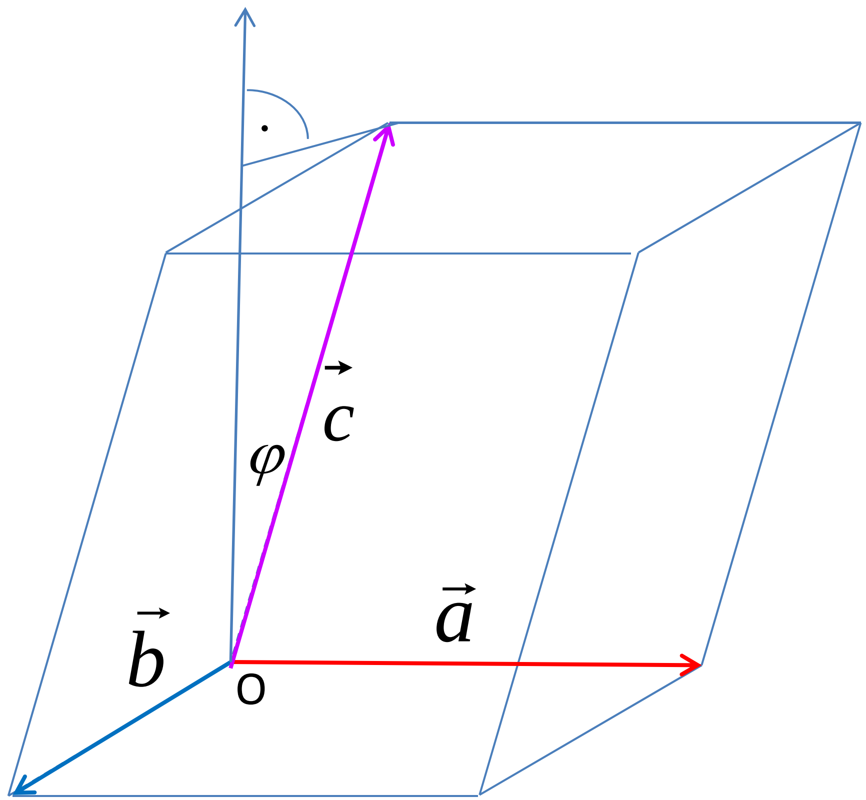
MEŠOVITI PROIZVOD VEKTORA



Mašoviti proizvod tri vektora

$\vec{a}, \vec{b}, \vec{c}$

je skalarni proizvod

vektora $\vec{a} \times \vec{b}$ i vektora $\vec{c}$:

$$[\vec{a}, \vec{b}, \vec{c}] = (\vec{a} \times \vec{b}) \cdot \vec{c}$$

$$\left[\vec{a}, \vec{b}, \vec{c}\right] = \left(\vec{a} \times \vec{b}\right) \cdot \vec{c} = \left|\vec{a} \times \vec{b}\right| \cdot \left|\vec{c}\right| \cdot \cos\varphi = B \cdot \left(\pm H\right) = \pm V$$

MEŠOVITI PROIZVOD VEKTORA

Mešoviti prozvod tri vektora $\vec{a}, \vec{b}, \vec{c}$ je skalar,
čija je apsolutna vrednost jednaka zapremini paralelopipeda konstruisanog nad tim vektoraima.

Znak mešovitog proizvoda označava orjentaciju trijedra koji grade ti vektori u odnosu na orjentaciju koordinatnog sistema. Pozitivan rezultat označava da je trijedar orjentisan isto kao i koordinatni sistem, a negativan rezultat označava da je njegova orjentacija suprotna od orjentacije koordinatnog sistema.

MEŠOVITI PROIZVOD VEKTORA

Cikličkom permutacijom vektora, vrednost mešovitog proizvoda se ne menja.

$$[\vec{a},\vec{b},\vec{c}]=[\vec{b},\vec{c},\vec{a}]=[\vec{c},\vec{a},\vec{b}]$$

$$(\vec{a}\times\vec{b})\cdot\vec{c}=(\vec{b}\times\vec{c})\cdot\vec{a}=(\vec{c}\times\vec{a})\cdot\vec{b}$$

MEŠOVITI PROIZVOD VEKTORA

$$\vec{a} = \left(a_x, a_y, a_z\right)$$

$$\vec{b} = \left(b_x, b_y, b_z\right)$$

$$\vec{c} = \left(c_x, c_y, c_z\right)$$

$$[\vec{a}, \vec{b}, \vec{c}] = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}$$

MEŠOVITI PROIZVOD VEKTORA – USLOV KOMPLANARNOSTI

Tri vektora $\vec{a}, \vec{b}, \vec{c}$ su komplanarni ako i samo ako je njihov mešoviti proizvod jednak nuli.

USLOV KOMPLANARNOSTI TRI VEKTORA

$$\vec{a} = (a_x, a_y, a_z)$$

$$\vec{b} = (b_x, b_y, b_z)$$

$$\vec{c} = (c_x, c_y, c_z)$$

$$\boxed{[\vec{a}, \vec{b}, \vec{c}] = (\vec{a} \times \vec{b}) \cdot \vec{c} = 0}$$

$$[\vec{a}, \vec{b}, \vec{c}] = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix} = 0$$

PRIMER

Vektori $\vec{a}=(1,-2,1)$ $\vec{b}=(2,2,-2)$ $\vec{c}=(0,2,-1)$ $[\vec{a},\vec{b},\vec{c}]=2$

su nekomplanarni zato sto je

$$[\vec{a},\vec{b},\vec{c}]=\begin{vmatrix}1 & -2 & 1\\ 2 & 2 & -2\\ 0 & 2 & -1\end{vmatrix}=-2\cdot\begin{vmatrix}1 & 1\\ 2 & -2\end{vmatrix}-\begin{vmatrix}1 & -2\\ 2 & 2\end{vmatrix}=8-6=2\neq 0$$

Vektori $\vec{a}=(1,-2,3)$ $\vec{b}=(2,3,-2)$ $\vec{c}=(3,1,1)$

su komplanarni zato sto je

$$[\vec{a},\vec{b},\vec{c}]=(\vec{a}\times\vec{b})\cdot\vec{c}=\begin{vmatrix}1 & -2 & 3\\ 2 & 3 & -2\\ 3 & 1 & 1\end{vmatrix}=1\cdot\begin{vmatrix}3 & -2\\ 1 & 1\end{vmatrix}+2\cdot\begin{vmatrix}2 & -2\\ 3 & 1\end{vmatrix}+3\cdot\begin{vmatrix}2 & 3\\ 3 & 1\end{vmatrix}$$

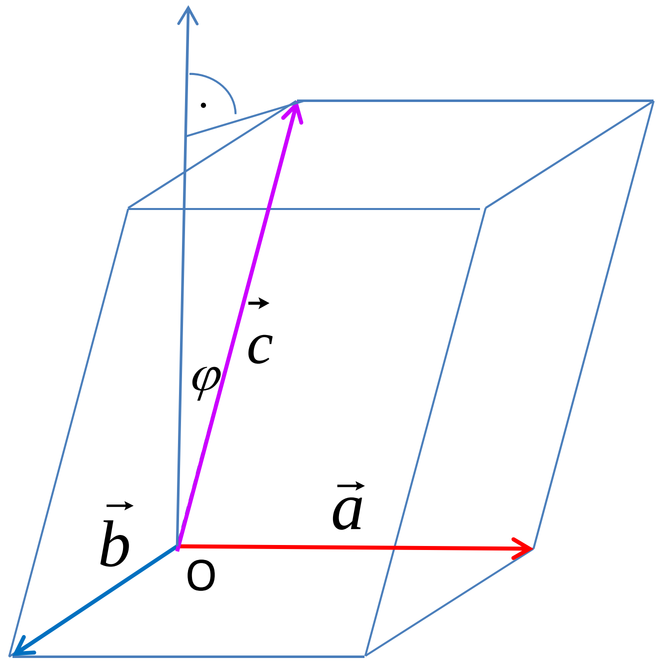
$$[\vec{a},\vec{b},\vec{c}]=5+16-21=0$$

PRIMER

Izračunati zapreminu paralelopipeda konstruisanog nad vektorima

$$\vec{a} = (1,-1,2) \qquad \vec{b} = (-1,1,0) \qquad \vec{c} = (1,2,0)$$

i odrediti orjentaciju u odnosu na orjentaciju koordinatnog sistema.



$$[\vec{a},\vec{b},\vec{c}] = (\vec{a}\times\vec{b})\cdot\vec{c} = \begin{vmatrix} 1 & -1 & 2 \\ -1 & 1 & 0 \\ 1 & 2 & 0 \end{vmatrix}$$

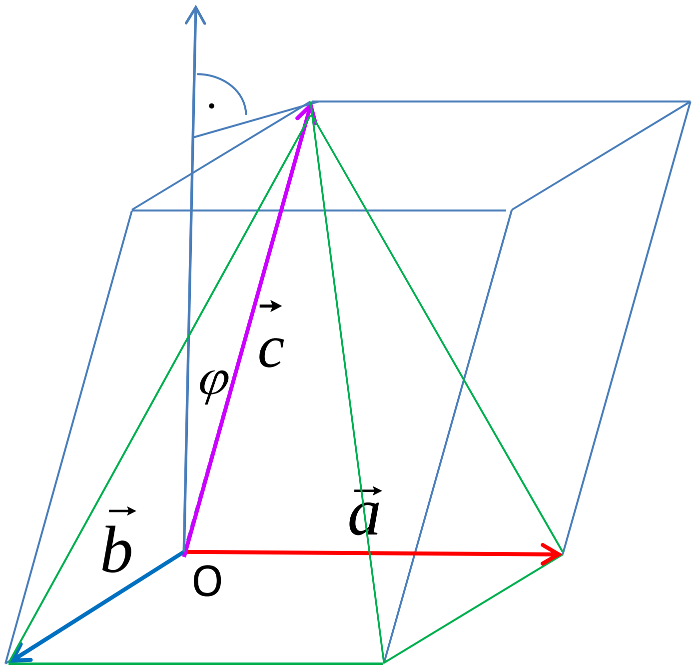
$$[\vec{a},\vec{b},\vec{c}] = 2\cdot\begin{vmatrix} -1 & 1 \\ 1 & 2 \end{vmatrix} = 2\cdot(-3) = -6$$

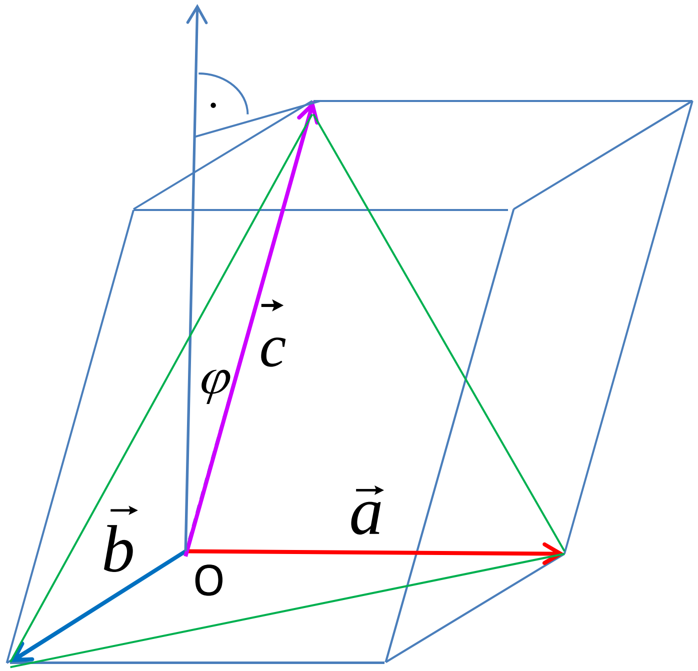
$$V = \left|[\vec{a},\vec{b},\vec{c}]\right| = |-6| = 6$$

$[\vec{a},\vec{b},\vec{c}] = -6 < 0$ Trijedar $\vec{a},\vec{b},\vec{c}$ suprotne orjentacije u odnosu na koordinatni sistem.

PRIMER

$$V_{paralelopipeda} = \left|[\vec{a}, \vec{b}, \vec{c}]\right| \qquad V_{paralelopipeda} = BH$$





$$V_{piramide} = \frac{1}{3}BH = \frac{1}{3}V_{paralelopipeda} = \frac{1}{3}\left|[\vec{a}, \vec{b}, \vec{c}]\right|$$

$$V_{tetraedra} = \frac{1}{3}B_1 H = \frac{1}{3}\frac{1}{2}BH = \frac{1}{6}V_{paralelopipeda} = \frac{1}{6}\left|[\vec{a}, \vec{b}, \vec{c}]\right|$$

PRIMER

$$\vec{a} = (1, -1, 2) \qquad \vec{b} = (-1, 1, 0) \qquad \vec{c} = (1, 2, 0)$$

$$V_{paralelopipeda} = \left|[\vec{a}, \vec{b}, \vec{c}]\right| = |-6| = 6$$

$$V_{piramide} = \frac{1}{3} V_{paralelopipeda} = \frac{1}{3}\left|[\vec{a}, \vec{b}, \vec{c}]\right| = \frac{1}{3} \cdot 6 = 2$$

$$V_{tetraedra} = \frac{1}{6} V_{paralelopipeda} = \frac{1}{6}\left|[\vec{a}, \vec{b}, \vec{c}]\right| = \frac{1}{6} \cdot 6 = 1$$