica sistema jednačina

$$A = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{bmatrix}$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \ldots \\ x_n \end{bmatrix} \qquad B = \begin{bmatrix} b_1 \\ b_2 \\ \ldots \\ b_n \end{bmatrix}$$

MATRICE I SISTEMI JEDNAČINA

$$AX = B$$

$$\begin{aligned} a_{11}\, x_1 + a_{12}\, x_2 + \ldots + a_{1n}\, x_n &= b_1 \\ a_{21}\, x_1 + a_{22}\, x_2 + \ldots + a_{2n}\, x_n &= b_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots&\ldots\ldots \\ a_{n1}\, x_1 + a_{n2}\, x_2 + \ldots + a_{nn}\, x_n &= b_n \end{aligned}$$

$$\begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \ldots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \ldots \\ b_n \end{bmatrix}$$

Matrica sistema jednačina

$$A = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{bmatrix}$$

Proširena matrica

$$A_{prosirena} = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} & b_1 \\ a_{21} & a_{22} & \ldots & a_{2n} & b_2 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ a_{m1} & a_{m2} & \ldots & a_{mn} & b_m \end{bmatrix}$$

MATRICE I SISTEMI JEDNAČINA - primer

$$2x + 3y - 4z = 7$$
$$x - 2y - 5z = 3$$

$$A = \begin{bmatrix} 2 & 3 & -4 \\ 1 & -2 & -5 \end{bmatrix} \qquad A_{prosirena} = \begin{bmatrix} 2 & 3 & -4 & 7 \\ 1 & -2 & -5 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 & -4 \\ 1 & -2 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \end{bmatrix} \qquad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad B = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$

$$AX = B$$

MATRICE I SISTEMI JEDNAČINA - primer

$$\begin{aligned} x + 2y - 3z &= 4 \\ x + 3y + z &= 11 \\ 2x + 5y - 4z &= 13 \\ 2x + 6y + 2z &= 22 \end{aligned}$$

$$\begin{bmatrix} 1 & 2 & -3 \\ 1 & 3 & 1 \\ 2 & 5 & -4 \\ 2 & 6 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 11 \\ 13 \\ 22 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & -3 \\ 1 & 3 & 1 \\ 2 & 5 & -4 \\ 2 & 6 & 2 \end{bmatrix} \qquad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad B = \begin{bmatrix} 4 \\ 11 \\ 13 \\ 22 \end{bmatrix} \qquad AX = B$$

MATRICE I SISTEMI JEDNAČINA - primer

$$
\begin{aligned}
x + 2y - 3z &= 4 \\
x + 3y + \ \ z &= 11 \\
2x + 5y - 4z &= 13 \\
2x + 6y + 2z &= 22
\end{aligned}
$$

$$A = \begin{bmatrix} 1 & 2 & -3 \\ 1 & 3 & 1 \\ 2 & 5 & -4 \\ 2 & 6 & 2 \end{bmatrix} \qquad A_{prosirena} = \begin{bmatrix} 1 & 2 & -3 & 4 \\ 1 & 3 & 1 & 11 \\ 2 & 5 & -4 & 13 \\ 2 & 6 & 2 & 22 \end{bmatrix}$$

MATRICE I SISTEMI JEDNAČINA – EKVIVALENTNE TRANSFORMACIJE

Sistem jednačina

$$
\begin{aligned}
a_{11}\ x_1 + a_{12}\ x_2 + \ldots + a_{1n}\ x_n &= b_1 \\
a_{21}\ x_1 + a_{22}\ x_2 + \ldots + a_{2n}\ x_n &= b_2 \\
\cdots\cdots\cdots\cdots\cdots&\cdots\cdots\cdots \\
a_{n1}\ x_1 + a_{n2}\ x_2 + \ldots + a_{nn}\ x_n &= b_n
\end{aligned}
$$

Proširena matrica sistema

$$
A_{prosirena} = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} & b_1 \\ a_{21} & a_{22} & \ldots & a_{2n} & b_2 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ a_{m1} & a_{m2} & \ldots & a_{mn} & b_m \end{bmatrix}
$$

Sistemi jednačina su **ekvivalentni** ako imaju ista rešenja.

Transformacija datog sistema linearnih jednačina u neki nov sistem linearnih jednačina je ***ekvivalentna transformacija*** ako je tom transformacijom dobijen sistem ekvivalentan datom sistemu jednačina.

MATRICE I SISTEMI JEDNAČINA – EKVIVALENTNE TRANSFORMACIJE

Sistem jednačina

$$a_{11}\ x_1 + a_{12}\ x_2 + \ldots + a_{1n}\ x_n = b_1$$

$$a_{21}\ x_1 + a_{22}\ x_2 + \ldots + a_{2n}\ x_n = b_2$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$a_{n1}\ x_1 + a_{n2}\ x_2 + \ldots + a_{nn}\ x_n = b_n$$

Proširena matrica sistema

$$A_{prosirena} = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} & b_1 \\ a_{21} & a_{22} & \ldots & a_{2n} & b_2 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ a_{m1} & a_{m2} & \ldots & a_{mn} & b_m \end{bmatrix}$$

1) medjusobna zamena mesta jednačina L_i i L_j

$$L_i \leftrightarrow L_j$$

1) medjusobna zamena vrsta proširene matrice L_i i L_j

MATRICE I SISTEMI JEDNAČINA – EKVIVALENTNE TRANSFORMACIJE

Sistem jednačina

$$a_{11}\ x_1 + a_{12}\ x_2 + \ldots + a_{1n}\ x_n = b_1$$

$$a_{21}\ x_1 + a_{22}\ x_2 + \ldots + a_{2n}\ x_n = b_2$$

. . . .

$$a_{n1}\ x_1 + a_{n2}\ x_2 + \ldots + a_{nn}\ x_n = b_n$$

Proširena matrica sistema

$$A_{prosirena} = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} & b_1 \\ a_{21} & a_{22} & \ldots & a_{2n} & b_2 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ a_{m1} & a_{m2} & \ldots & a_{mn} & b_m \end{bmatrix}$$

2) množenje bilo koje jednačine proizvoljnim brojem $\lambda \neq 0$

$$L_i \to \lambda L_i$$

2) Množenje bilo koje vrste proširene matrice proizvoljnim brojem $\lambda \neq 0$

MATRICE I SISTEMI JEDNAČINA – EKVIVALENTNE TRANSFORMACIJE

Sistem jednačina

$$a_{11}\ x_1 + a_{12}\ x_2 + \ldots + a_{1n}\ x_n = b_1$$
$$a_{21}\ x_1 + a_{22}\ x_2 + \ldots + a_{2n}\ x_n = b_2$$
$$. \ .$$
$$a_{n1}\ x_1 + a_{n2}\ x_2 + \ldots + a_{nn}\ x_n = b_n$$

Proširena matrica sistema

$$A_{prosirena} = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} & b_1 \\ a_{21} & a_{22} & \ldots & a_{2n} & b_2 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ a_{m1} & a_{m2} & \ldots & a_{mn} & b_m \end{bmatrix}$$

3) sabiranje proizvoljne jednačine i neke druge jednačine, prethodno pomnožene proizvoljnim realnim brojem $\lambda \in R$

$$L_i \to L_i + \lambda L_j$$

3) sabiranje proizvoljne vrste proširene matrice i neke druge njene vrste, prethodno pomnožene proizvoljnim realnim brojem

MATRICE I SISTEMI JEDNAČINA – EKVIVALENTNE TRANSFORMACIJE

Sistem jednačina

$$\begin{aligned} a_{11}\ x_1 + a_{12}\ x_2 + \ldots + a_{1n}\ x_n &= b_1 \\ a_{21}\ x_1 + a_{22}\ x_2 + \ldots + a_{2n}\ x_n &= b_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{n1}\ x_1 + a_{n2}\ x_2 + \ldots + a_{nn}\ x_n &= b_n \end{aligned}$$

Proširena matrica sistema

$$A_{prosirena} = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} & b_1 \\ a_{21} & a_{22} & \ldots & a_{2n} & b_2 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ a_{m1} & a_{m2} & \ldots & a_{mn} & b_m \end{bmatrix}$$

4) Sabiranje dve proizvoljne jednačine prethodno pomnožene proizvoljnim realnim brojevima $\lambda_i, \lambda_j \in R, \lambda_i \neq 0$

$$L_i \to \lambda_i L_i + \lambda_j L_j$$

4) Sabiranje dve proizvoljne vrste proširene matrice, prethodno pomnožene proizvoljnim realnim brojevima $\lambda_i, \lambda_j \in R, \lambda_i \neq 0$

GAUSOV POSTUPAK ZA REŠAVANJE SISTEMA JEDNAČINA - MATRIČNI PRIKAZ

Algoritam primene ekvivalentnih transformacija u okviru Gausovog postupka izvodi se u dva koraka.

1. Jednačine ili vrste proširene matrice medjusobno menjaju mesta tako da je element u novoj matrici $a_{11} \neq 0$

2. Za $\forall i \in N, \quad 1 < i \leq m$ vrši se transformacija

$$L_i \rightarrow a_{11} L_i - a_{i1} L_1$$

Potom se prva jednačina prepisuje i postupak se ponavlja Nad preostalim jednačinama.

GAUSOV POSTUPAK ZA REŠAVANJE SISTEMA JEDNAČINA - MATRIČNI PRIKAZ - primer

$$\begin{array}{cccc|l}
x & +2y & -3z & =4 & L_1 \to L_1 \\
x & +3y & +z & =11 & L_2 \to L_2 - L_1 \\
2x & +5y & -4z & =13 & L_3 \to L_3 - 2L_1 \\
2x & +6y & +2z & 22 & L_4 \to L_4 - 2L_1
\end{array}$$

$$\begin{array}{cccc}
x & +2y & -3z & =4 \\
 & y & +4z & =7 \\
 & y & +2z & =5 \\
 & 2y & +8z & =14
\end{array}$$

GAUSOV POSTUPAK ZA REŠAVANJE SISTEMA JEDNAČINA - MATRIČNI PRIKAZ - primer

$$
\begin{array}{llll}
x & +2y & -3z & =4 \\
 & y & +4z & =7 \\
 & y & +2z & =5 \\
 & 2y & +8z & =14
\end{array}
\qquad
\begin{array}{l}
L_1 \to L_1 \\
L_2 \to L_2 \\
L_3 \to L_3 - L_2 \\
L_4 \to L_4 - 2L_2
\end{array}
$$

$$
\begin{array}{llll}
x & +2y & -3z & =4 \\
 & y & +4z & =7 \\
 & & -2z & =-2 \\
 & & 0 & =0
\end{array}
\qquad \uparrow \qquad
\begin{array}{l}
z=1 \\
y=7-4z=3 \\
x=4+3z-2y=1
\end{array}
$$

,

GAUSOV POSTUPAK ZA REŠAVANJE SISTEMA JEDNAČINA - MATRIČNI PRIKAZ - primer

,

$$x + 2y - 3z = 4$$

$$x + 3y + z = 11$$

$$2x + 5y - 4z = 13$$

$$2x + 6y + 2z = 22$$

$$A_{prosirena} = \begin{bmatrix} 1 & 2 & -3 & 4 \\ 1 & 3 & 1 & 11 \\ 2 & 5 & -4 & 13 \\ 2 & 6 & 2 & 22 \end{bmatrix} \qquad \begin{array}{l} L_1 \to L_1 \\ L_2 \to L_2 - L_1 \\ L_3 \to L_3 - 2L_1 \\ L_4 \to L_4 - 2L_1 \end{array} \qquad \begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & 1 & 4 & 7 \\ 0 & 1 & 2 & 5 \\ 0 & 2 & 8 & 14 \end{bmatrix}$$

GAUSOV POSTUPAK ZA REŠAVANJE SISTEMA JEDNAČINA - MATRIČNI PRIKAZ - primer

$$\begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & 1 & 4 & 7 \\ 0 & 1 & 2 & 5 \\ 0 & 2 & 8 & 14 \end{bmatrix} \quad \begin{matrix} L_1 \to L_1 \\ L_2 \to L_2 \\ L_3 \to L_3 - L_2 \\ L_4 \to L_4 - 2L_2 \end{matrix} \quad \begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & 1 & 4 & 7 \\ 0 & 0 & -2 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

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$$A_{prosirena}=\begin{bmatrix}1 & 2 & -3 & 4\\ 0 & 1 & 4 & 7\\ 0 & 0 & -2 & -2\\ 0 & 0 & 0 & 0\end{bmatrix}\qquad A=\begin{bmatrix}1 & 2 & -3\\ 0 & 1 & 4\\ 0 & 0 & -2\\ 0 & 0 & 0\end{bmatrix}\qquad X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}\qquad B=\begin{bmatrix}4\\ 11\\ 13\\ 22\end{bmatrix}$$

$$\begin{bmatrix}1 & 2 & -3\\ 0 & 1 & 4\\ 0 & 0 & -2\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}4\\ 11\\ 13\\ 22\end{bmatrix}$$

$$\begin{aligned}x+2y-3z&=4\\ y+4z&=7\\ -2z&=-2\\ 0&=0\end{aligned}$$

$$A_{prosirena} = \begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & 1 & 4 & 7 \\ 0 & 0 & -2 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 1 & 4 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} x + 2y - 3z &= 4 \\ y + 4z &= 7 \\ -2z &= -2 \\ 0 &= 0 \end{aligned}$$

$$z = 1$$

$$y = 7 - 4z = 3$$

$$x = 4 + 3z - 2y = 1$$

INVERZNA MATRICA - izračunavanje

$$a_{11}\ x_1 + a_{12}\ x_2 + \ldots + a_{1n}\ x_n = b_1$$

$$a_{21}\ x_1 + a_{22}\ x_2 + \ldots + a_{2n}\ x_n = b_2$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$a_{n1}\ x_1 + a_{n2}\ x_2 + \ldots + a_{nn}\ x_n = b_n$$

$$\begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \ldots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \ldots \\ b_n \end{bmatrix}$$

$$AX = B$$

Matrica sistema jednačina

$$A = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{bmatrix}$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \ldots \\ x_n \end{bmatrix} \qquad B = \begin{bmatrix} b_1 \\ b_2 \\ \ldots \\ b_n \end{bmatrix}$$

INVERZNA MATRICA - izračunavanje

$$AX = B$$

$$A^{-1}(AX) = A^{-1}B$$

$$(A^{-1}A)\,X = A^{-1}B$$

$$I\,X = A^{-1}B$$

$$X = A^{-1}B$$

Ekvivalentne transformacije

$$A\,X = I\,B \xrightarrow{\text{Ekvivalentne transformacije}} I\,X = A^{-1}B$$

$$\left[\begin{array}{cccc|cccc} a_{11} & a_{21} & \dots & a_{n1} & 1 & 0 & \dots & 0 \\ a_{12} & a_{22} & \dots & a_{n2} & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & 0 \\ a_{1n} & a_{2n} & \dots & a_{nn} & 0 & 0 & \dots & 1 \end{array}\right]$$

Ekvivalentne transformacije

$$\left[\begin{array}{cccc|cccc} 1 & 0 & \dots & 0 & c_{11} & c_{21} & \dots & c_{n1} \\ 0 & 1 & \dots & 0 & c_{12} & c_{22} & \dots & c_{n2} \\ \dots & \dots & \dots & 0 & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 & c_{1n} & c_{2n} & \dots & c_{nn} \end{array}\right]$$

Ekvivalentne transformacije

1) medjusobna zamena vrsta proširene matrice L_i i L_j

$$L_i \leftrightarrow L_j$$

2) Množenje bilo koje vrste proširene matrice proizvoljnim brojem $\lambda \neq 0$

$$L_i \rightarrow \lambda L_i$$

3) sabiranje proizvoljne vrste proširene matrice i neke druge njene vrste, prethodno pomnožene proizvoljnim realnim brojem

$$L_i \rightarrow L_i + \lambda L_j$$

4) Sabiranje dve proizvoljne vrste proširene matrice, prethodno pomnožene proizvoljnim realnim brojevima $\lambda_i, \lambda_j \in R, \lambda_i \neq 0$

$$L_i \rightarrow \lambda_i L_i + \lambda_j L_j$$

INVERZNA MATRICA - izračunavanje

Ekvivalentne transformacije

$$\left[\begin{array}{cccc|cccc} a_{11} & a_{21} & \dots & a_{n1} & 1 & 0 & \dots & 0 \\ a_{12} & a_{22} & \dots & a_{n2} & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & 0 \\ a_{1n} & a_{2n} & \dots & a_{nn} & 0 & 0 & \dots & 1 \end{array}\right] \longrightarrow \left[\begin{array}{cccc|cccc} 1 & 0 & \dots & 0 & c_{11} & c_{21} & \dots & c_{n1} \\ 0 & 1 & \dots & 0 & c_{12} & c_{22} & \dots & c_{n2} \\ \dots & \dots & \dots & 0 & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 & c_{1n} & c_{2n} & \dots & c_{nn} \end{array}\right]$$

$$A^{-1} = C = \begin{bmatrix} c_{11} & c_{21} & \dots & c_{n1} \\ c_{12} & c_{22} & \dots & c_{n2} \\ \dots & \dots & \dots & \dots \\ c_{1n} & c_{2n} & \dots & c_{nn} \end{bmatrix}$$

INVERZNA MATRICA - primer

$$A=\begin{bmatrix}1&1&1\\1&-1&2\\2&1&-1\end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc}1&1&1&1&0&0\\1&-1&2&0&1&0\\2&1&-1&0&0&1\end{array}\right]\xrightarrow{\substack{L_1\to L_1\\ L_2\to L_2-L_1\\ L_3\to L_3-2L_1}}\left[\begin{array}{ccc|ccc}1&1&1&1&0&0\\0&-2&1&-1&1&0\\0&-1&-3&-2&0&1\end{array}\right]\xrightarrow{\substack{L_1\to L_1\\ L_2\to L_2\\ L_3\to -2L_3+L_2}}\left[\begin{array}{ccc|ccc}1&1&1&1&0&0\\0&-2&1&-1&1&0\\0&0&7&3&1&-2\end{array}\right]\to$$

$$\xrightarrow{\substack{L_1\to 7L_1-L_3\\ L_2\to 7L_2-L_3\\ L_3\to L_3}}\left[\begin{array}{ccc|ccc}7&7&0&4&-1&2\\0&-14&0&-10&6&2\\0&0&7&3&1&-2\end{array}\right]\xrightarrow{\substack{L_1\to 2L_1+L_2\\ L_2\to L_2\\ L_3\to L_3}}\left[\begin{array}{ccc|ccc}14&0&0&-2&4&6\\0&-14&0&-10&6&2\\0&0&7&3&1&-2\end{array}\right]\xrightarrow{\substack{L_1\to \frac{1}{14}L_1\\ L_2\to -\frac{1}{14}L_2\\ L_3\to \frac{1}{7}L_3}}$$

$$\longrightarrow\left[\begin{array}{ccc|ccc}1&0&0&-\frac{1}{7}&\frac{2}{7}&\frac{3}{7}\\0&1&0&\frac{5}{7}&-\frac{3}{7}&-\frac{1}{7}\\0&0&1&\frac{3}{7}&\frac{1}{7}&-\frac{2}{7}\end{array}\right]$$

$$A^{-1}=\begin{bmatrix}-\frac{1}{7}&\frac{2}{7}&\frac{3}{7}\\\frac{5}{7}&-\frac{3}{7}&-\frac{1}{7}\\\frac{3}{7}&\frac{1}{7}&-\frac{2}{7}\end{bmatrix}$$