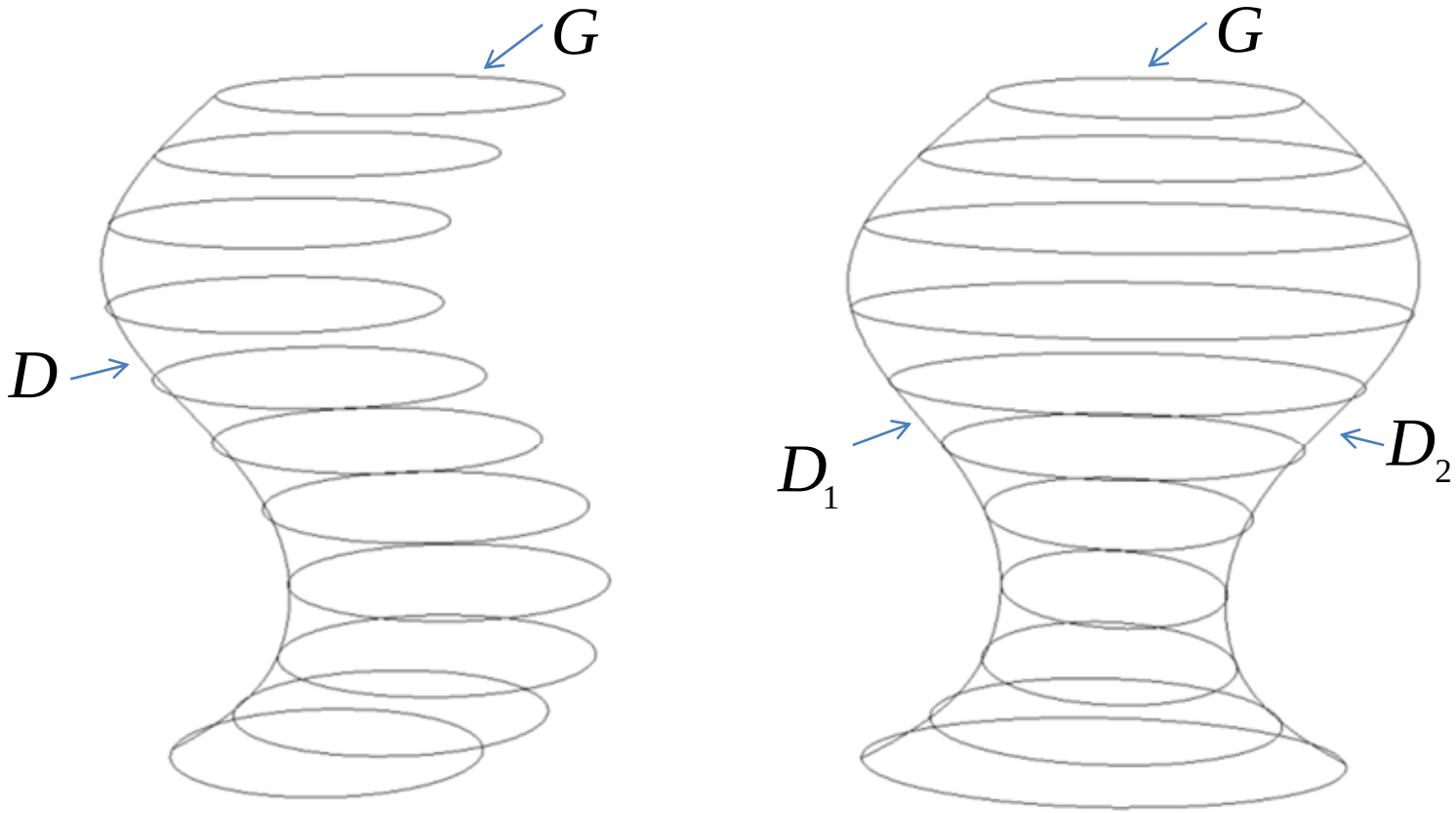
POVRŠI U PROSTORU

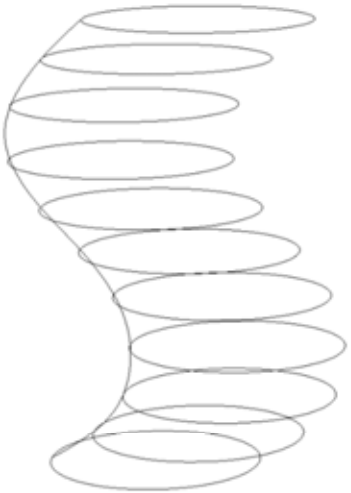
Površ kao trag pokretne krive

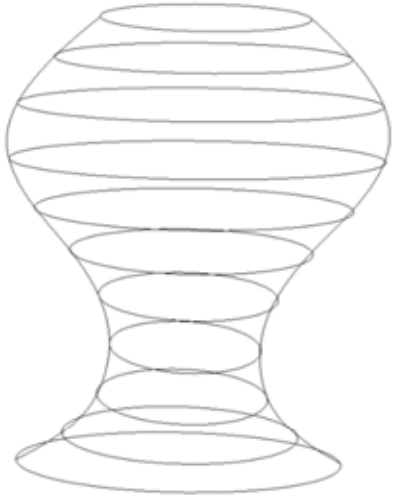


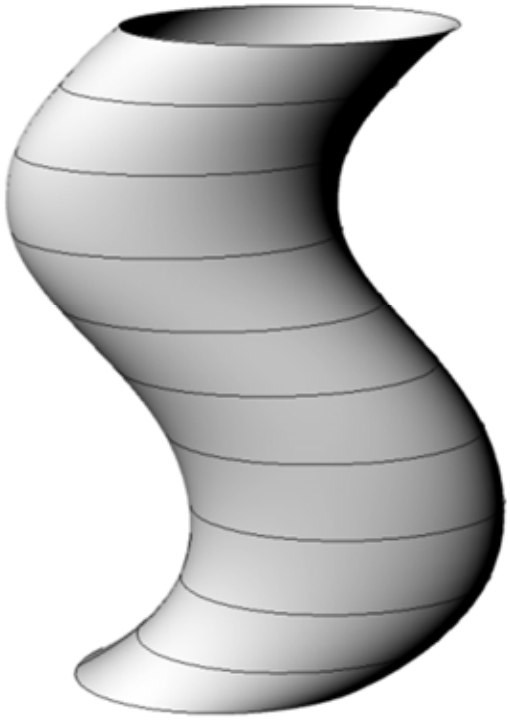
Linija svojim kretanjem u prostoru proizvodi - generiše površ. Ta linija se naziva generatrisom. Linija duž koje se vrši kretanje naziva se direktrisom.

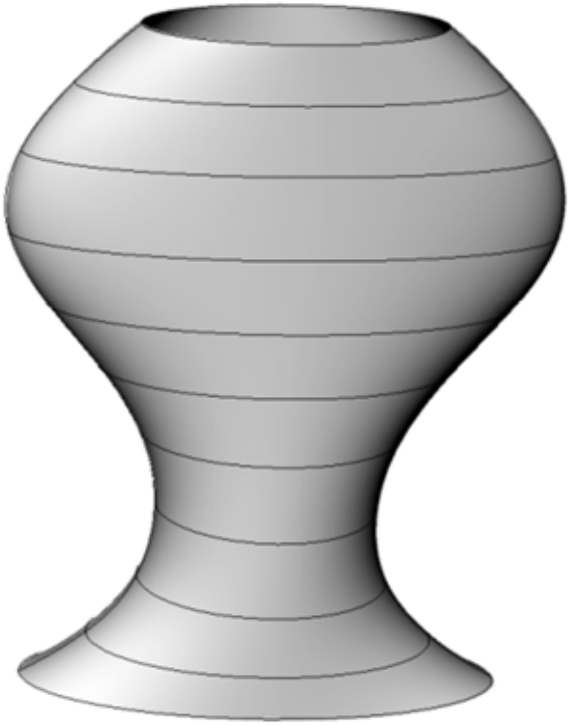
POVRŠI U PROSTORU

Površ kao trag pokretne krive



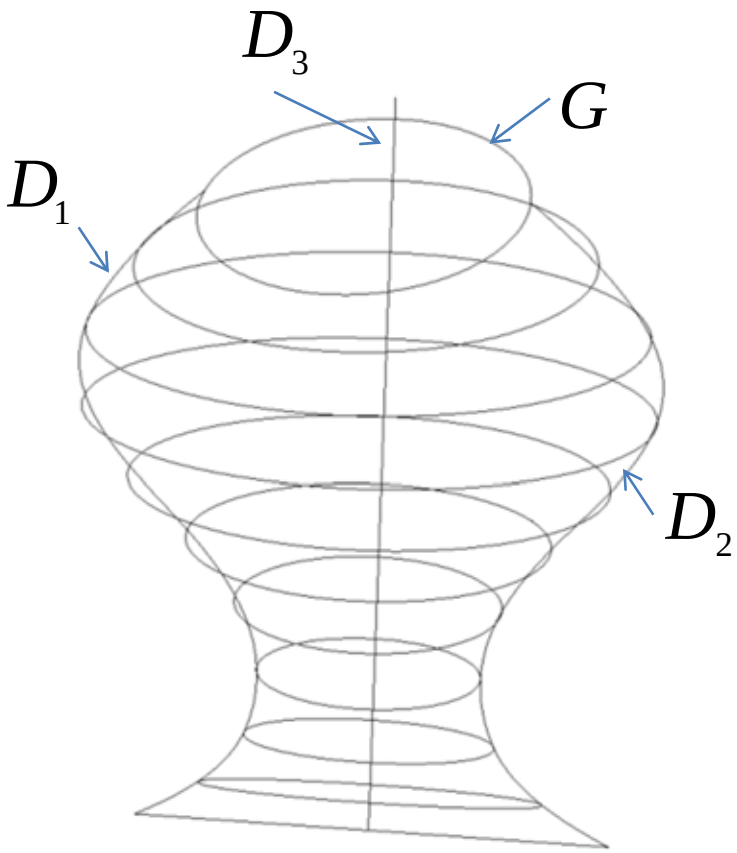


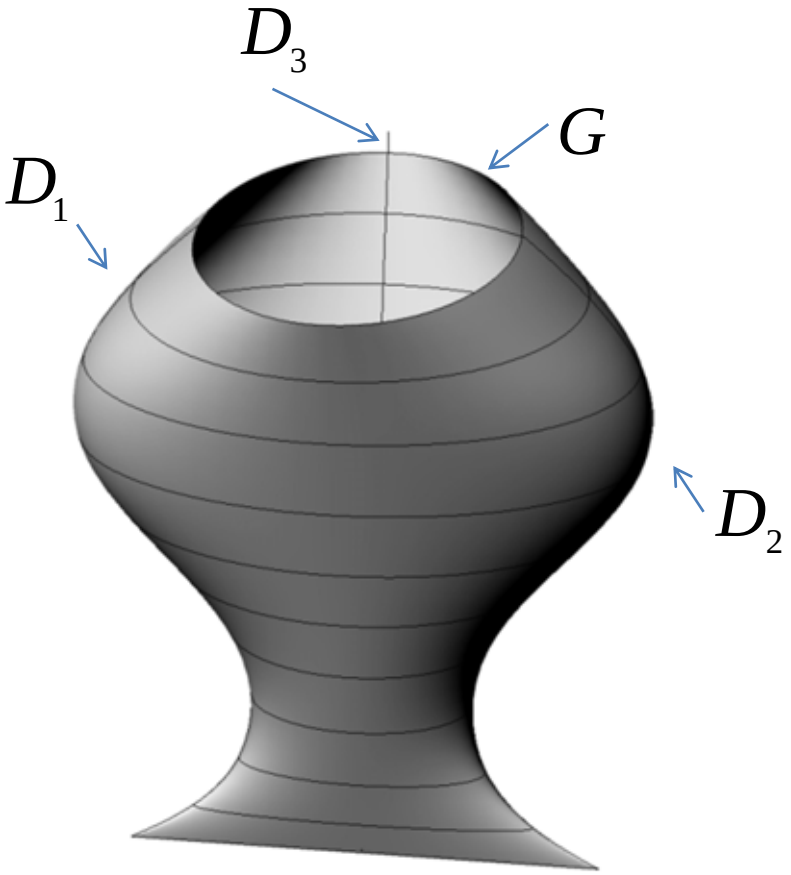




POVRŠI U PROSTORU

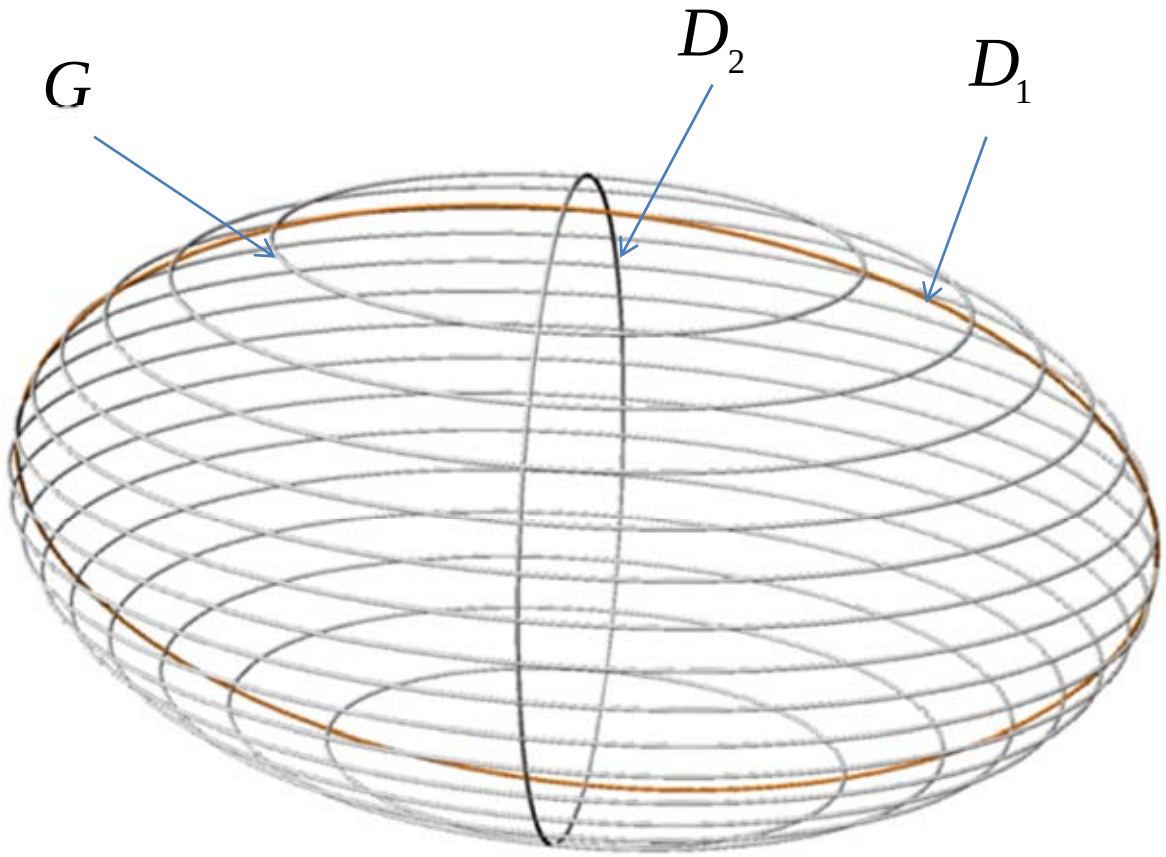
Površ kao trag pokretne krive





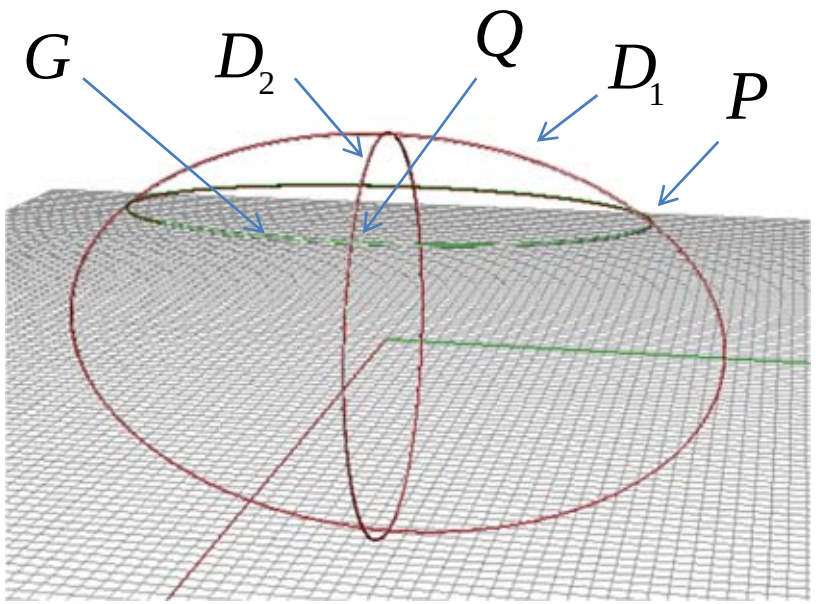
POVRŠI U PROSTORU

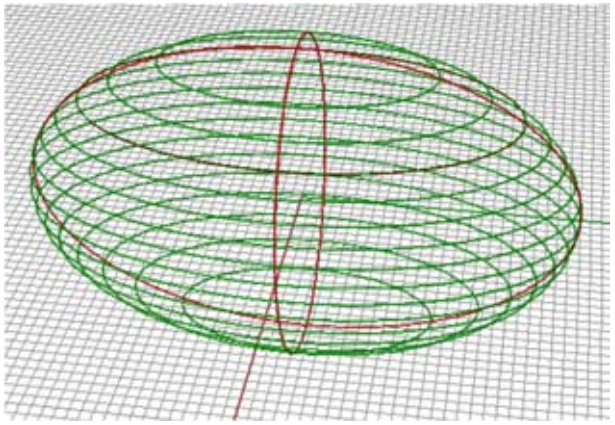
Površ kao trag pokretne krive - elipsoid



POVRŠI U PROSTORU

Površ kao trag pokretne krive - elipsoid





$$D_1 : \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1, \quad x = 0$$

$$D_2 : \frac{x^2}{a^2} + \frac{z^2}{c^2} = 1, \quad y = 0$$

$$G : \frac{x^2}{\alpha^2} + \frac{y^2}{\gamma^2} = 1, \quad z = \gamma$$

$$P = D_1 \cap G$$

$$P(0, \beta, \gamma) \in D_1$$

$$Q = D_2 \cap G$$

$$Q(\alpha, \beta, 0) \in D_2$$

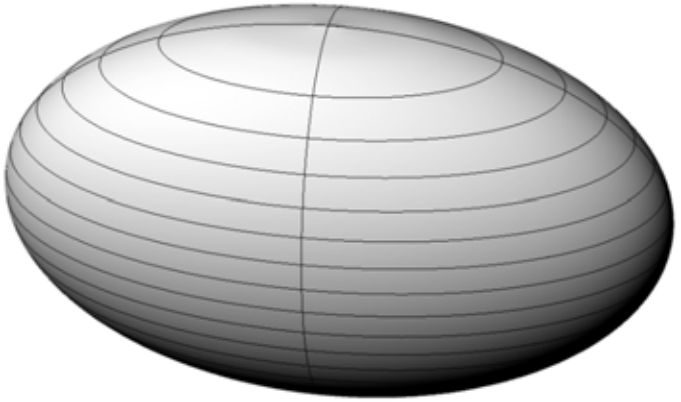
POVRŠI U PROSTORU

Površ kao trag pokretne krive - elipsoid

$$G: \frac{x^2}{\alpha^2}+\frac{y^2}{\beta^2}=1, \quad z=\gamma$$

$$D_1: \frac{y^2}{b^2}+\frac{z^2}{c^2}=1, \quad x=0$$

$$P(0,\beta,\gamma)\in D_1 \Rightarrow \quad \frac{\beta^2}{b^2}+\frac{\gamma^2}{c^2}=1$$



$$D_2: \frac{x^2}{a^2}+\frac{z^2}{c^2}=1, \quad y=0$$

$$Q(\alpha,0,\gamma)\in D_2 \Rightarrow \quad \frac{\alpha^2}{a^2}+\frac{\gamma^2}{c^2}=1$$

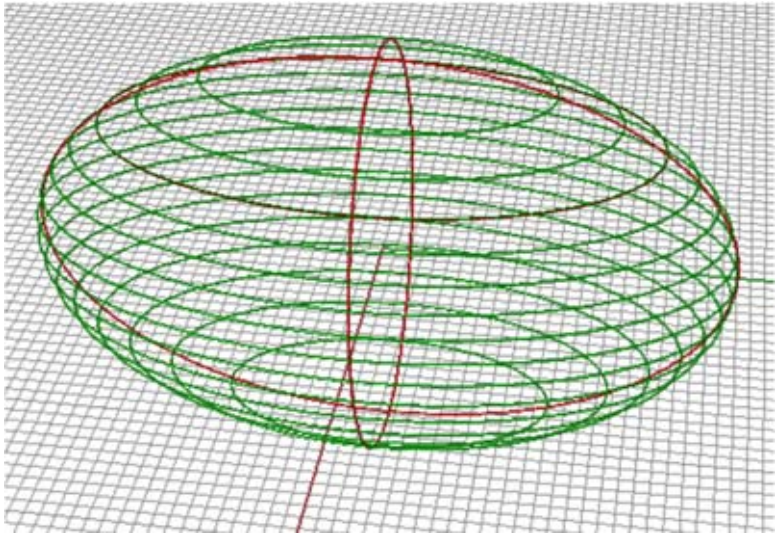
$$\frac{y^2}{b^2}+\frac{z^2}{c^2}=1 \qquad \frac{x^2}{a^2}+\frac{z^2}{c^2}=1 \qquad \frac{\beta^2}{b^2}+\frac{\gamma^2}{c^2}=1 \qquad \frac{\alpha^2}{a^2}+\frac{\gamma^2}{c^2}=1$$

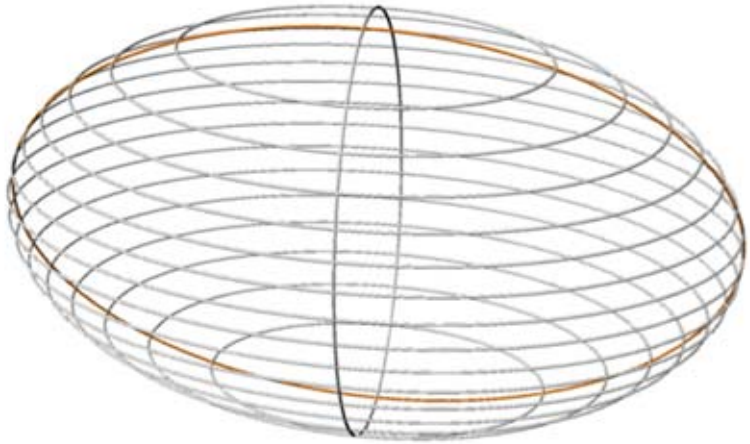
$$\Rightarrow \quad \frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

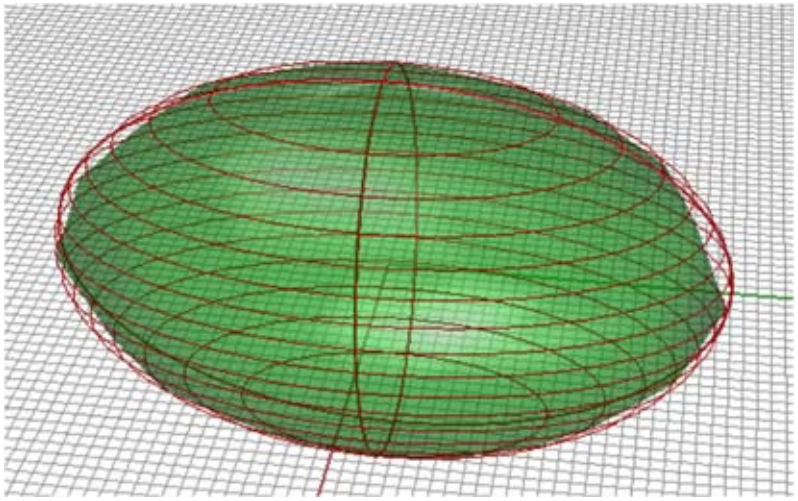
jednačina elipsoida

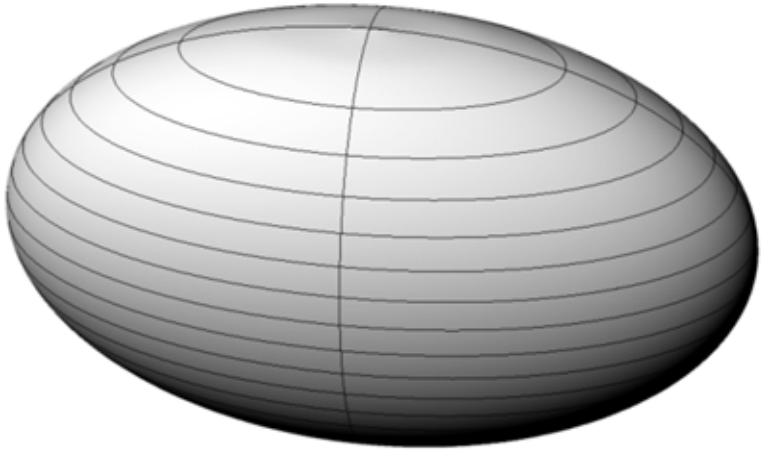
POVRŠI U PROSTORU

Površ kao trag pokretne krive – elipsoid – geometrijska interpretacija



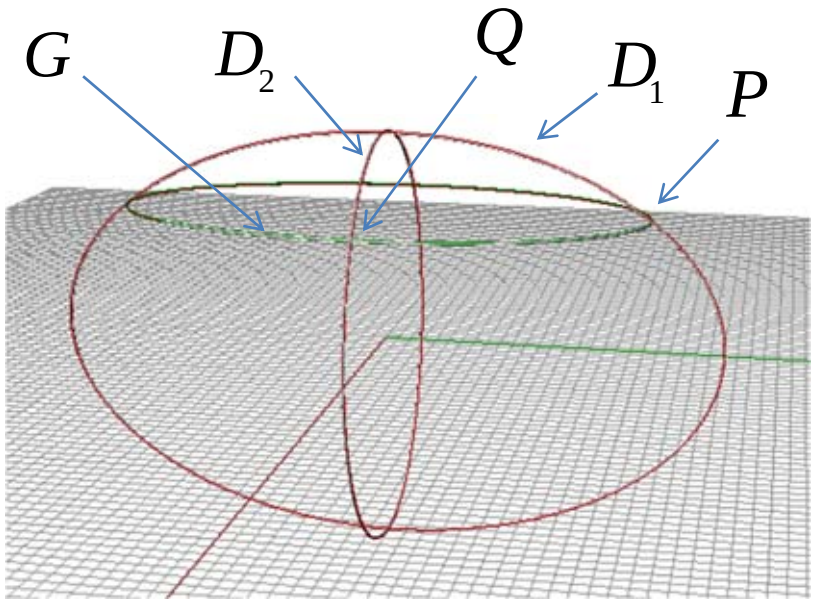


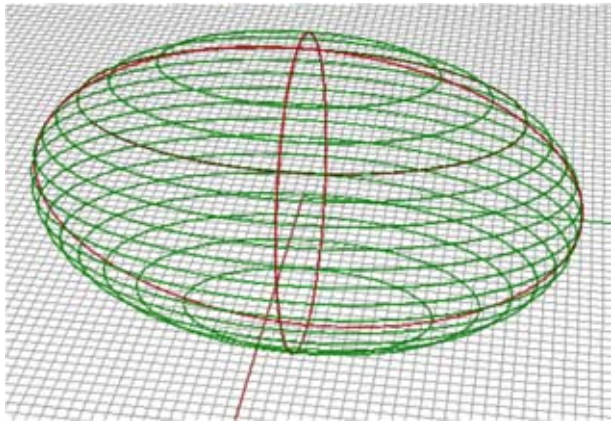




POVRŠI U PROSTORU

Površ kao trag pokretne krive - elipsoid





$$D_1:\ x=0,\ y=b\cos\theta,\ z=c\sin\theta$$

$$D_2:\ x=a\cos\theta,\ y=0,\ z=c\sin\theta$$

$$G:\ x=\alpha\cos\varphi,\ y=\beta\sin\varphi,\ z=\gamma$$

$$P=D_1\cap G$$

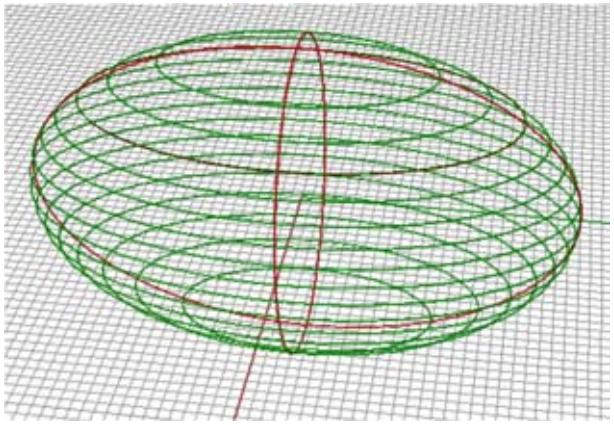
$$P(0,\beta,\gamma)\in D_1$$

$$Q=D_2\cap G$$

$$Q(\alpha,\beta,0)\in D_2$$

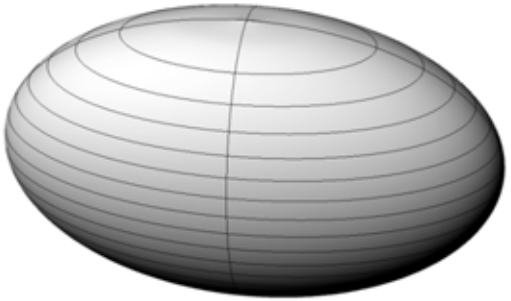
POVRŠI U PROSTORU

Površ kao trag pokretne krive - elipsoid



$$D_1:\ x=0,\ y=b\cos\theta,\ z=c\sin\theta$$

$$P(0,\beta,\gamma)\in D_1 \Rightarrow \quad b\cos\theta=\beta,\ c\sin\theta=\gamma$$



$$D_2:\ x=a\cos\theta,\ y=0,\ z=c\sin\theta$$

$$Q(\alpha,\beta,0)\in D_2 \Rightarrow \quad a\cos\theta=\alpha,\ c\sin\theta=\gamma$$

$$G:\ x=\alpha\cos\varphi,\ y=\beta\sin\varphi,\ z=\gamma$$

$$b\cos\theta=\beta,\ c\sin\theta=\gamma$$

$$a\cos\theta=\alpha,\ c\sin\theta=\gamma$$

Parametarske jednačine elipsoida

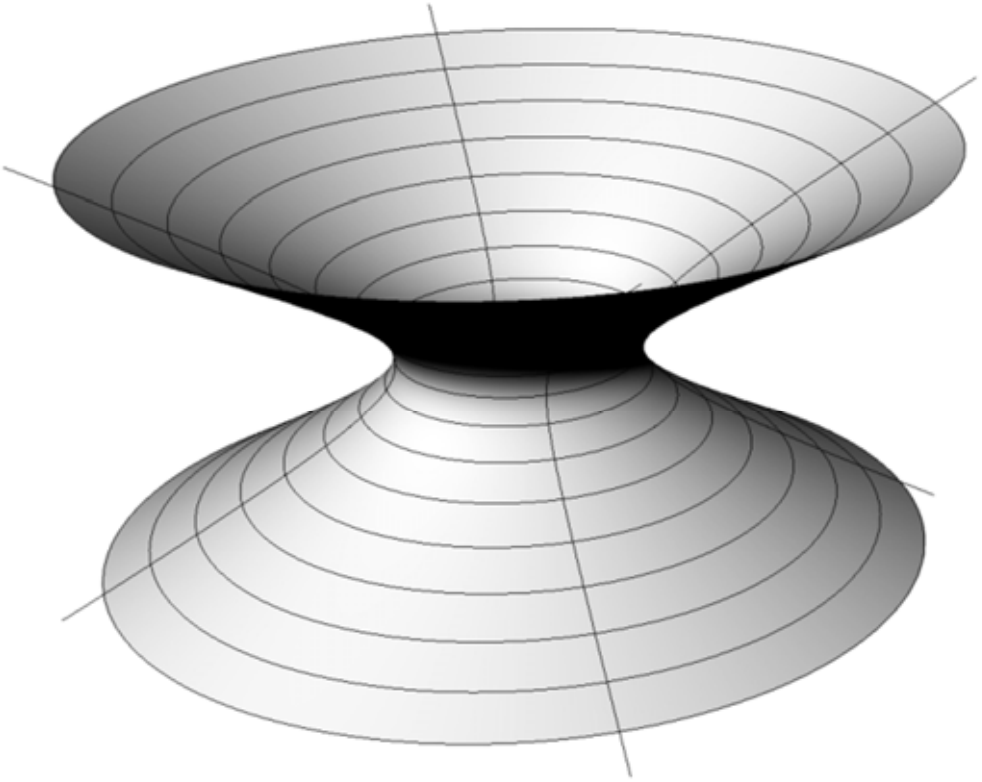
$$x=a\cos\theta\cos\varphi$$

$$y=\beta\cos\theta$$

$$z=c\sin\theta$$

POVRŠI U PROSTORU

Površ kao trag pokretne krive – jednograni hiperboloid



$$D_1 : \frac{x^2}{a^2} - \frac{z^2}{c^2} = 1, \quad y = 0$$

$$D_2 : \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1, \quad x = 0$$

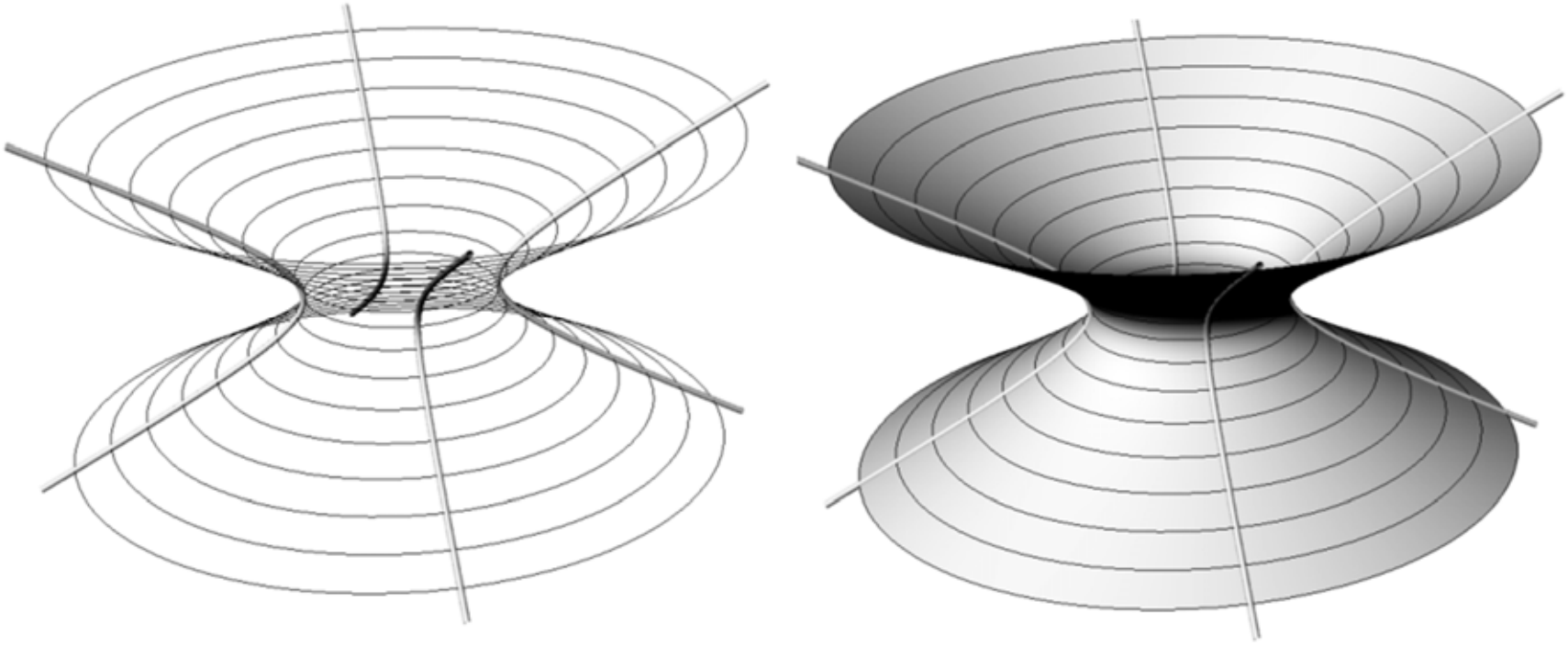
$$G : \frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} = 1, \quad z = \gamma$$

Jednačina jednogranog hiperpoloida:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

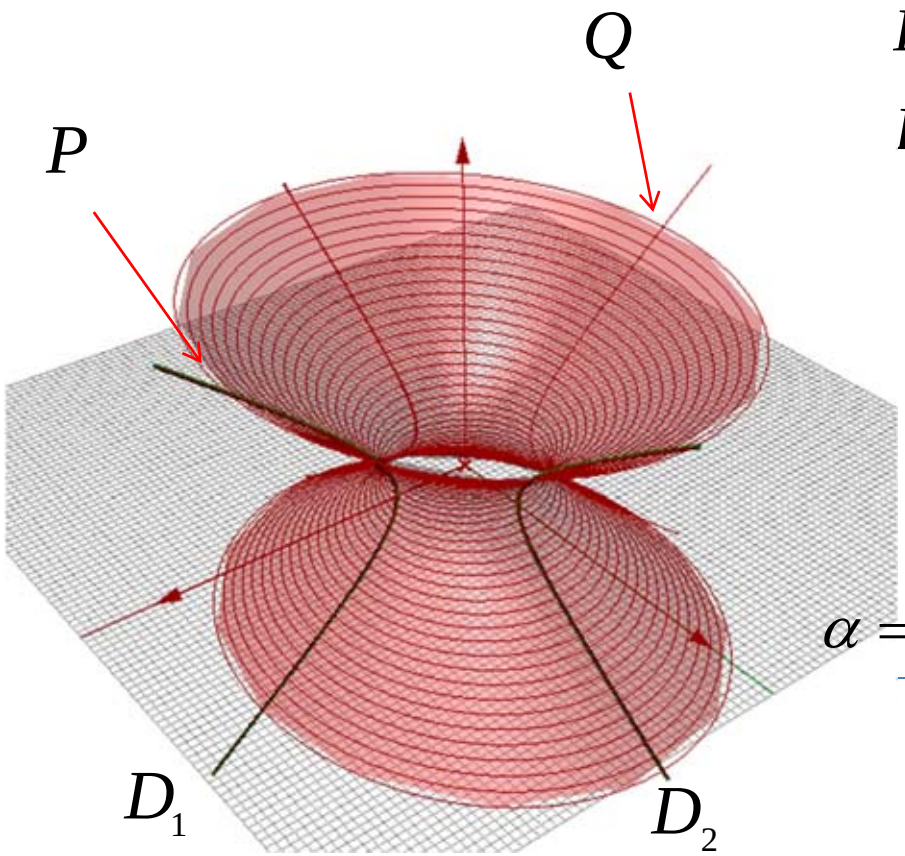
POVRŠI U PROSTORU

Površ kao trag pokretne krive – jednograni hiperboloid



POVRŠI U PROSTORU

Površ kao trag pokretne krive – jednograni hiperboloid - parametrizacija



$$D_1 : x = a\cosh\theta,\ y = 0,\ z = c\sinh\theta$$

$$D_2 :\ x = 0,\ y = b\cosh\theta,\ z = c\sinh\theta$$

$$G : x = \alpha\cos\varphi, y = \beta\sin\varphi, z = \gamma$$

$$P = D_1 \cap G \qquad Q = D_2 \cap G$$

$$P(\alpha,0,\gamma) \in D_1 \qquad Q(0,\beta,\gamma) \in D_2$$

$$\alpha = a\cosh\theta,\ \beta = b\cosh\theta,\ \gamma = c\sinh\theta$$

$$x = a\cosh\theta\cos\varphi$$

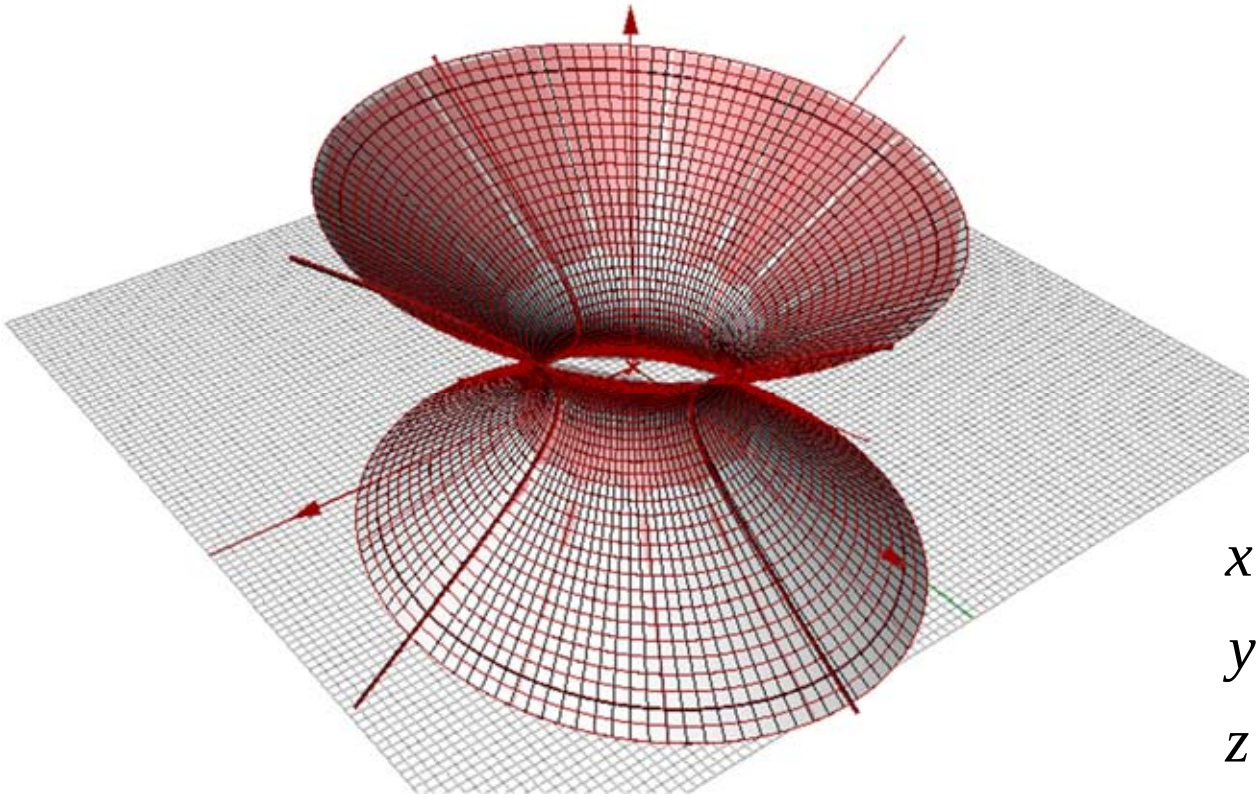
$$y = b\cosh\theta\sin\varphi$$

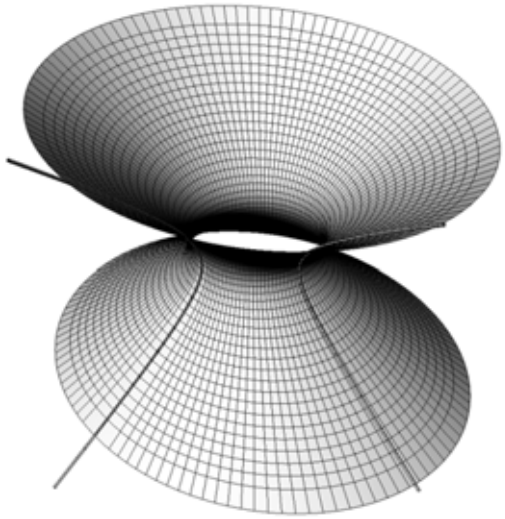
$$z = c\sinh\theta$$

$$0 \le \varphi \le 2\pi,\ -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$$

POVRŠI U PROSTORU

Površ kao trag pokretne krive – jednograni hiperboloid – parametrizacija
Izoparametarske linije





$$x = a\cosh\theta\cos\varphi$$

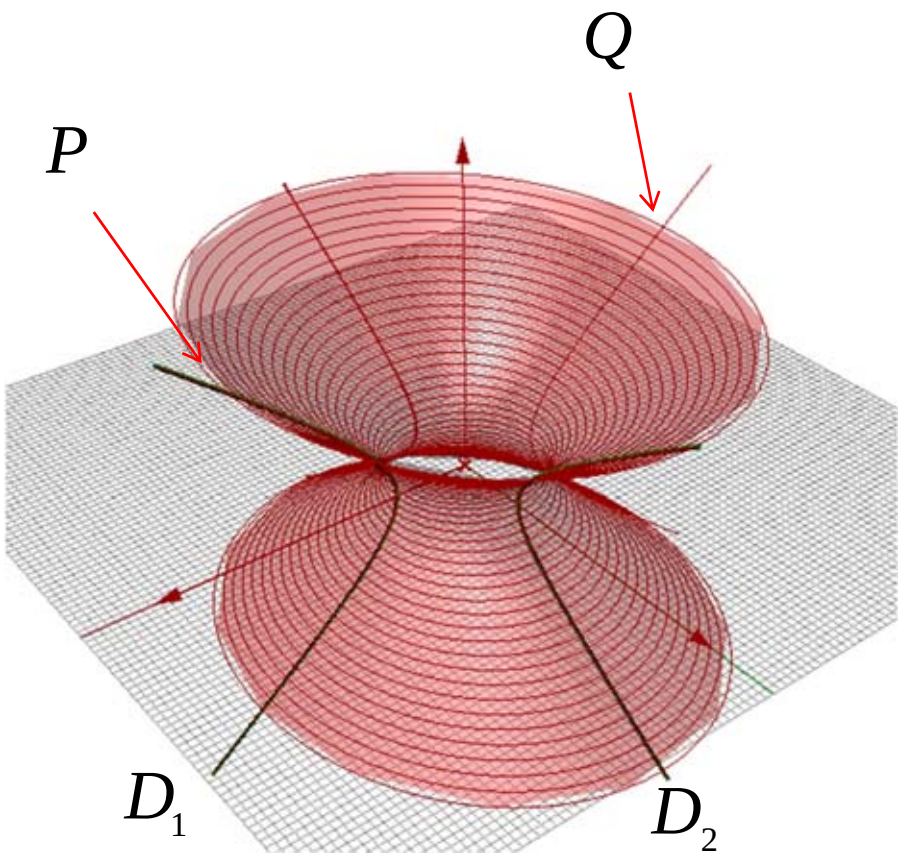
$$y = b\cosh\theta\sin\varphi$$

$$z = c\sinh\theta$$

$$0 \le \varphi \le 2\pi,\ -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$$

POVRŠI U PROSTORU

Površ kao trag pokretne krive – jednograni hiperboloid - parametrizacija



$$D_1 : x = a\sec\theta,\ y = 0,\ z = c\tan\theta$$

$$D_2 :\ x = 0,\ y = b\sec\theta,\ z = c\tan\theta$$

$$G : x = \alpha\cos\varphi, y = \beta\sin\varphi, z = \gamma$$

$$x = a\sec\theta\cos\varphi$$

$$y = b\sec\theta\sin\varphi$$

$$z = c\tan\theta$$

$$0 \le \varphi \le 2\pi,\ -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$$

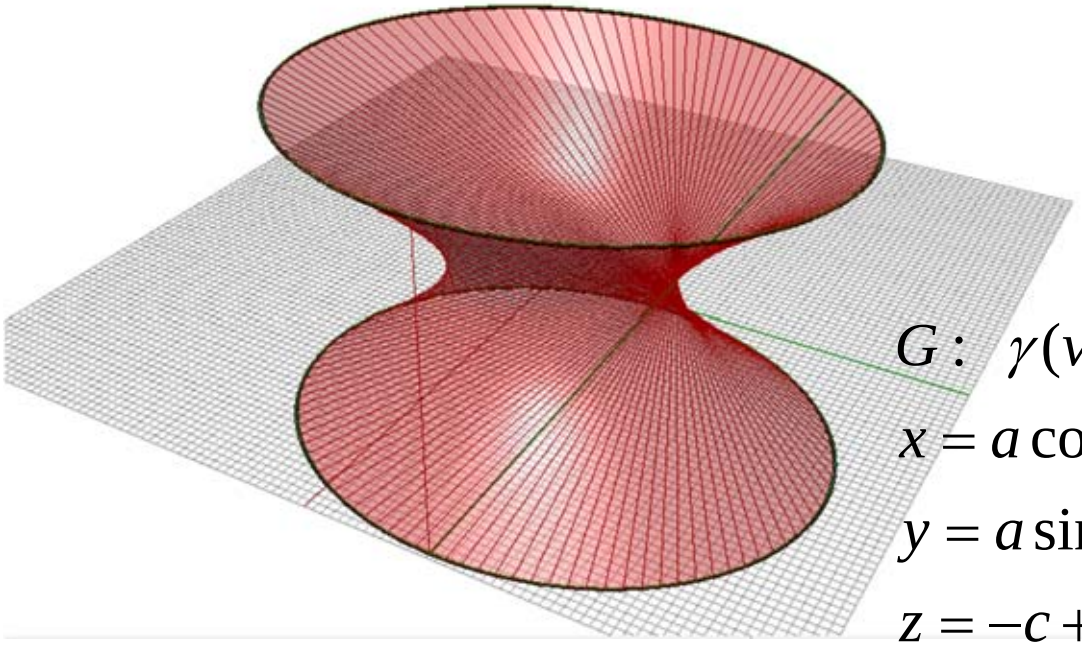
POVRŠI U PROSTORU

Površ kao trag pokretne krive – jednograni hiperboloid – parametrizacija

$$D_1 - \alpha = \alpha(u): x = a\cos u,\ y = b\sin u,\ z = -c$$

$$D_2 - \beta = \beta(u): x = a\cos u,\ y = b\sin u,\ z = c$$

$$0 \le u \le 2\pi$$



$$G: \ \gamma(v) = \alpha(u) + v(\beta(u+t) - \alpha(u)):$$

$$x = a\cos u + av(\cos(u+t) - \cos u)$$

$$y = a\sin u + av(\sin(u+t) - \sin u)$$

$$z = -c + 2cv$$

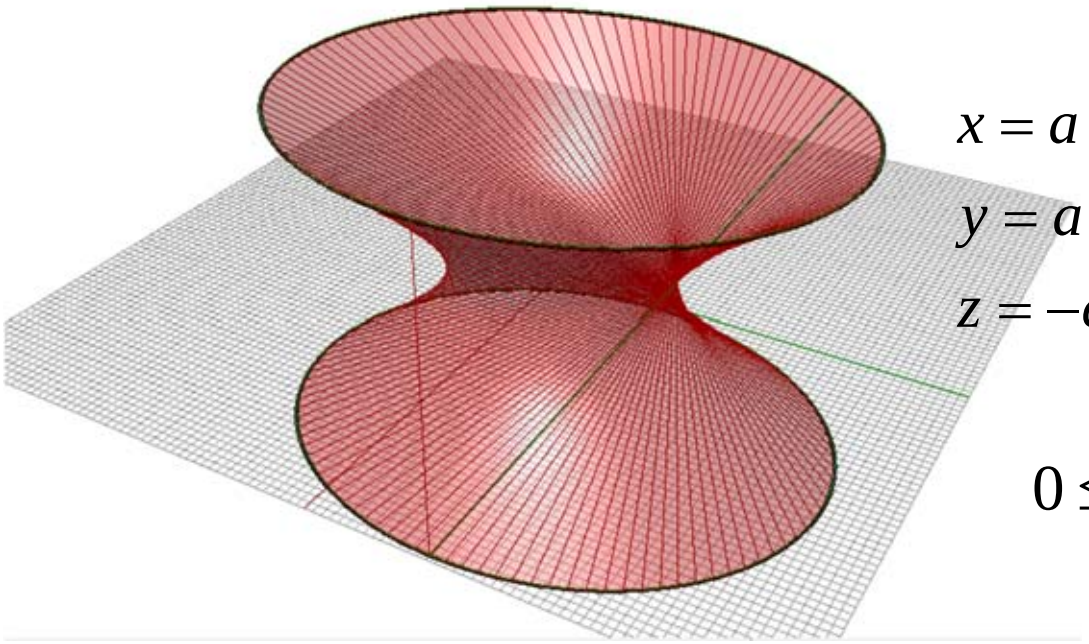
$$0 \le t \le 2\pi$$

POVRŠI U PROSTORU

Površ kao trag pokretne krive – jednograni hiperboloid – parametrizacija

$$0 \le t \le 2\pi$$

Parametarske jednačine:



$$x = a\cos u + av(\cos(u+t) - \cos u)$$

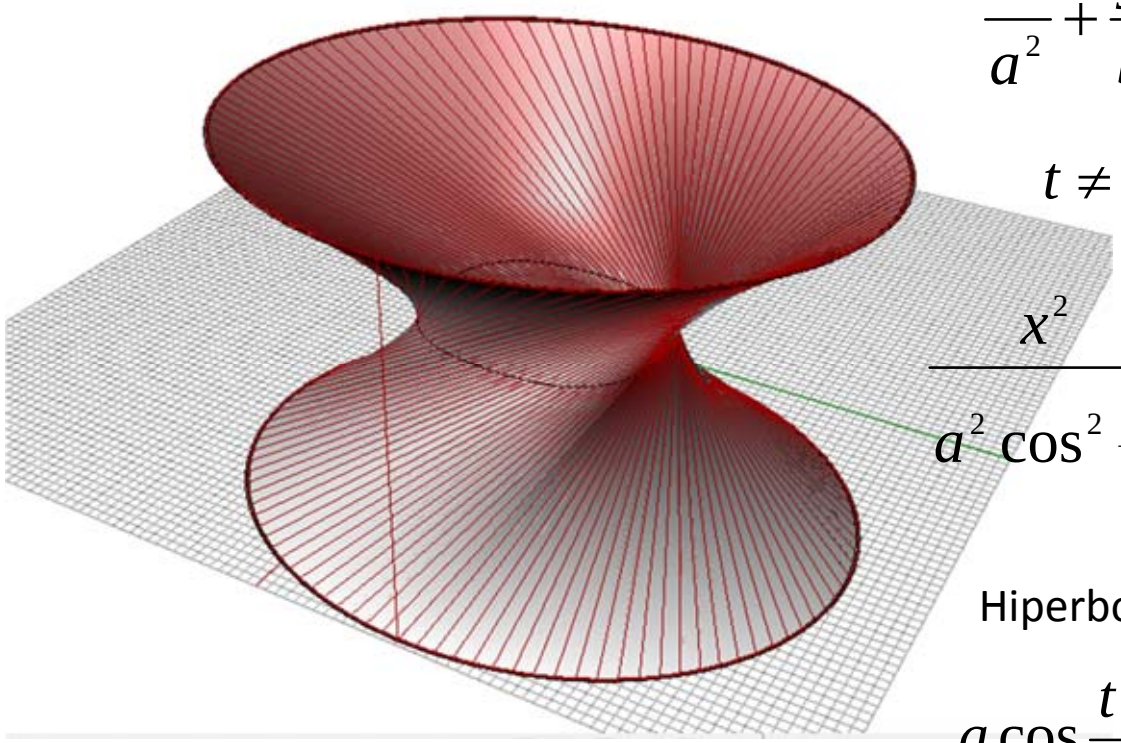
$$y = a\sin u + av(\sin(u+t) - \sin u)$$

$$z = -c + 2cv$$

$$0 \le u \le 2\pi,\ v \in R$$

POVRŠI U PROSTORU

Površ kao trag pokretne krive – jednograni hiperboloid – parametrizacija



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\sin^2\frac{t}{2}\cdot\frac{z^2}{c^2}=\cos^2\frac{t}{2}$$

$$t\neq 0\wedge t\neq \pi$$

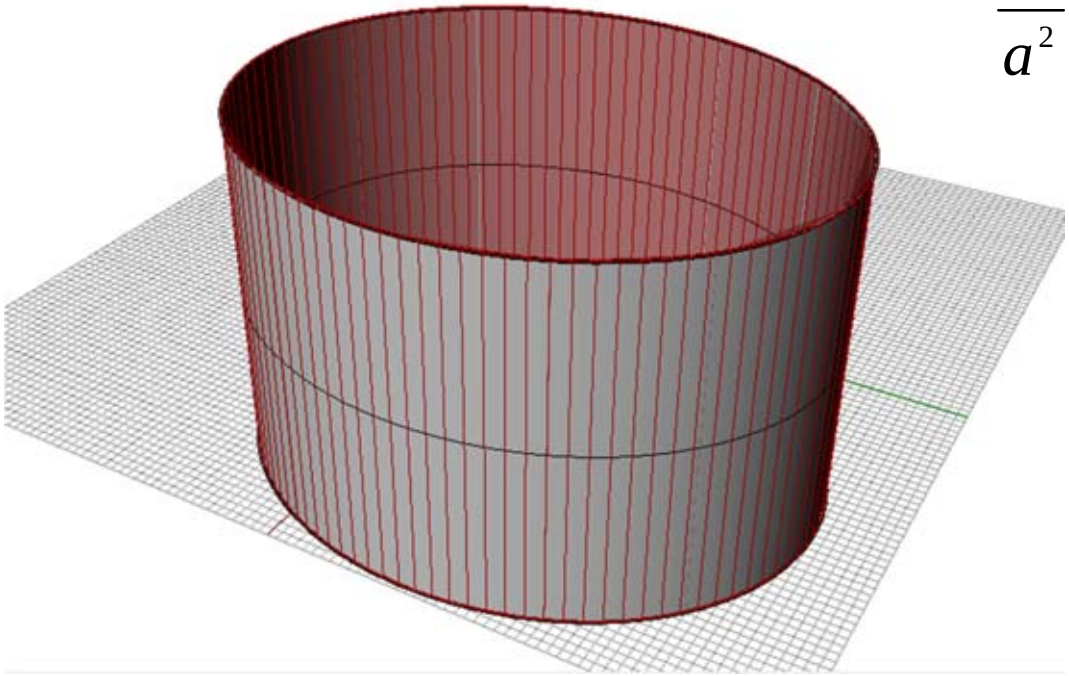
$$\frac{x^2}{a^2\cos^2\frac{t}{2}}+\frac{y^2}{b^2\cos^2\frac{t}{2}}-\frac{z^2}{c^2ctg^2\frac{t}{2}}=1$$

Hiperboloid sa poluosama:

$$a\cos\frac{t}{2},\ b\cos\frac{t}{2},\ c\cdot ctg\frac{t}{2}$$

POVRŠI U PROSTORU

Površ kao trag pokretne krive – cilindrična površ - parametrizacija



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\sin^2\frac{t}{2}\cdot\frac{z^2}{c^2}=\cos^2\frac{t}{2}$$

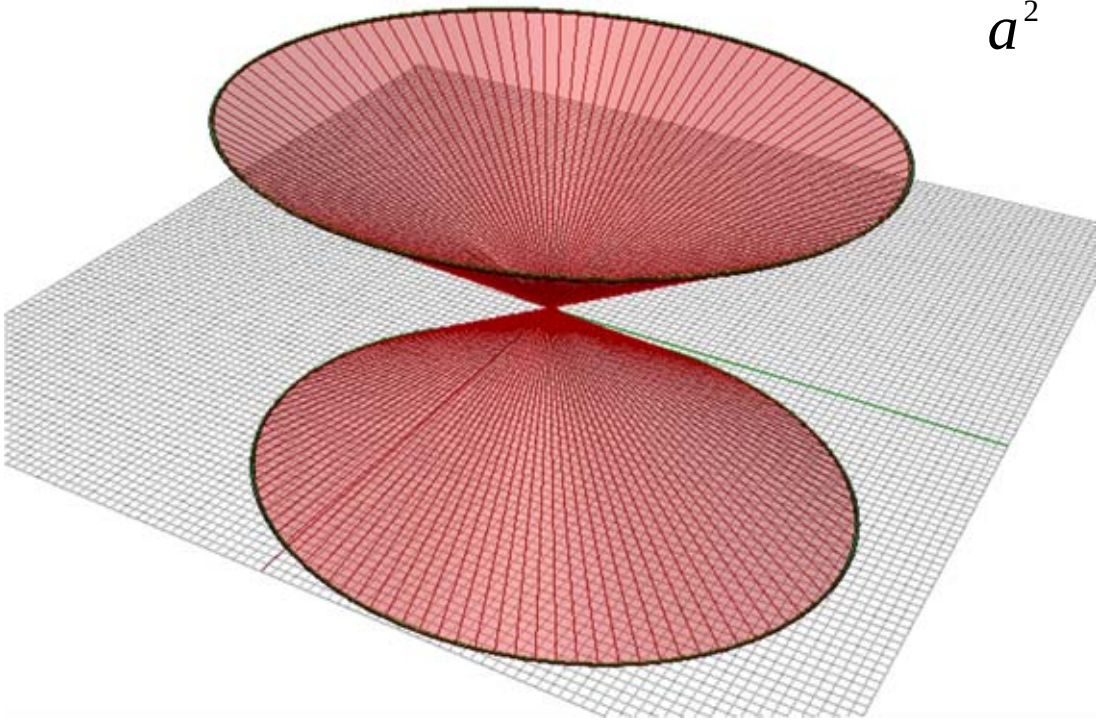
$$t=0$$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Cilindrična površ

POVRŠI U PROSTORU

Površ kao trag pokretne krive – konusna površ - parametrizacija



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\sin^2\frac{t}{2}\cdot\frac{z^2}{c^2}=\cos^2\frac{t}{2}$$

$$t=\pi$$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$$

Konusna površ

POVRŠI U PROSTORU

Površ kao trag pokretne krive – jednograni hiperboloid – parametrizacija
Izoparametarske linije

Parametarske jednačine:

$$x = a\cos u + av(\cos(u+t) - \cos u)$$

$$y = a\sin u + av(\sin(u+t) - \sin u)$$

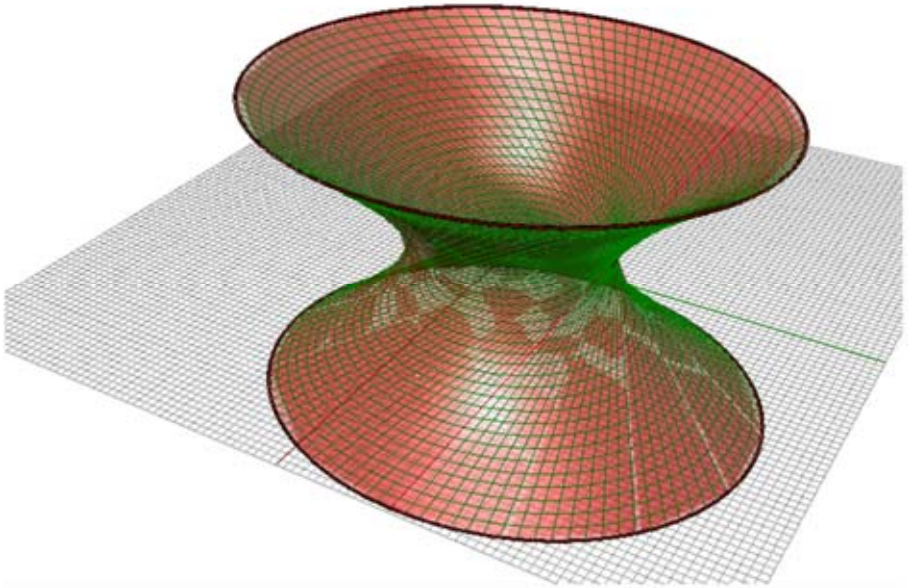
$$z = -c + 2cv$$

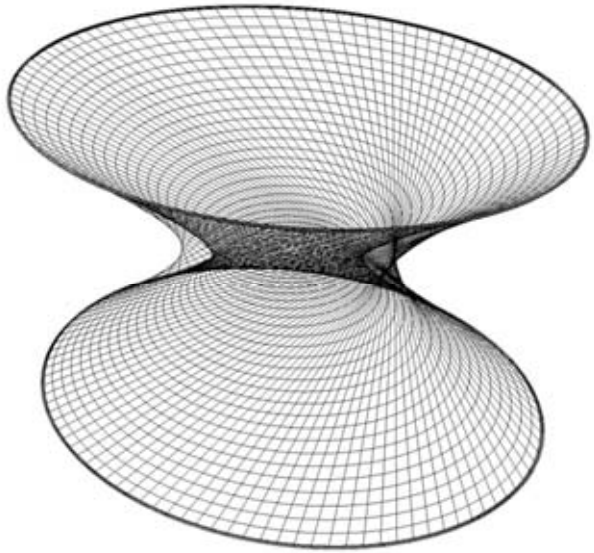
$$0 \le u \le 2\pi,\ v \in R$$

$$0 \le t \le 2\pi$$

$$t \ne 0 \wedge t \ne \pi$$

Jednograni hiperboloid





POVRŠI U PROSTORU

Površ kao trag pokretne krive – jednograni hiperboloid – parametrizacija
Izoparametarske linije

Parametarske jednačine:

$$x = a\cos u + av(\cos(u+t) - \cos u)$$

$$y = a\sin u + av(\sin(u+t) - \sin u)$$

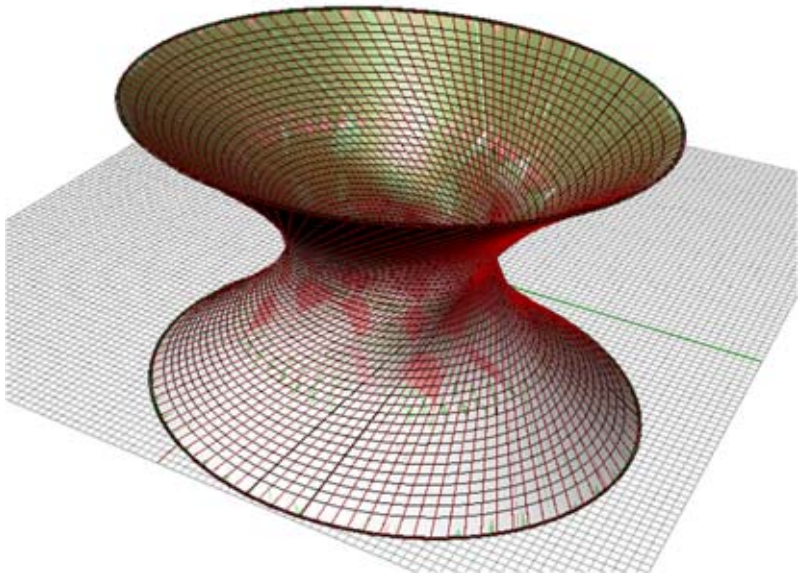
$$z = -c + 2cv$$

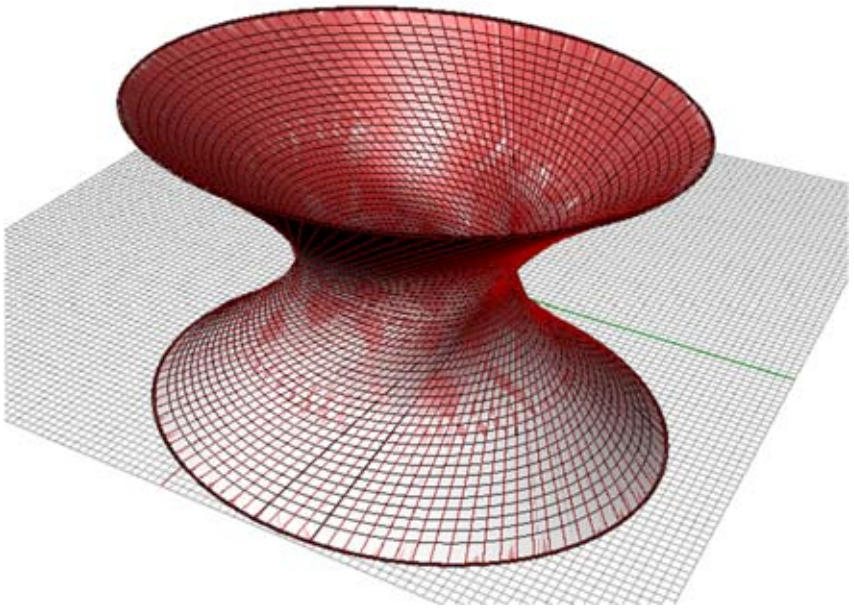
$$0 \le u \le 2\pi,\ v \in R$$

$$0 \le t \le 2\pi$$

$$t \ne 0 \wedge t \ne \pi$$

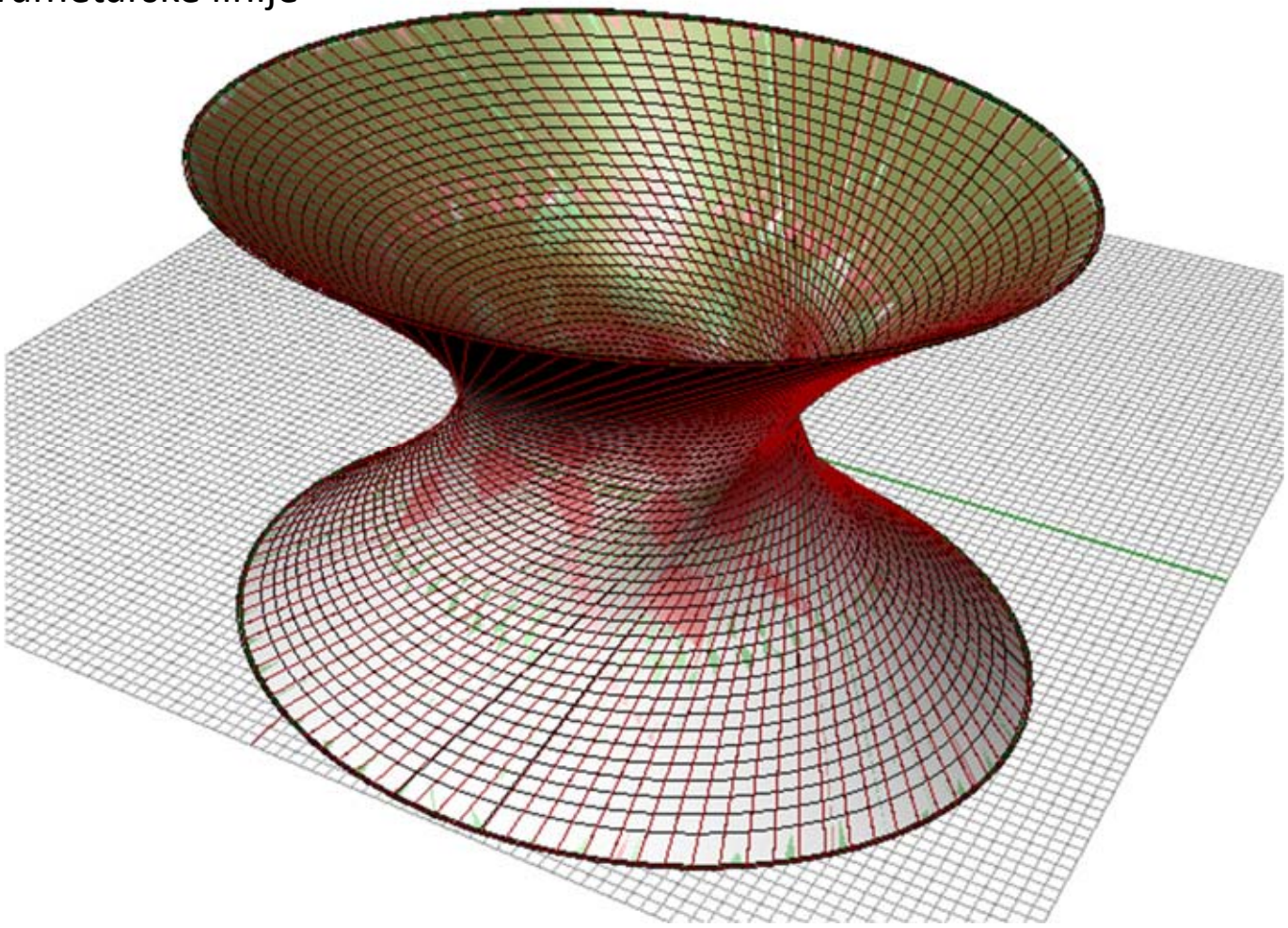
Jednograni hiperboloid





POVRŠI U PROSTORU

Površ kao trag pokretne krive – jednograni hiperboloid – parametrizacija
Izoparametarske linije



POVRŠI U PROSTORU

Površ kao trag pokretne krive – cilindrična površ – parametrizacija
Izoparametarske linije

Parametarske jednačine:

$$x = a\cos u + av(\cos(u+t) - \cos u)$$
$$y = a\sin u + av(\sin(u+t) - \sin u)$$
$$z = -c + 2cv$$

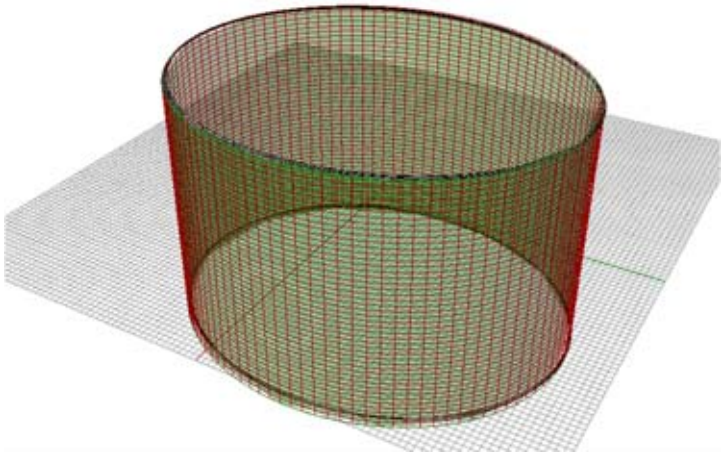
$$0 \le t \le 2\pi$$

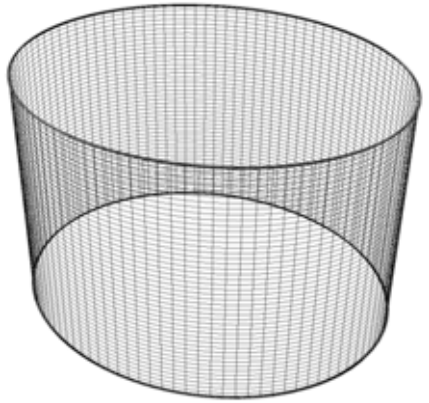
$$t = 0$$

Cilindrična površ

$$0 \le u \le 2\pi,\ v \in R$$

$$x = a\cos u, y = a\sin u, z = -c + 2cv \qquad 0 \le u \le 2\pi,\ v \in R$$





POVRŠI U PROSTORU

Površ kao trag pokretne krive – konusna površ – parametrizacija
Izoparametarske linije

Parametarske jednačine:

$$x = a\cos u + av(\cos(u+t) - \cos u)$$
$$y = a\sin u + av(\sin(u+t) - \sin u)$$
$$z = -c + 2cv$$

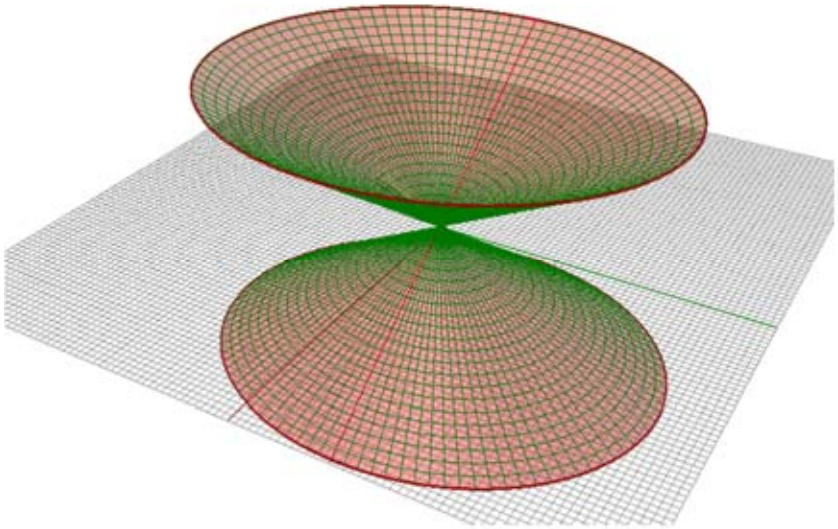
$0 \le t \le 2\pi$

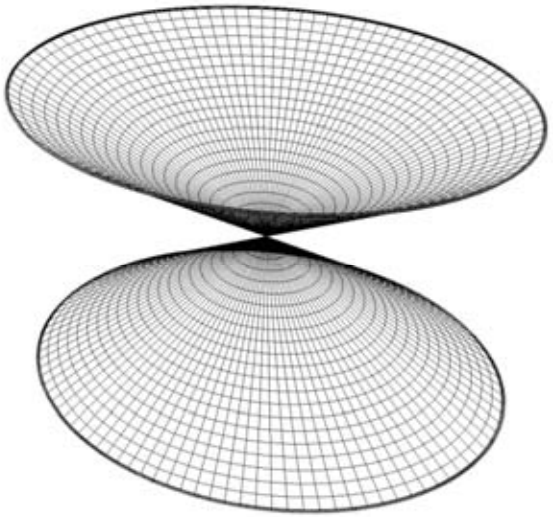
$t = \pi$

Konusna površ

$$0 \le u \le 2\pi,\ v \in R$$

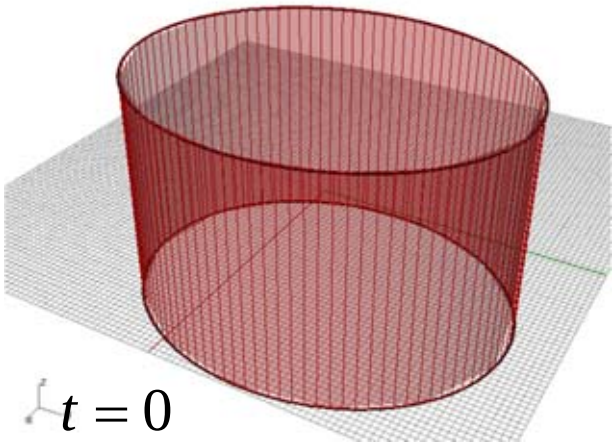
$$x = a\cos u - 2av\cos u,\ y = a\sin u - 2av\sin u,\ z = -c + 2cv$$

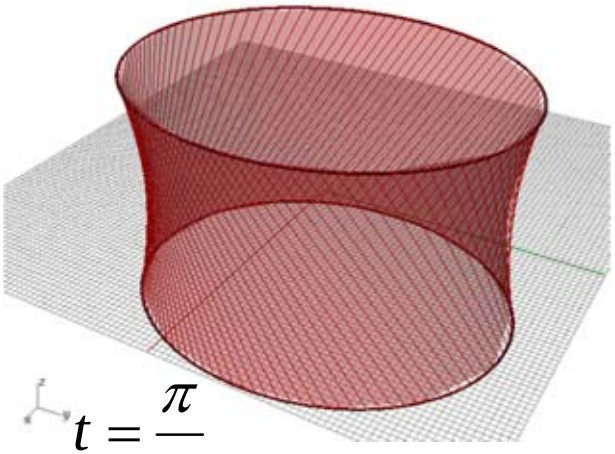


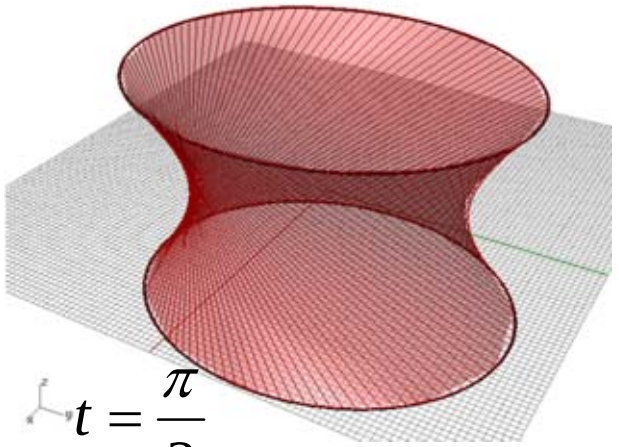


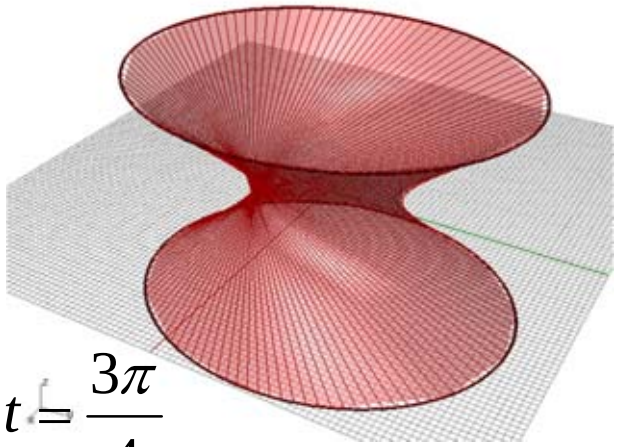
POVRŠI U PROSTORU

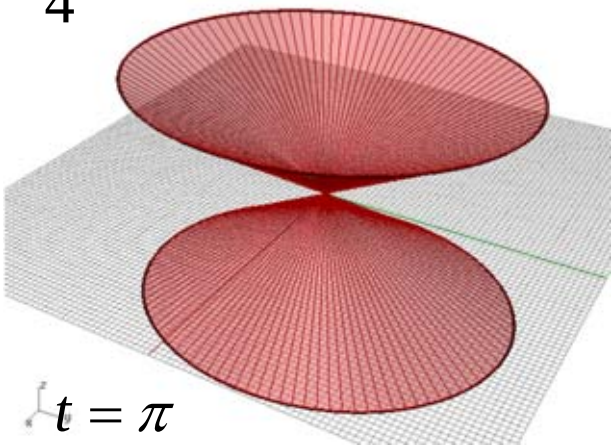
Površ kao trag pokretne krive – parametrizacija





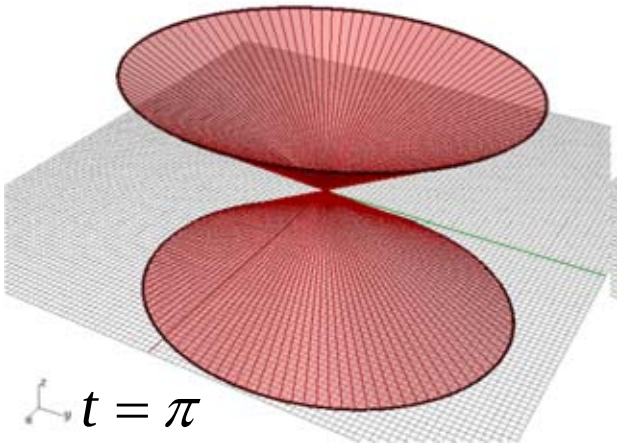


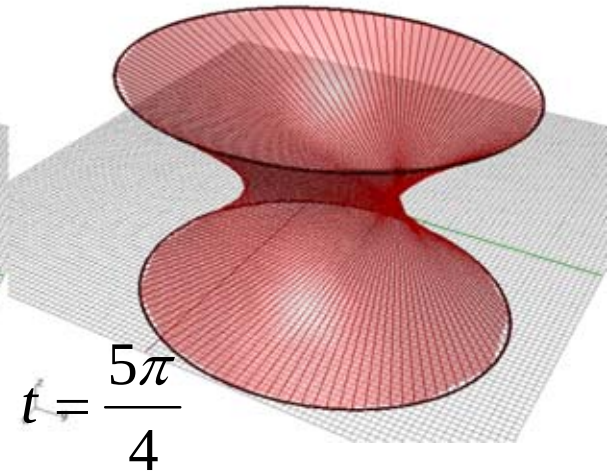


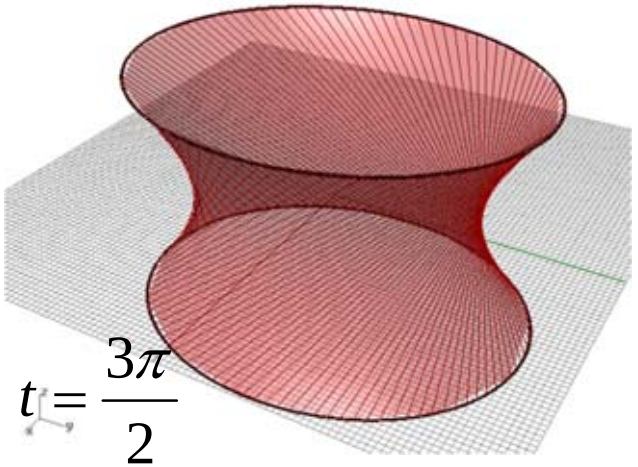


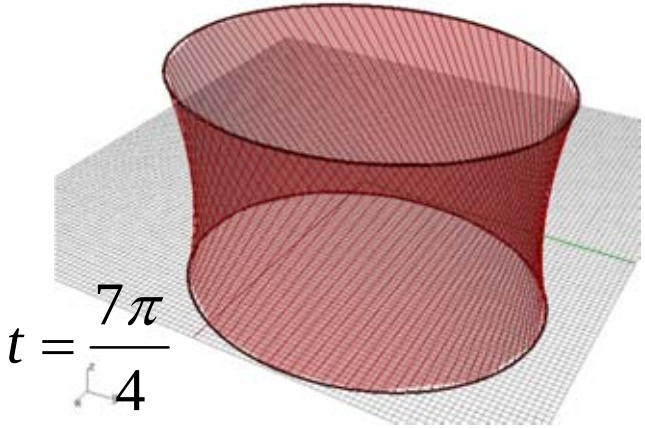
POVRŠI U PROSTORU

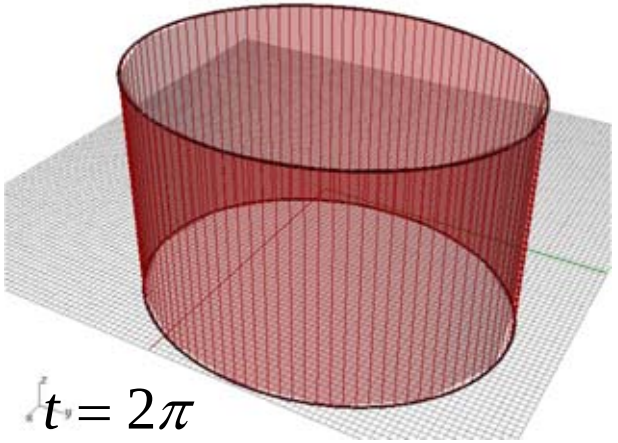
Površ kao trag pokretne krive – parametrizacija





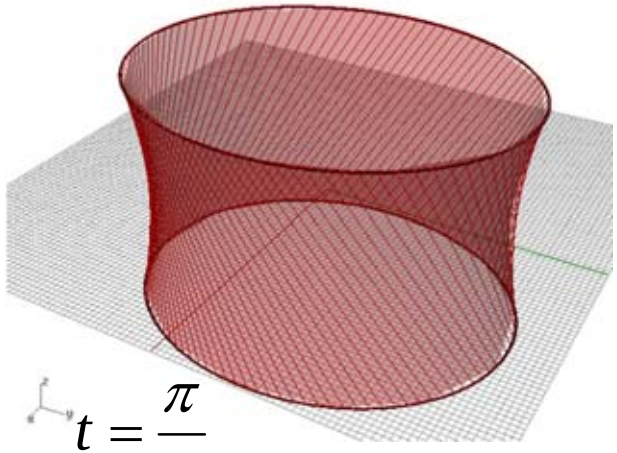


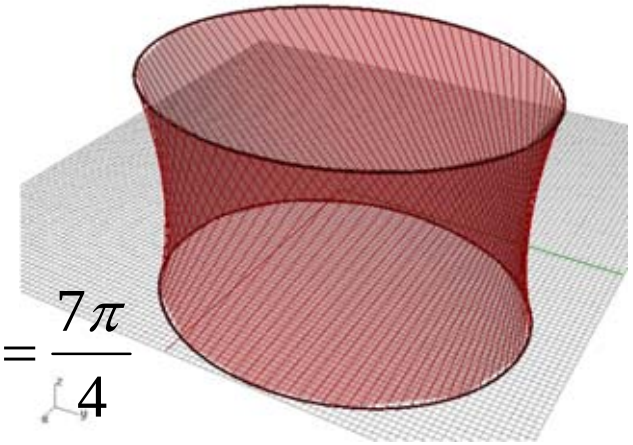


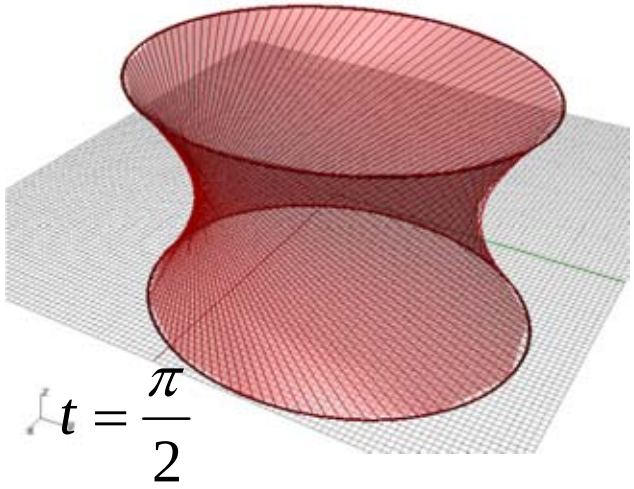


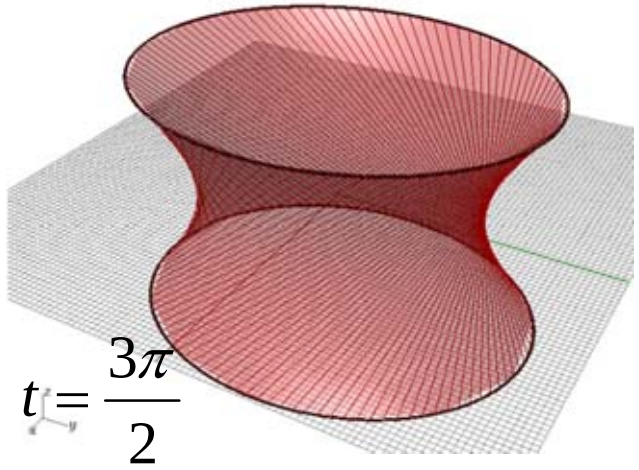
POVRŠI U PROSTORU

Površ kao trag pokretne krive – parametrizacija







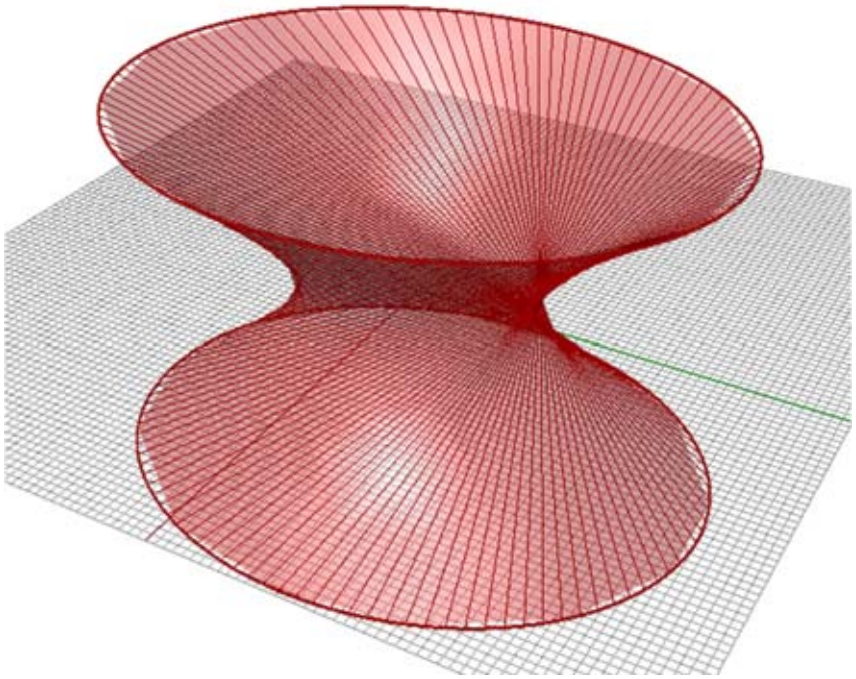


POVRŠI U PROSTORU

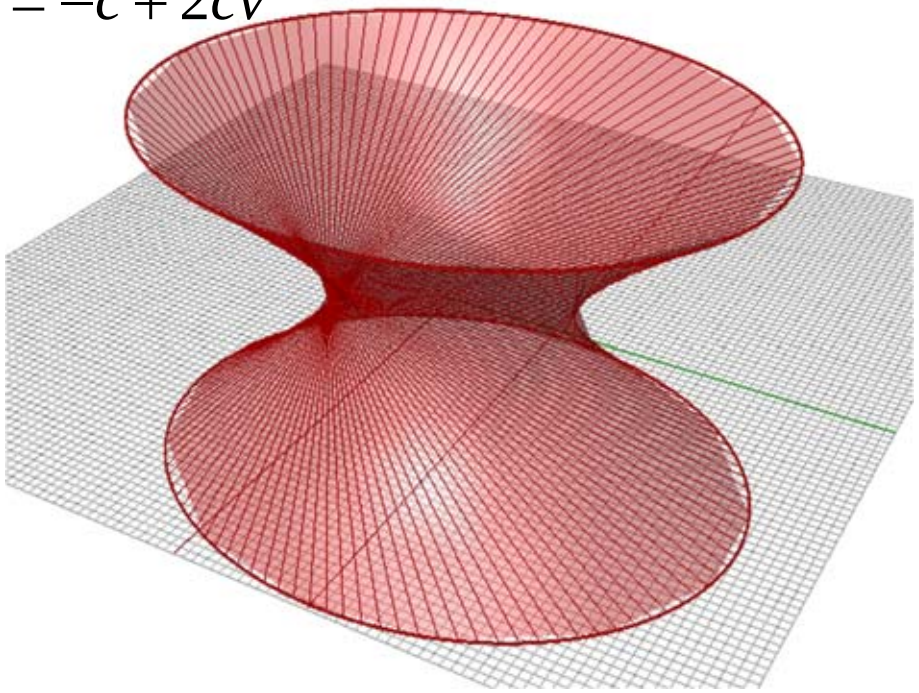
Površ kao trag pokretne krive – jednograni hiperboloid – parametrizacija

$$0 \le u \le 2\pi,\ v \in R$$

$$x = a\cos u + av(\cos(u+t) - \cos u)$$
$$y = a\sin u + av(\sin(u+t) - \sin u)$$
$$z = -c + 2cv$$



$t = 3.956$

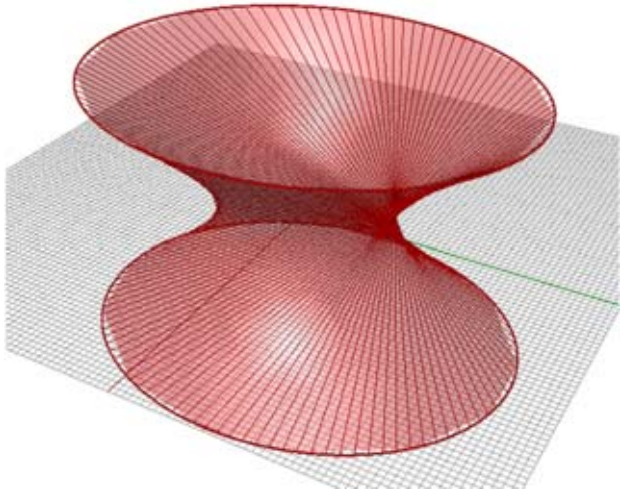


$t = 2\pi - 3.956 = 2.324$

POVRŠI U PROSTORU

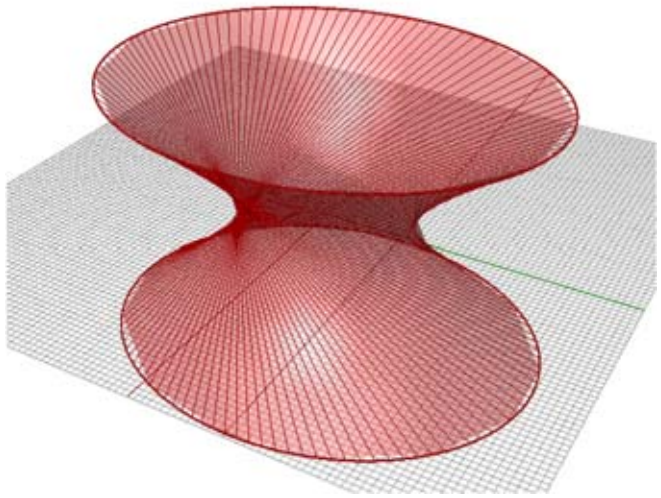
Površ kao trag pokretne krive – jednograni hiperboloid – parametrizacija

$$x = a\cos u + av(\cos(u+t) - \cos u)$$
$$y = a\sin u + av(\sin(u+t) - \sin u)$$
$$z = -c + 2cv$$
$$0 \le u \le 2\pi,\ v \in R$$



$$t = 3.956$$

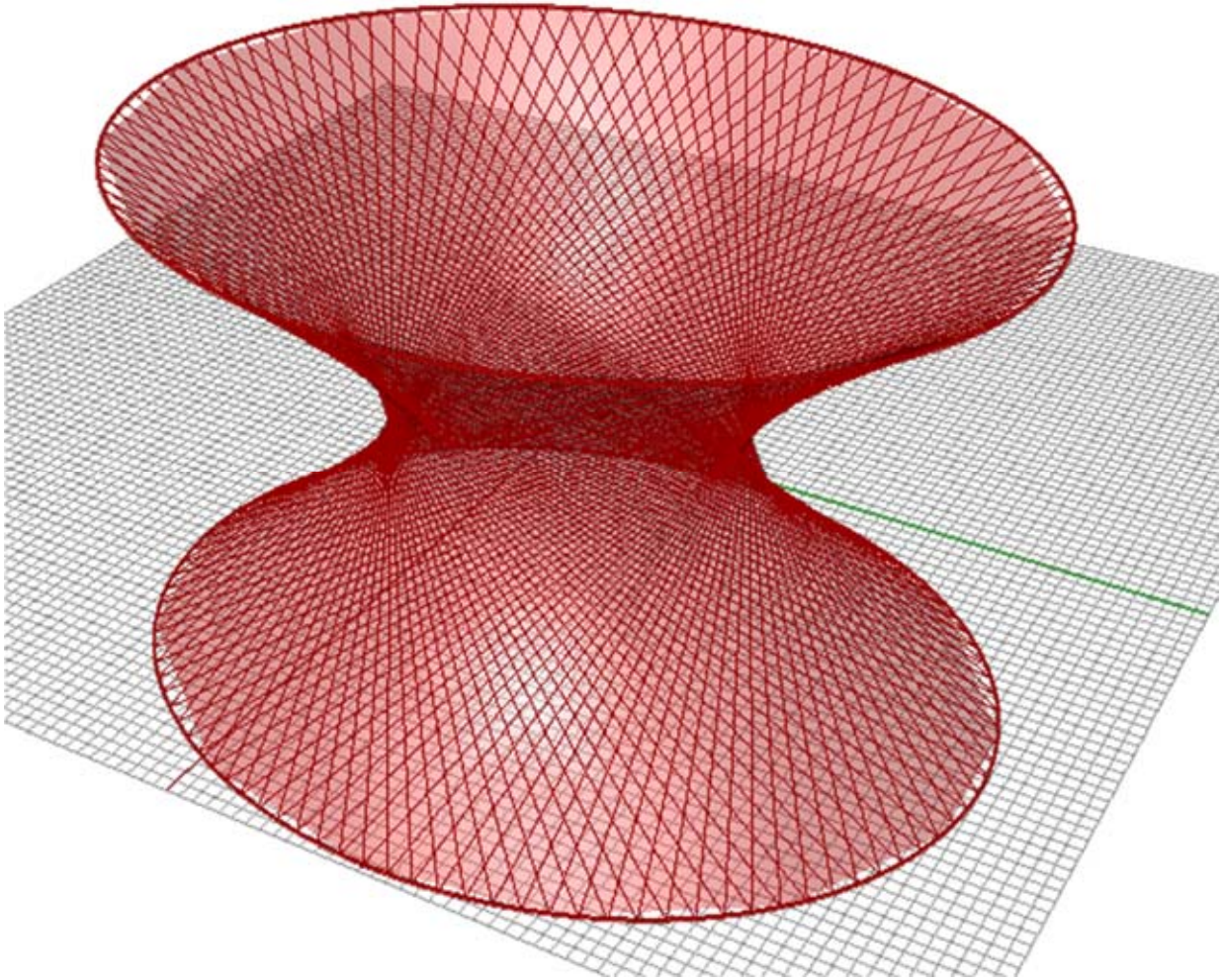
$$x = a\cos u + av(\cos(u-t) - \cos u)$$
$$y = a\sin u + av(\sin(u-t) - \sin u)$$
$$z = -c + 2cv$$
$$0 \le u \le 2\pi,\ v \in R$$



$$t = 2\pi - 3.956 = 2.324$$

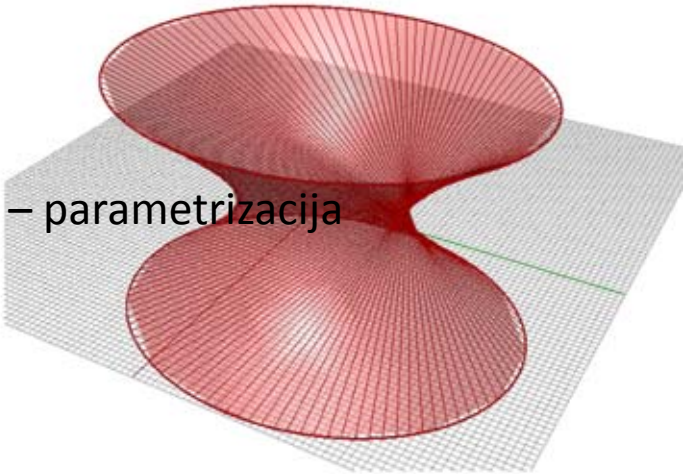
POVRŠI U PROSTORU

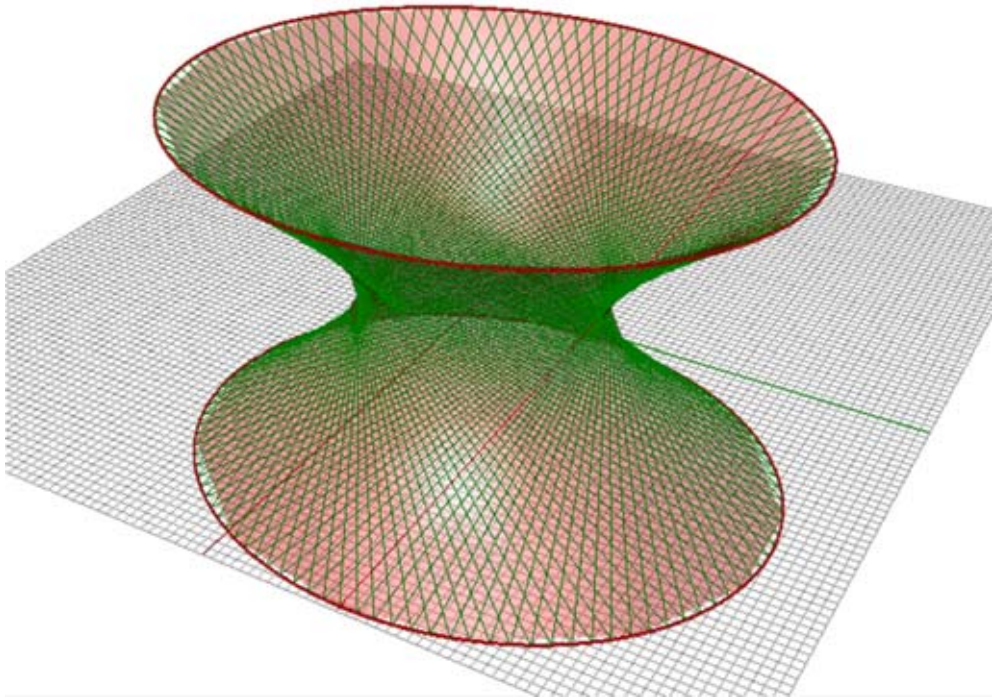
Površ kao trag pokretne krive – jednograni hiperboloid – parametrizacija

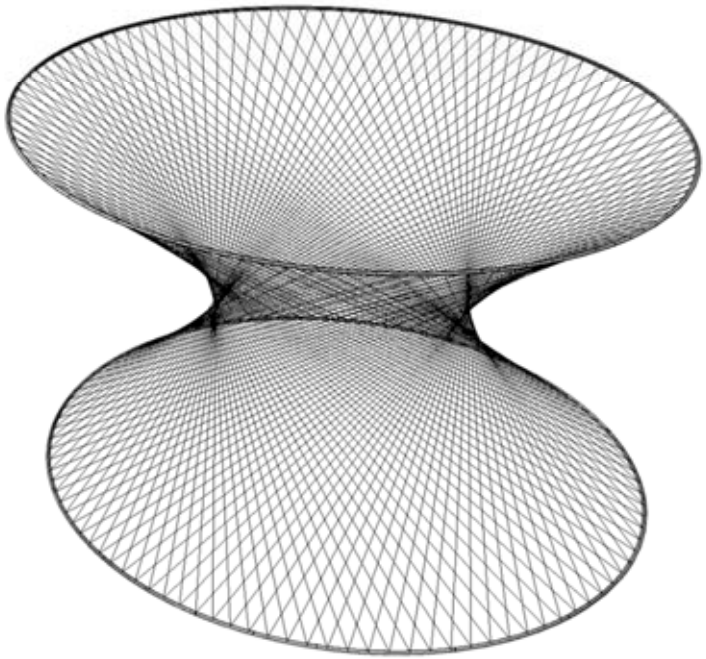


POVRŠI U PROSTORU

Površ kao trag pokretne krive – jednograni hiperboloid – parametrizacija
Izoparametarske linije

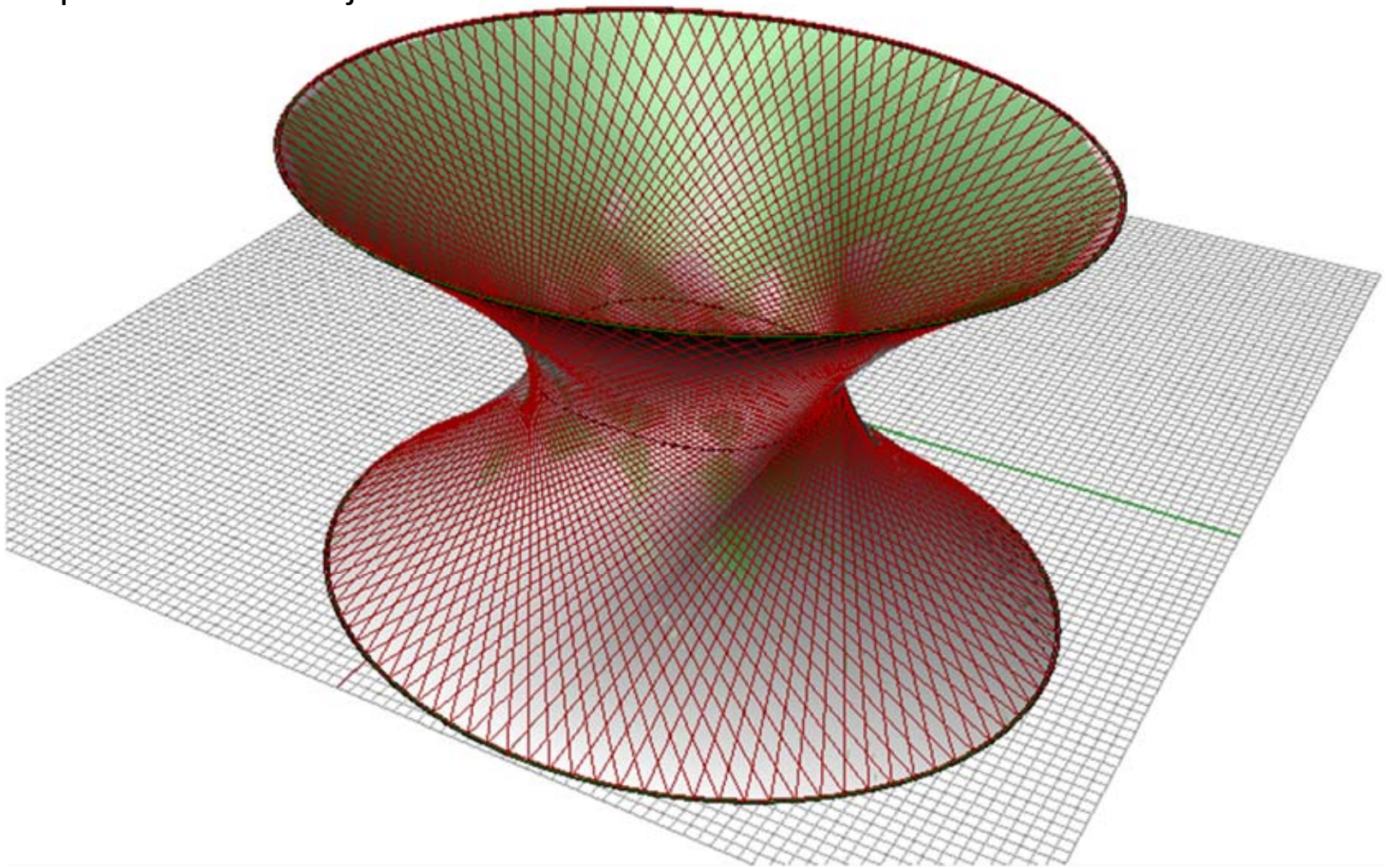






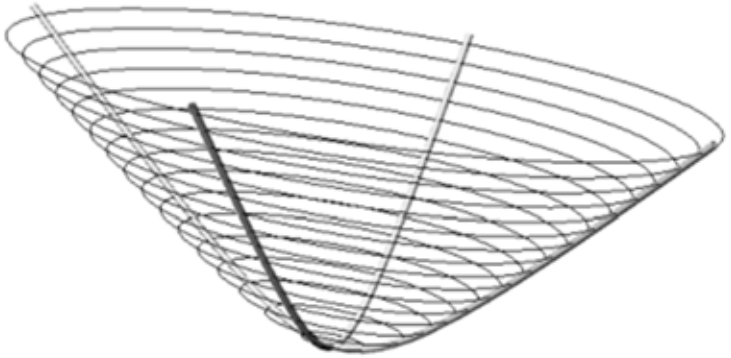
POVRŠI U PROSTORU

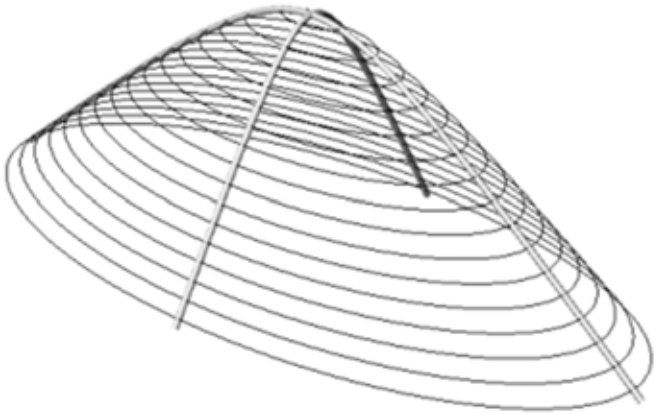
Površ kao trag pokretne krive – jednograni hiperboloid – parametrizacija
Izoparametarske linije



POVRŠI U PROSTORU

Površ kao trag pokretne krive – dvograni hiperboloid





$$D_1 : \frac{z^2}{b^2} - \frac{x^2}{a^2} = 1, \quad y = 0$$

$$D_2 : \frac{z^2}{c^2} - \frac{y^2}{b^2} = 1, \quad x = 0$$

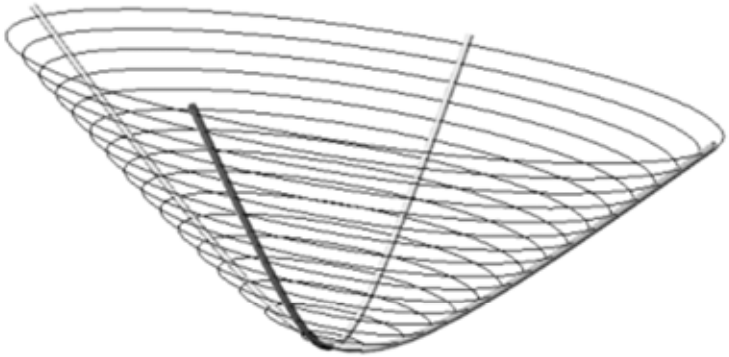
$$G : \frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} = 1, \quad z = \gamma$$

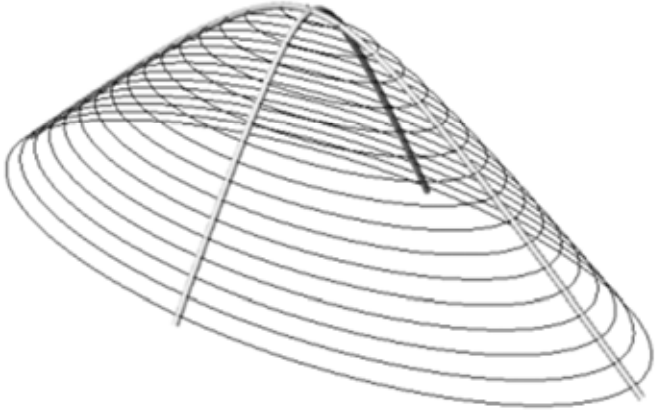
Jednačina dvogranog hiperboloida:

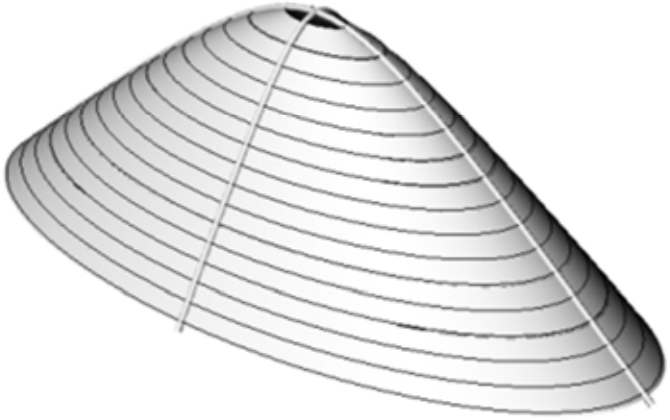
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = -1$$

POVRŠI U PROSTORU

Površ kao trag pokretne krive – dvograni hiperboloid

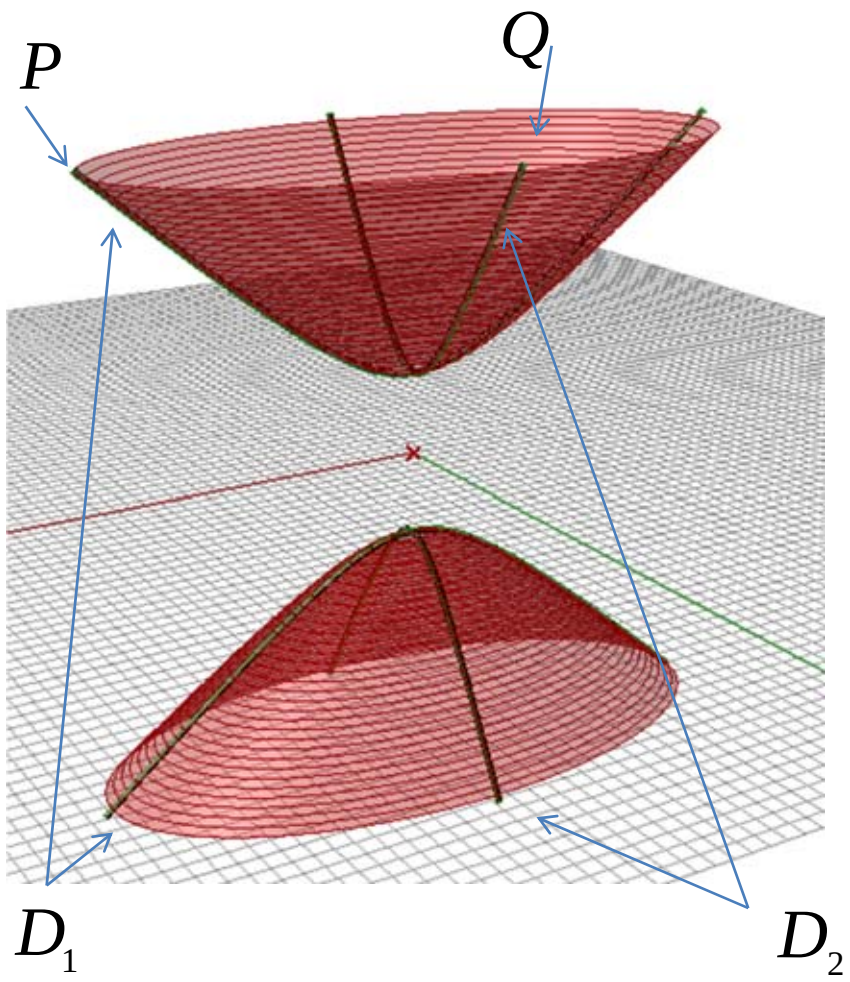






POVRŠI U PROSTORU

Površ kao trag pokretne krive – dvograni hiperboloid – parametrizacija



$$D_1 : x = a\sinh\theta,\ y = 0,\ z = c\cosh\theta$$

$$D_2 :\ x = 0,\ y = b\sinh\theta,\ z = c\cosh\theta$$

$$G : x = \alpha\cos\varphi, y = \beta\sin\varphi, z = \gamma$$

$$P(\alpha,0,\gamma),\ Q(0,\beta,\gamma)\ \in G$$

$$P \in D_1,\ Q \in D_2$$

$$x = a\sinh\theta\cos\varphi$$

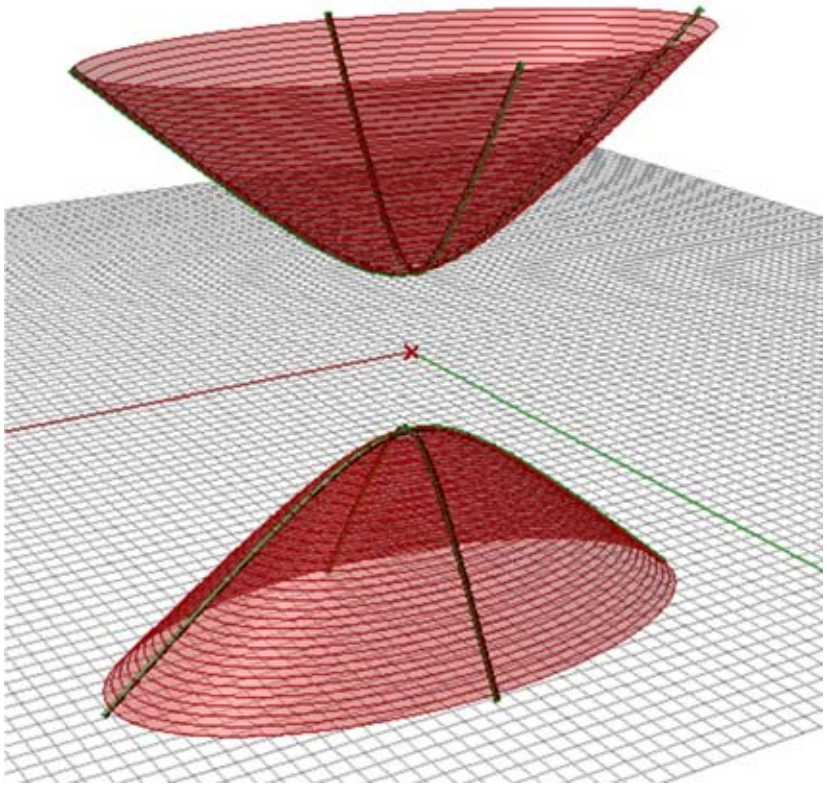
$$y = b\sinh\theta\sin\varphi$$

$$z = c\cosh\theta$$

$$0 \le \varphi \le 2\pi,\ -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$$

POVRŠI U PROSTORU

Površ kao trag pokretne krive – dvograni hiperboloid – parametrizacija



$$D_1 : x = a\tan\theta,\ y = 0,\ z = c\sec\theta$$

$$D_2 :\ x = 0,\ y = b\tan\theta,\ z = c\sec\theta$$

$$G : x = \alpha\cos\varphi, y = \beta\sin\varphi, z = \gamma$$

$$P(\alpha,0,\gamma),\ Q(0,\beta,\gamma)\ \in G$$

$$P \in D_1,\ Q \in D_2$$

$$x = a\tan\theta\cos\varphi$$

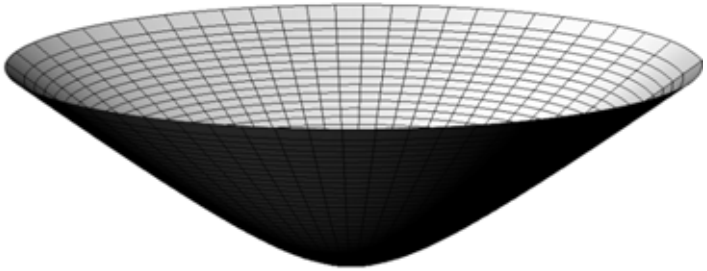
$$y = b\tan\theta\sin\varphi$$

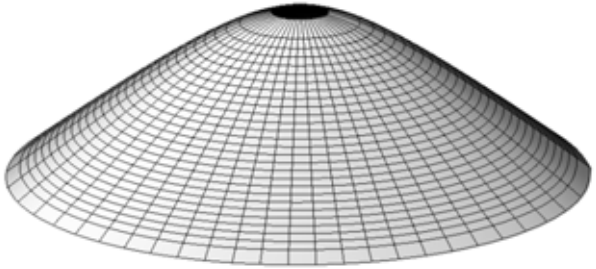
$$z = c\sec\theta$$

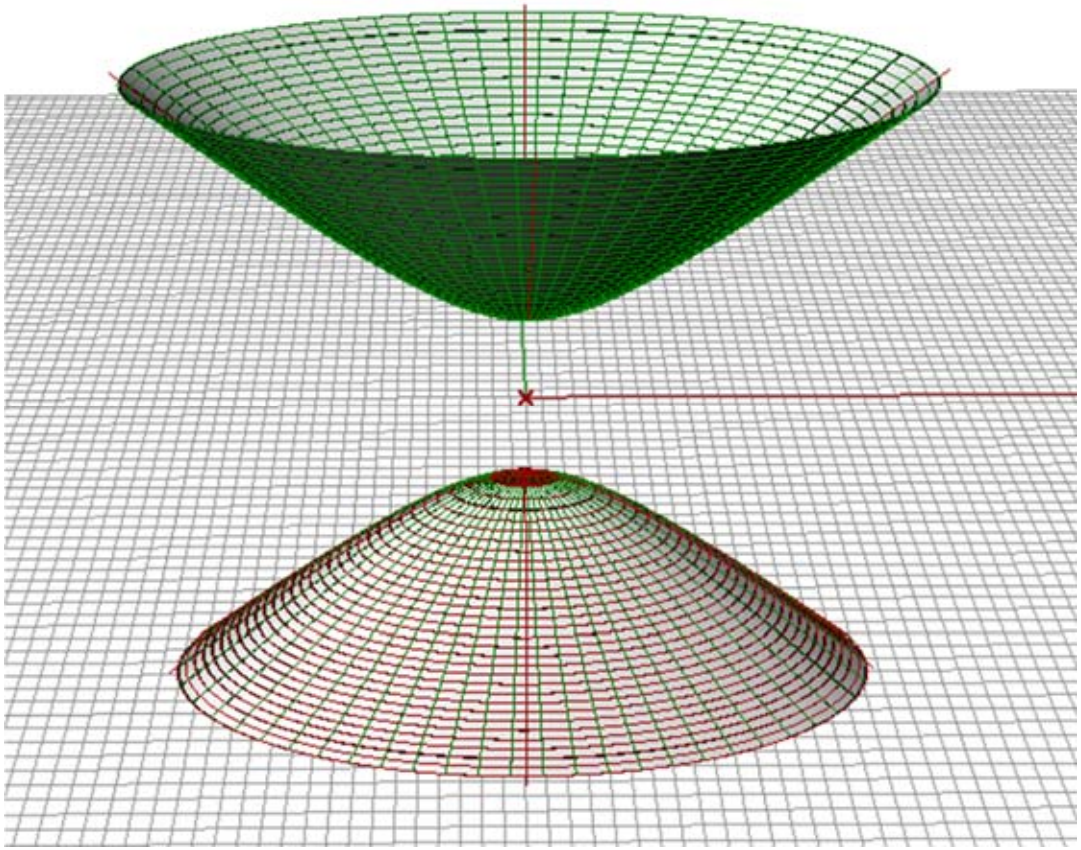
$$0 \le \varphi \le 2\pi,\ -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$$

POVRŠI U PROSTORU

Površ kao trag pokretne krive – dvograni hiperboloid -
Parametrizacija - Izoparametarske linije

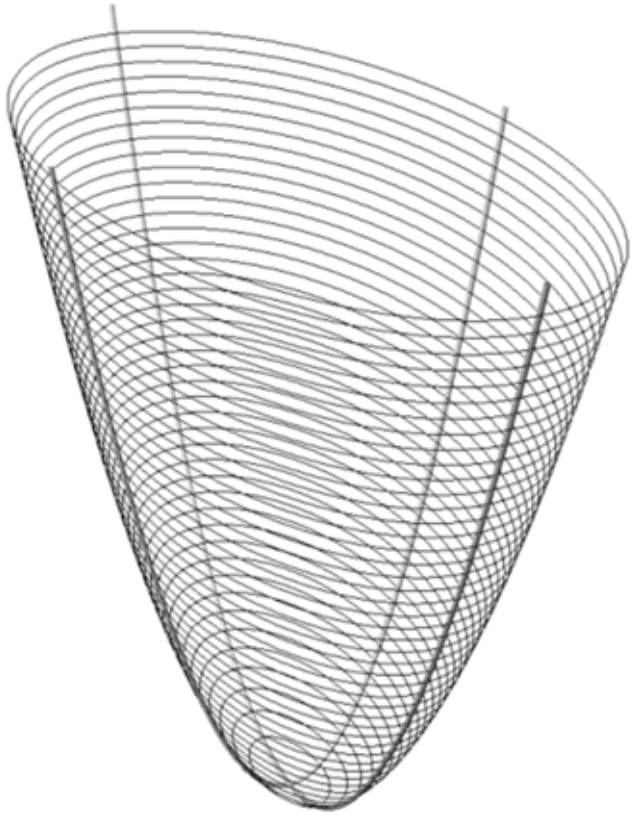






POVRŠI U PROSTORU

Površ kao trag pokretne krive – eliptički paraboloid



$$D_1 : x^2 = 2pz, \quad y = 0$$

$$D_2 : \ y^2 = 2qz, \quad x = 0$$

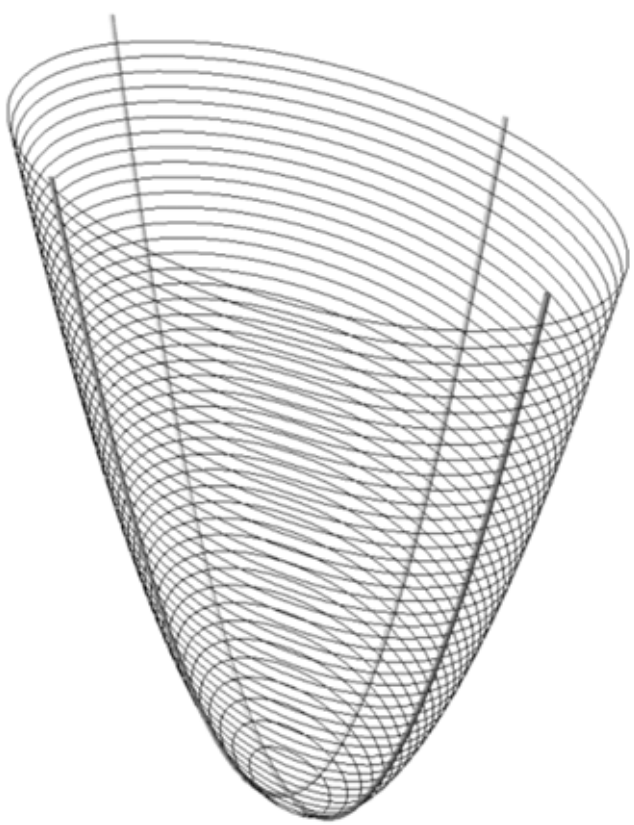
$$G : \frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} = 1, \quad z = \gamma$$

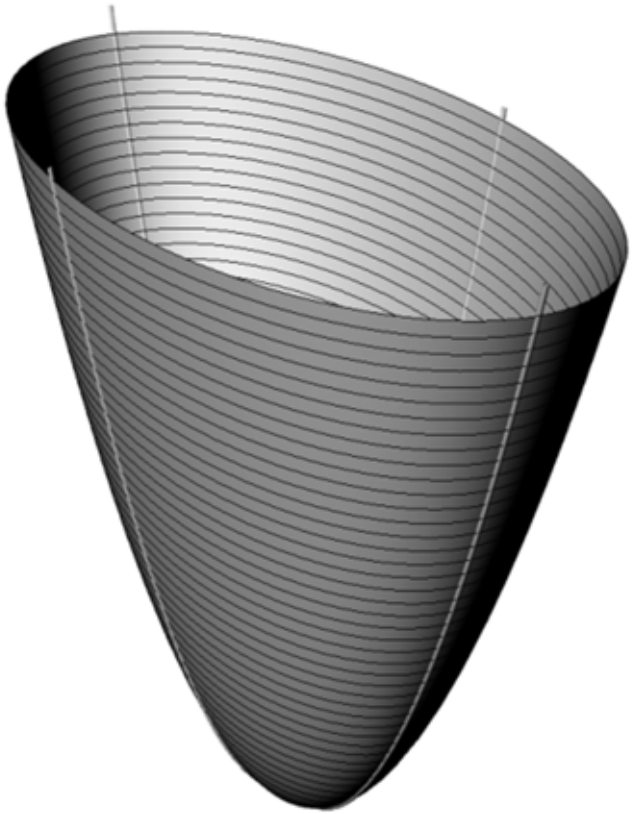
Jednačina eliptičkog paraboloida:

$$\frac{x^2}{p} + \frac{y^2}{q} = 2z$$

POVRŠI U PROSTORU

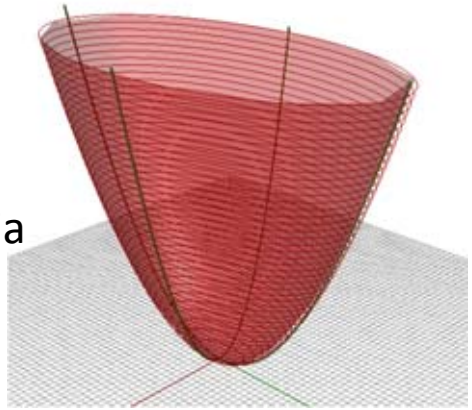
Površ kao trag pokretne krive – eliptički paraboloid

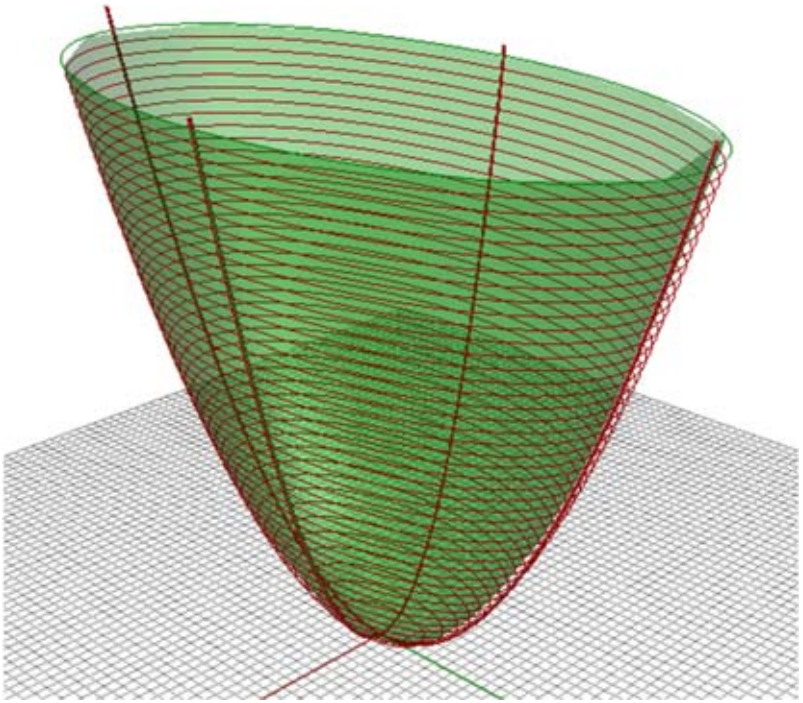




POVRŠI U PROSTORU

Površ kao trag pokretne krive – eliptički paraboloid - parametrizacija





$$D_1 : x^2 = 2pu, \quad y = 0, z = u$$

$$D_2 : \ y^2 = 2qu, \quad x = 0, z = u$$

$$G : x = a\cos\varphi, \quad y = b\sin\varphi, \ \ z = \gamma$$

$$P(\alpha,0,\gamma), \ \ Q(0,\beta,\gamma) \ \ \in G$$

$$P \in D_1, \ \ Q \in D_2$$

$$x = \sqrt{2pu}\cos\varphi$$

$$y = \sqrt{2qu}\sin\varphi$$

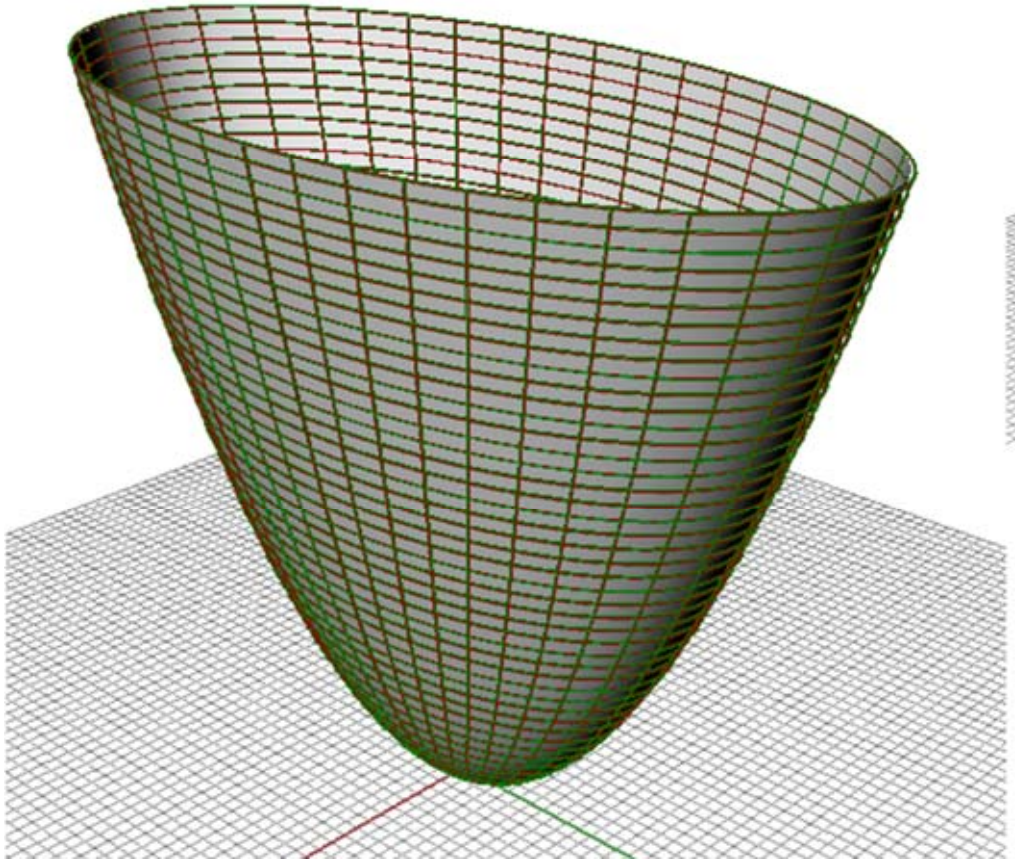
$$z = u$$

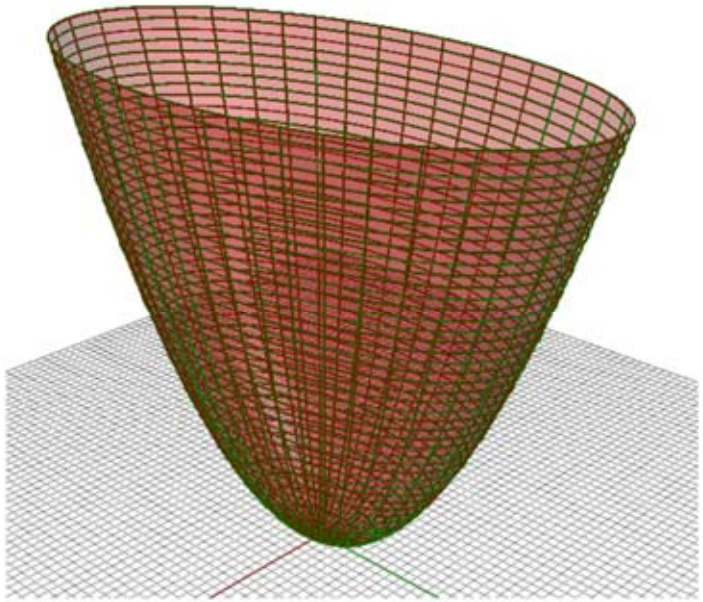
$$u \geq 0, \quad 0 \leq \varphi \leq 2\pi$$

POVRŠI U PROSTORU

Površ kao trag pokretne krive – eliptički paraboloid – parametrizacija

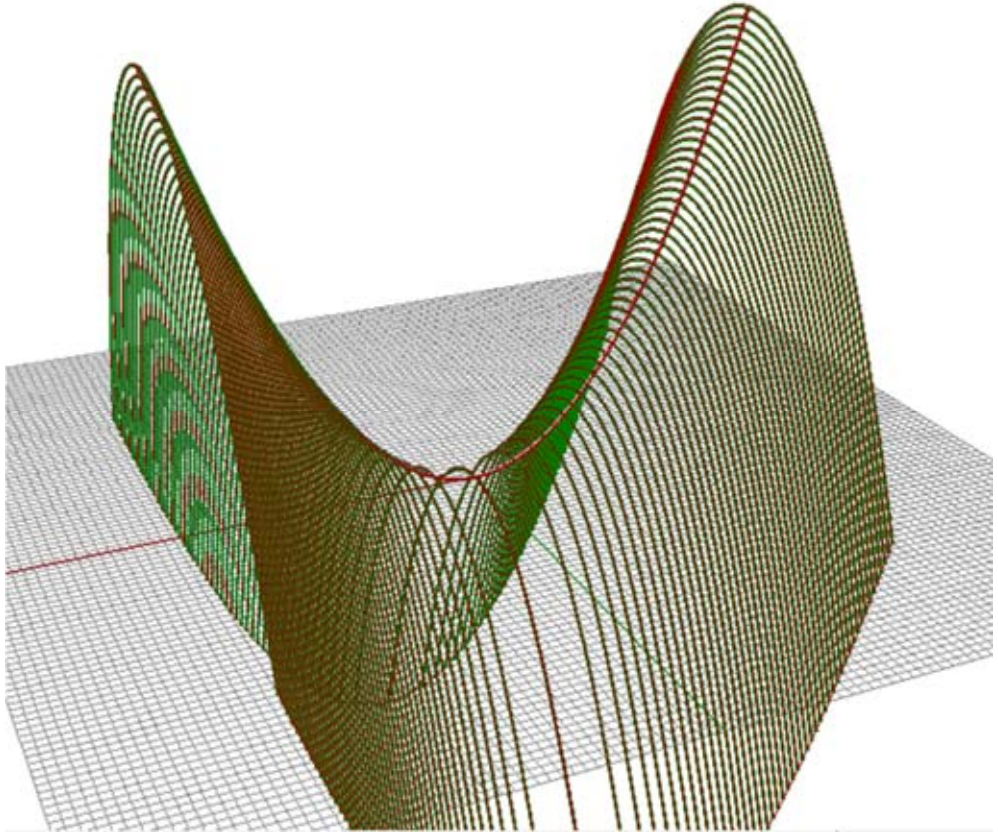
Izoparametarske linije

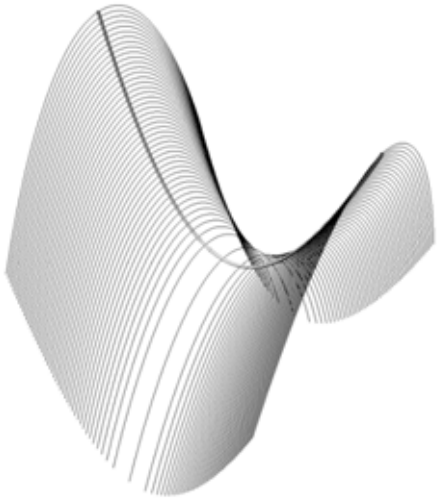




POVRŠI U PROSTORU

Površ kao trag pokretne krive – hiperbolički paraboloid





$$D: x^2 = 2pz, \quad y = 0$$

$$G: \ y^2 = -2q(z-\gamma), \quad x = \beta$$

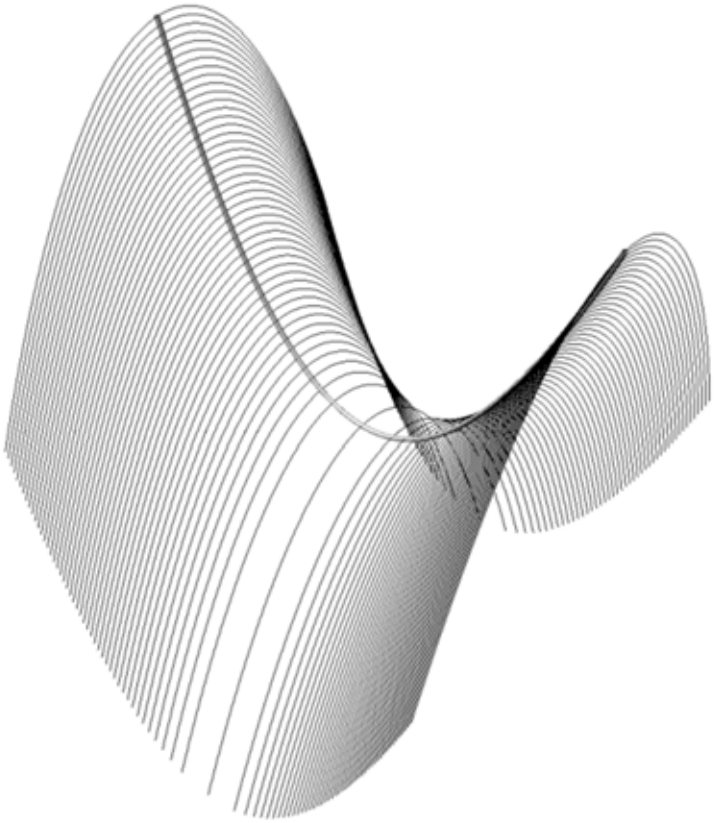
$$p, q > 0$$

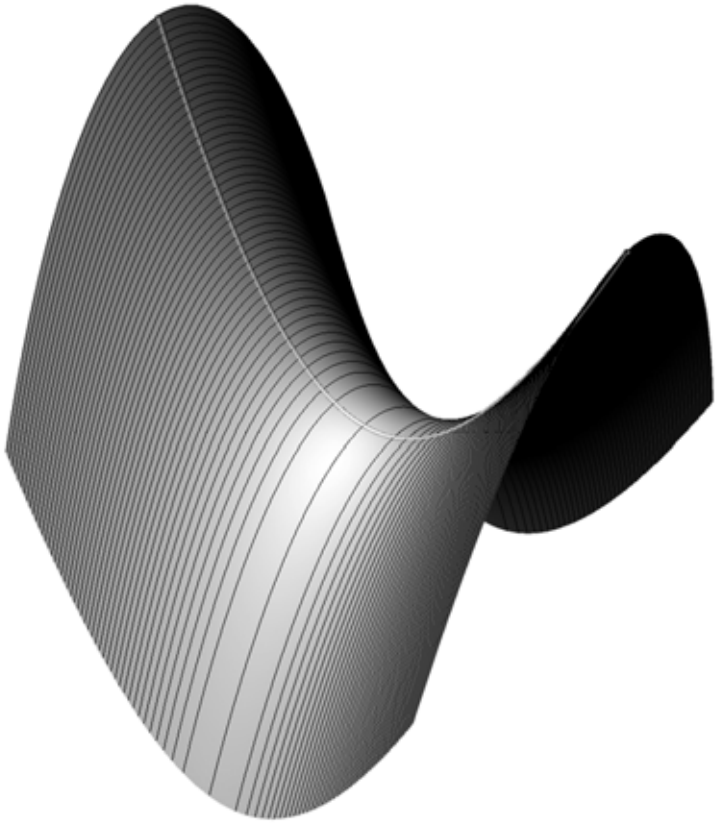
Jednačina hiperboličkog paraboloida:

$$\frac{x^2}{p} - \frac{y^2}{q} = 2z$$

POVRŠI U PROSTORU

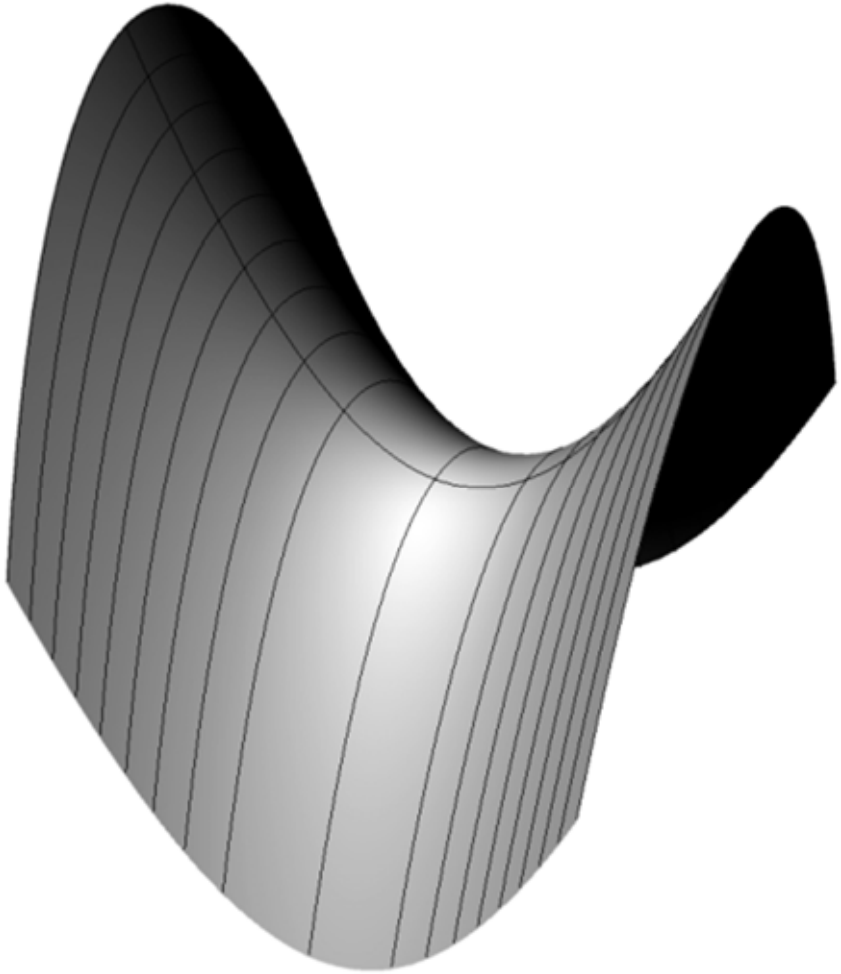
Površ kao trag pokretne krive – hiperbolički paraboloid





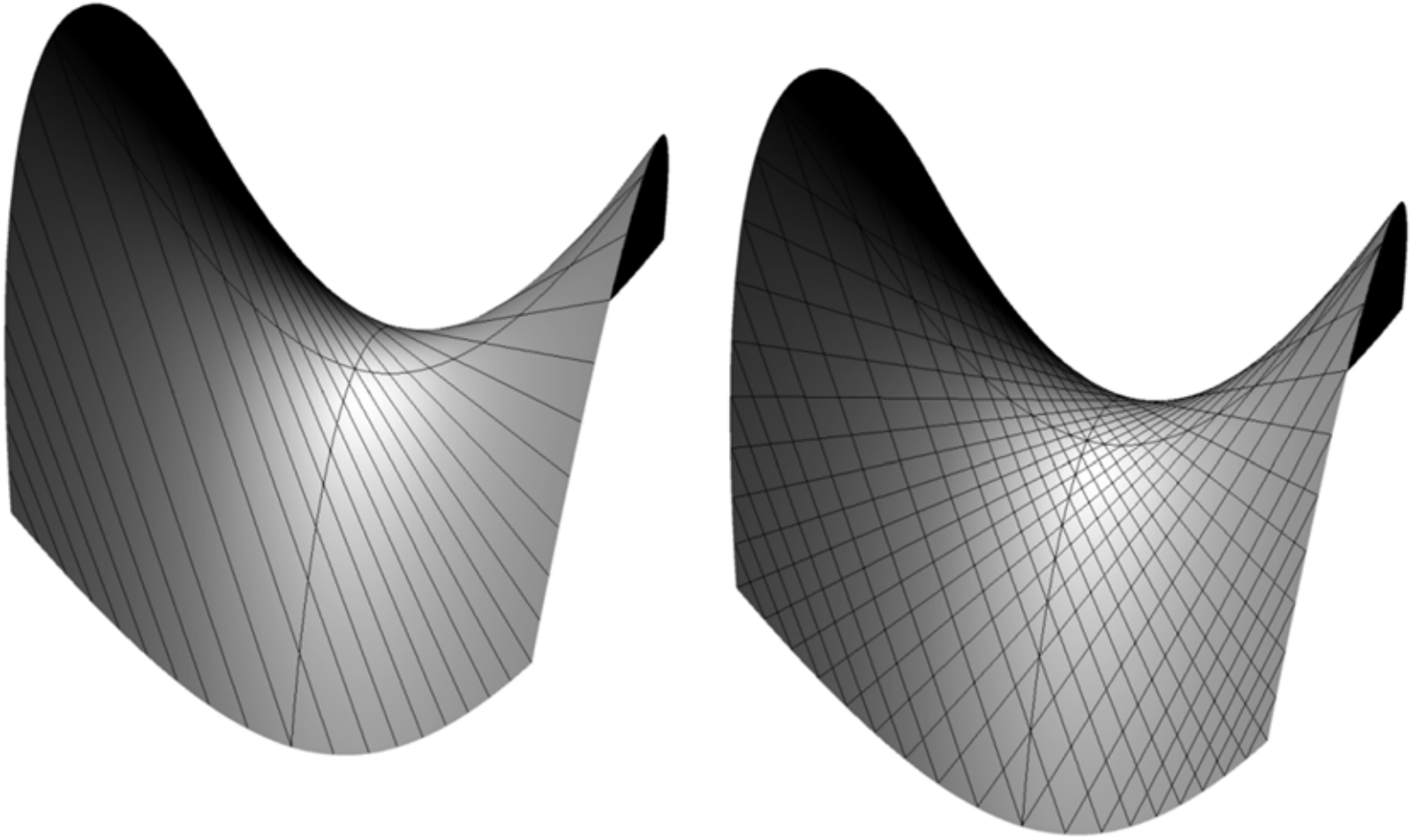
POVRŠI U PROSTORU

Površ kao trag pokretne krive – hiperbolički paraboloid



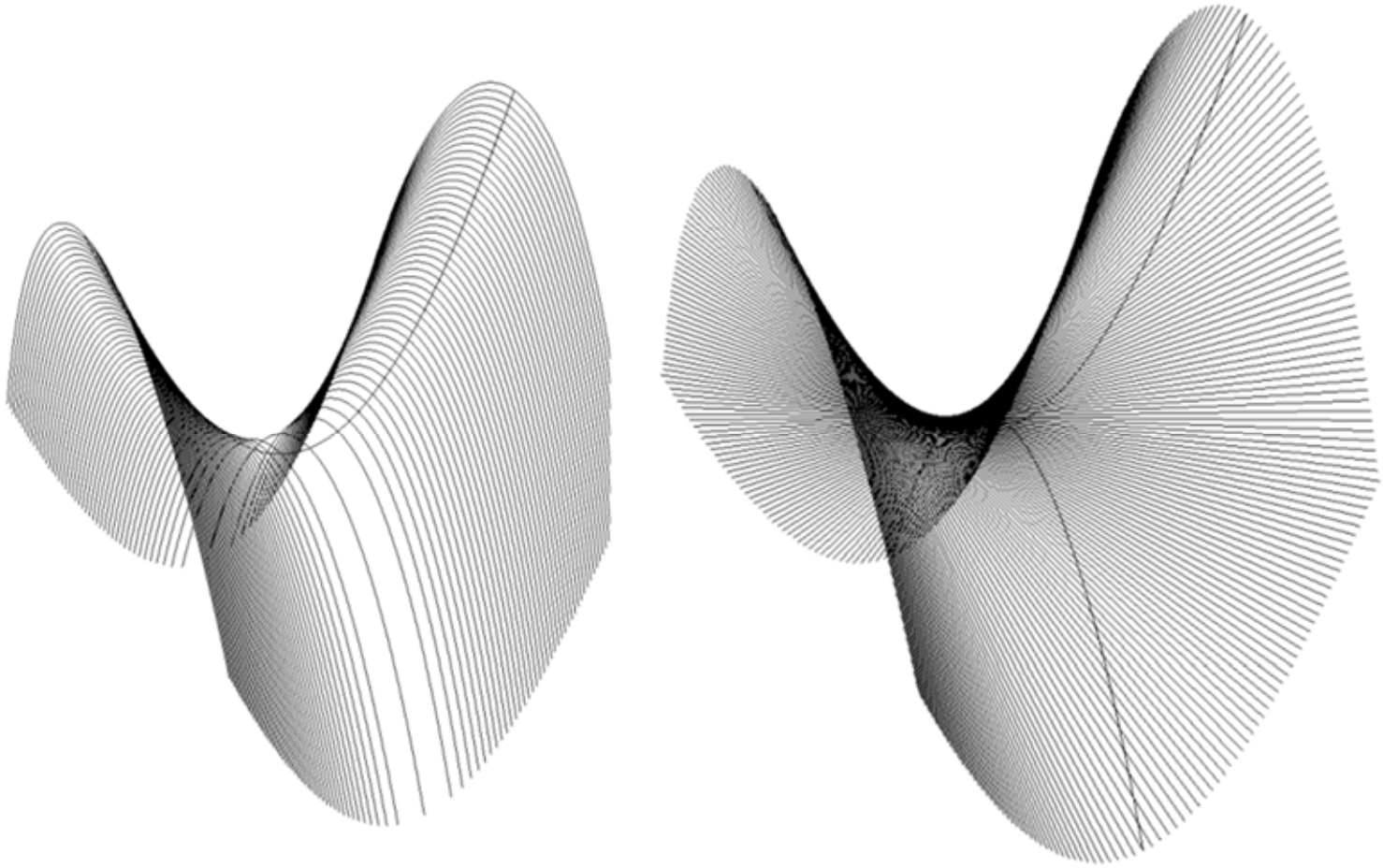
POVRŠI U PROSTORU

Površ kao trag pokretne krive – hiperbolički paraboloid



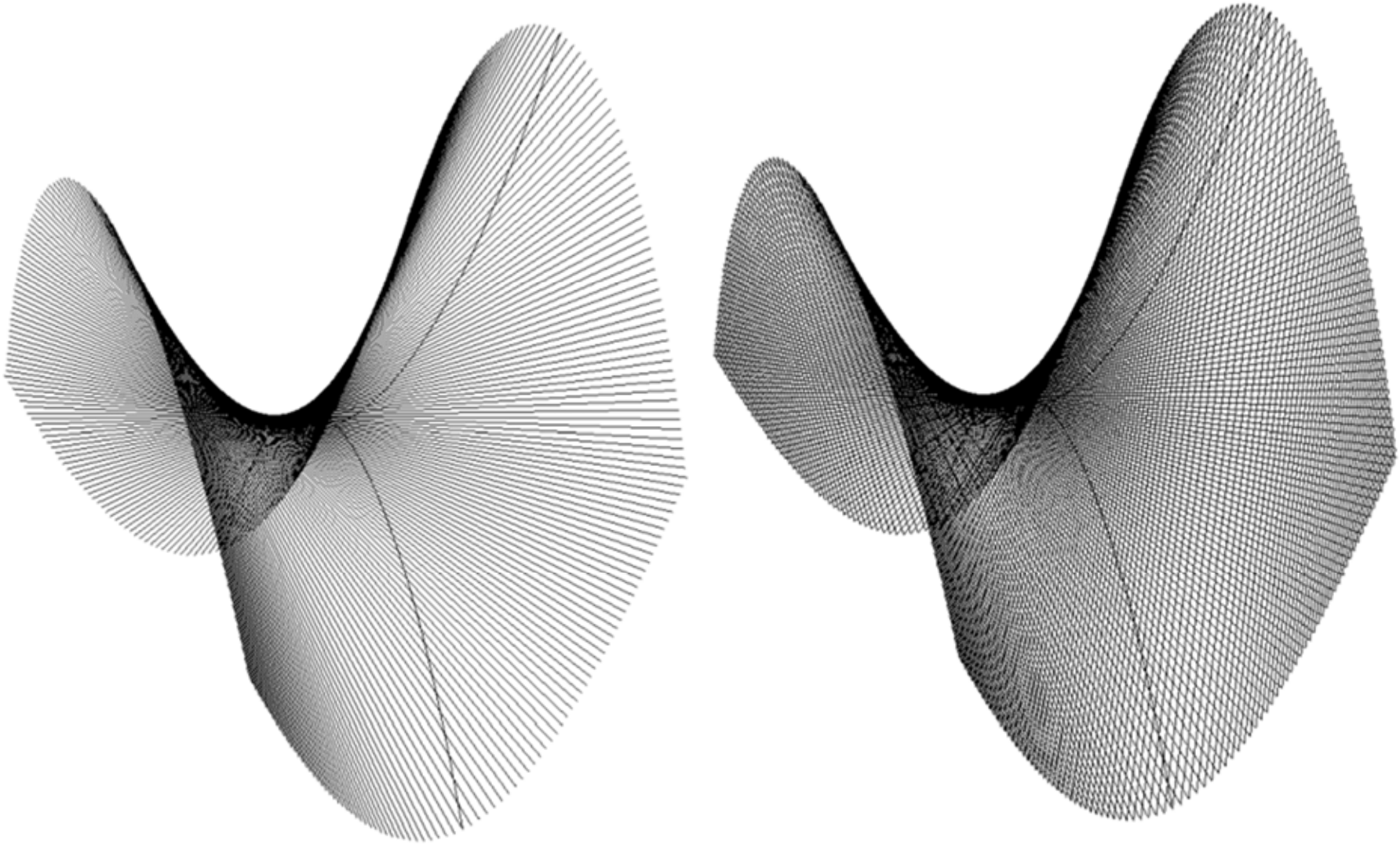
POVRŠI U PROSTORU

Površ kao trag pokretne krive – hiperbolički paraboloid



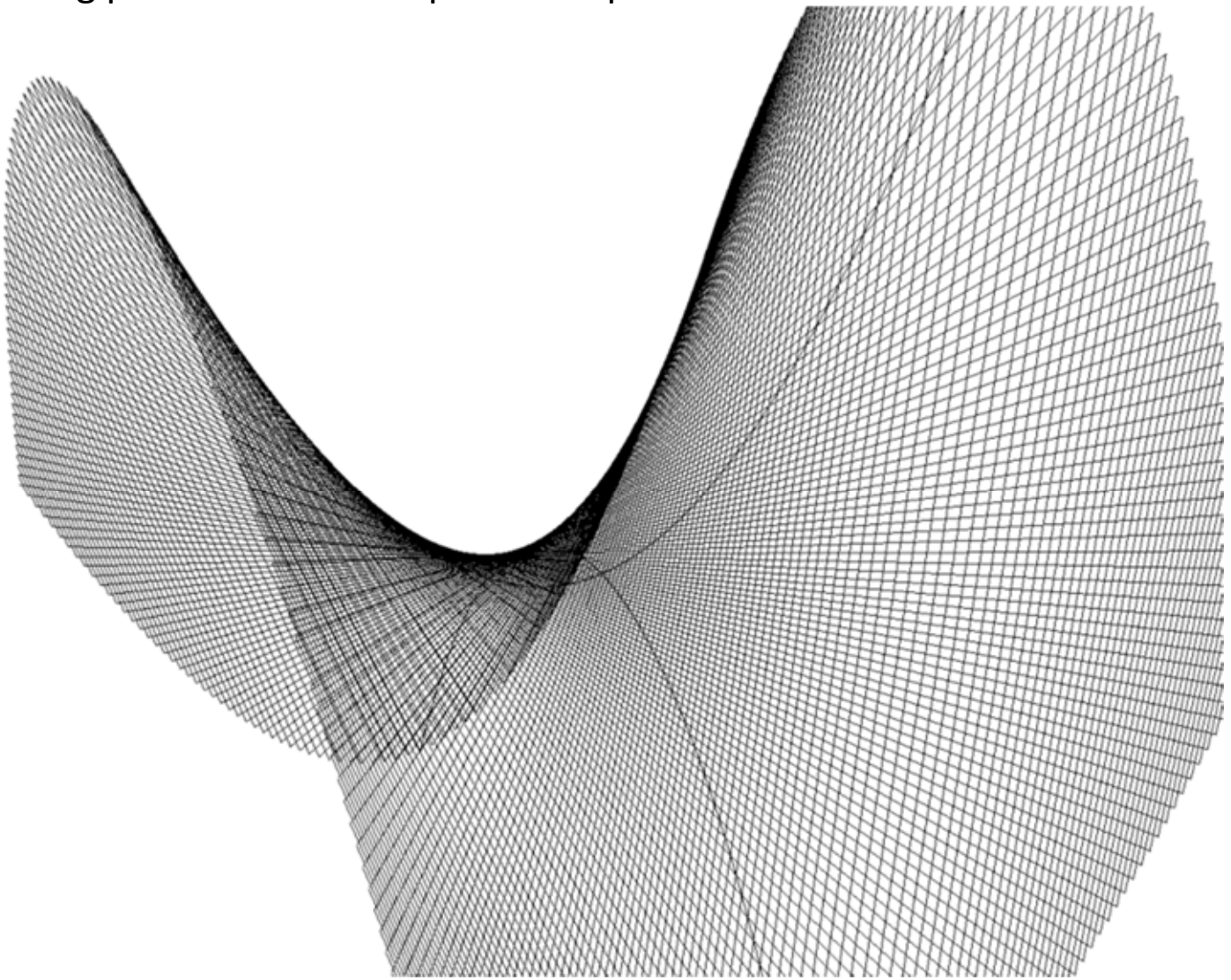
POVRŠI U PROSTORU

Površ kao trag pokretne krive – hiperbolički paraboloid



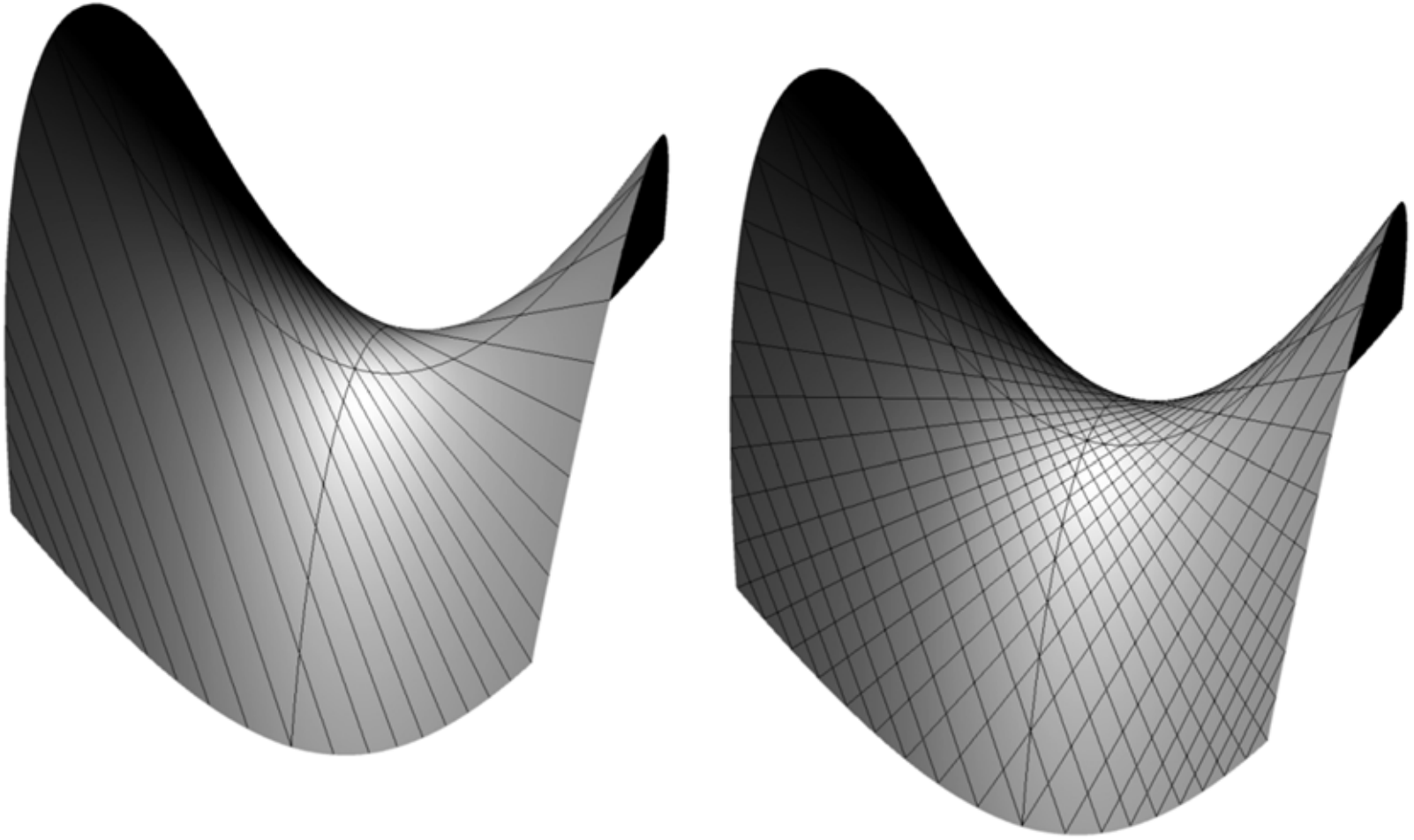
POVRŠI U PROSTORU

Površ kao trag pokretne krive – hiperbolički paraboloid



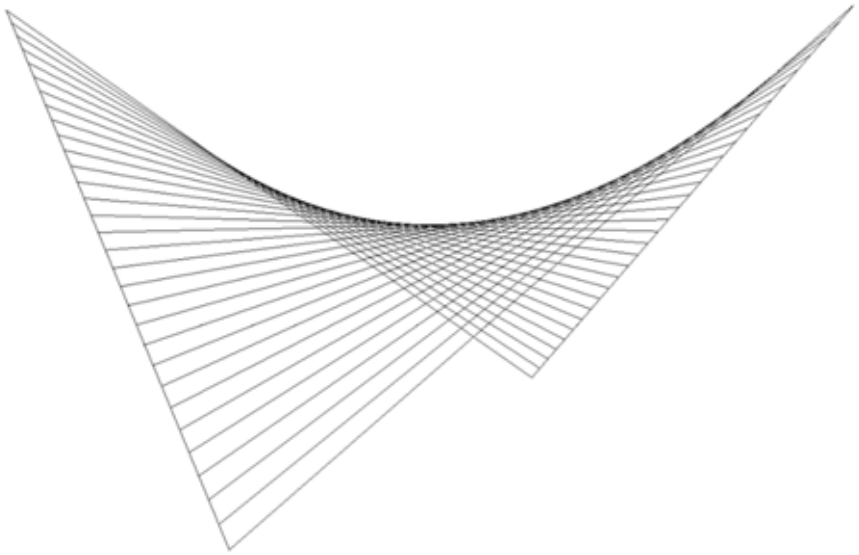
POVRŠI U PROSTORU

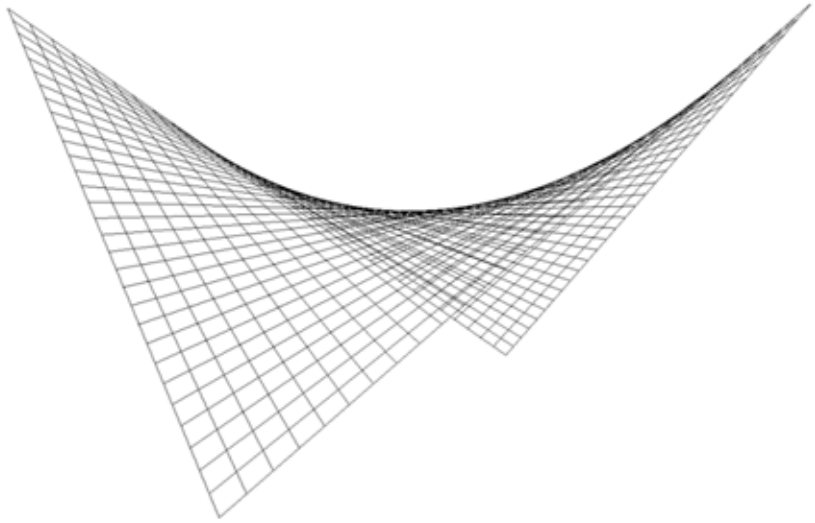
Površ kao trag pokretne krive – hiperbolički paraboloid



POVRŠI U PROSTORU

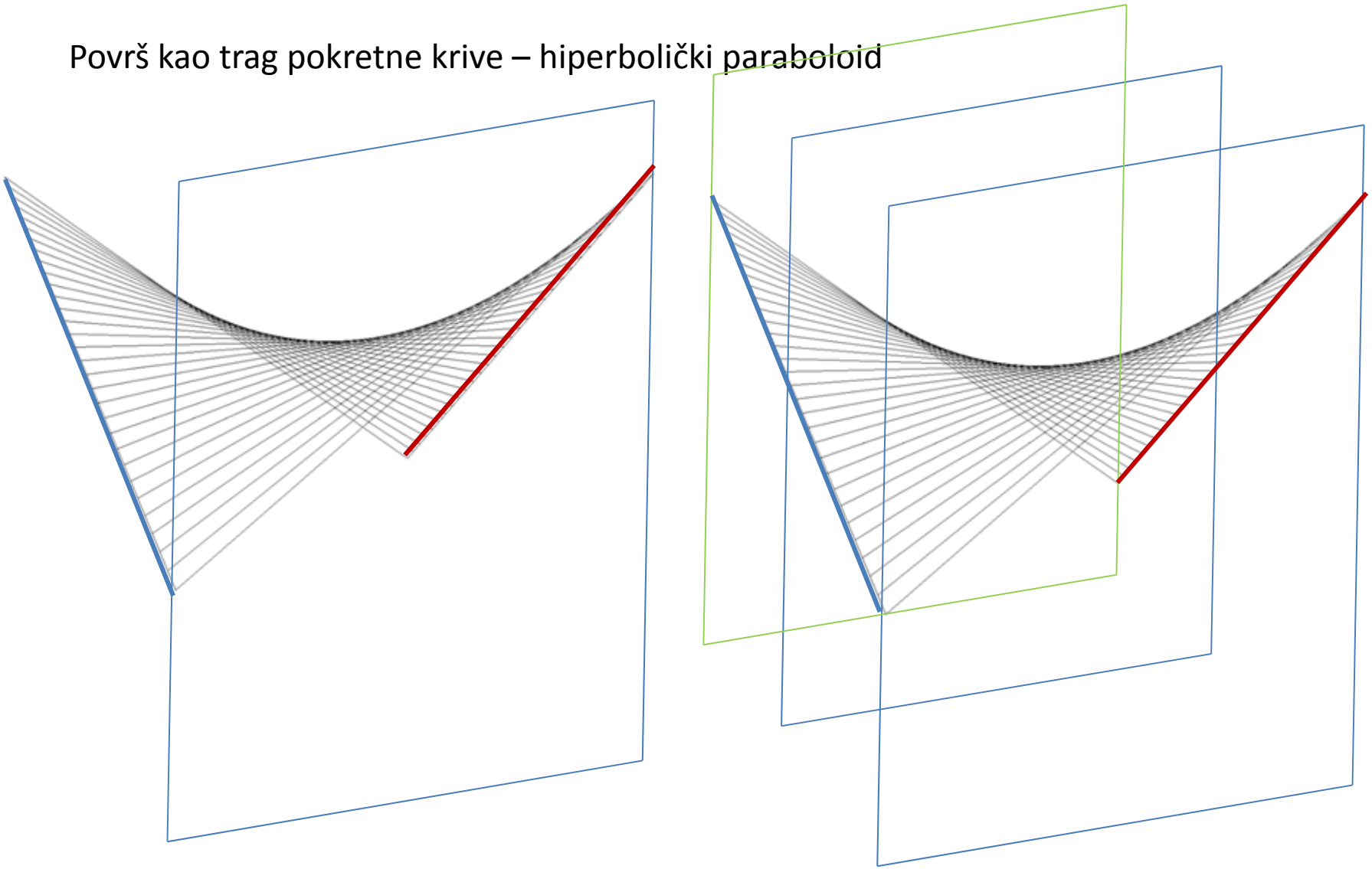
Površ kao trag pokretne krive – hiperbolički paraboloid





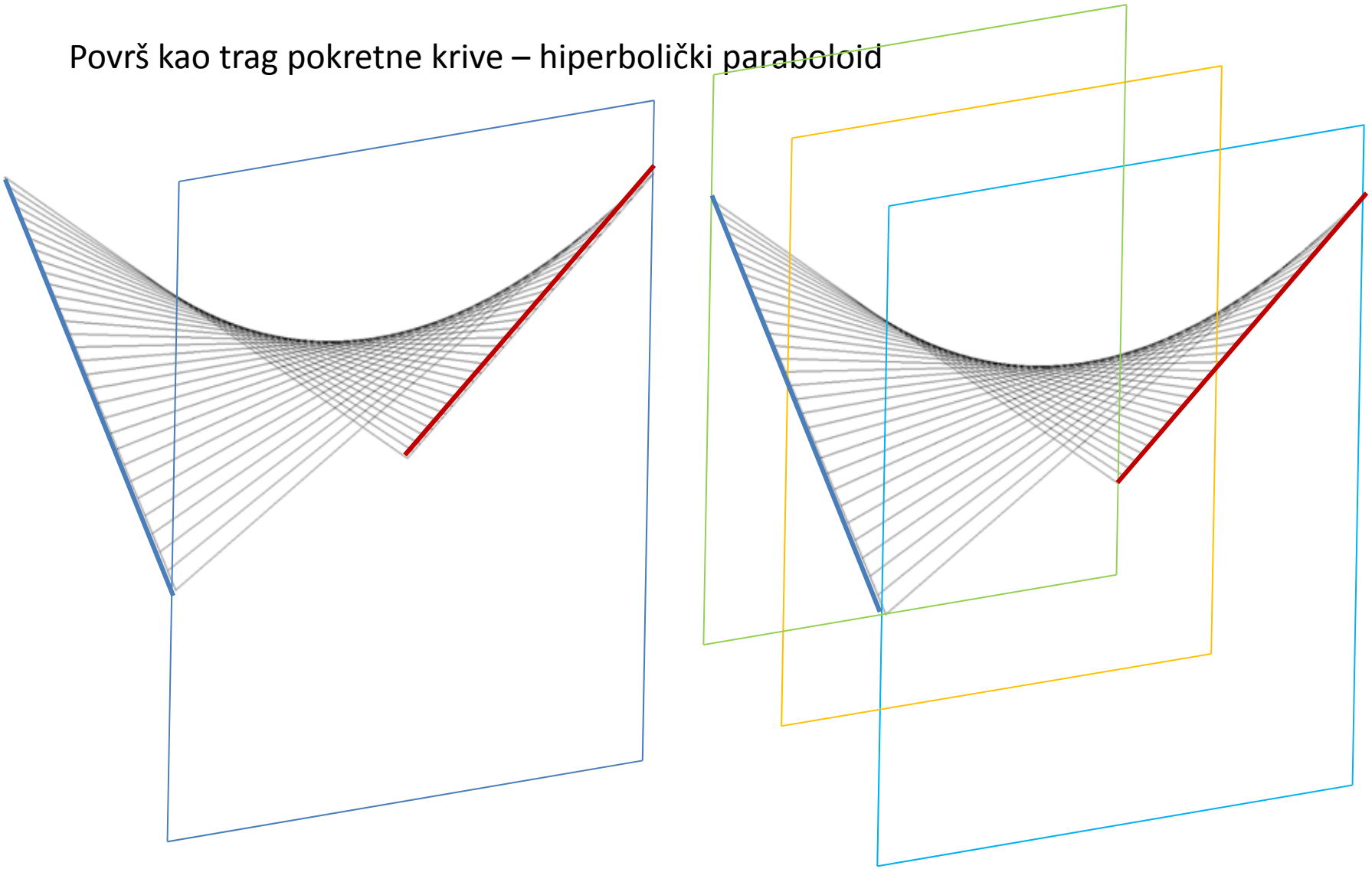
POVRŠI U PROSTORU

Površ kao trag pokretne krive – hiperbolički paraboloid



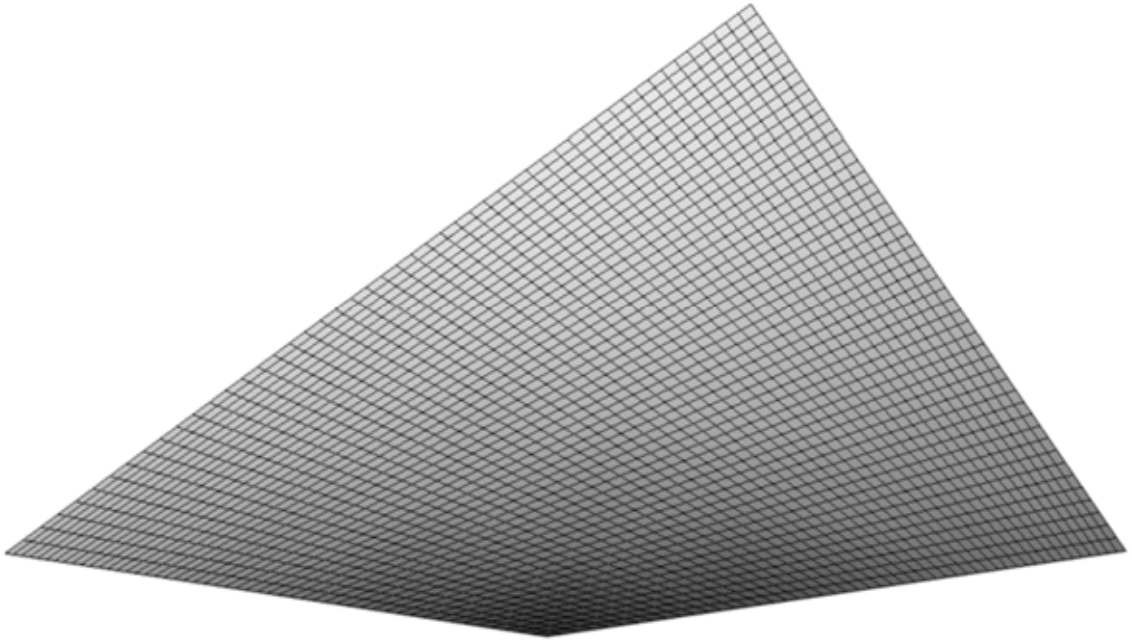
POVRŠI U PROSTORU

Površ kao trag pokretne krive – hiperbolički paraboloid



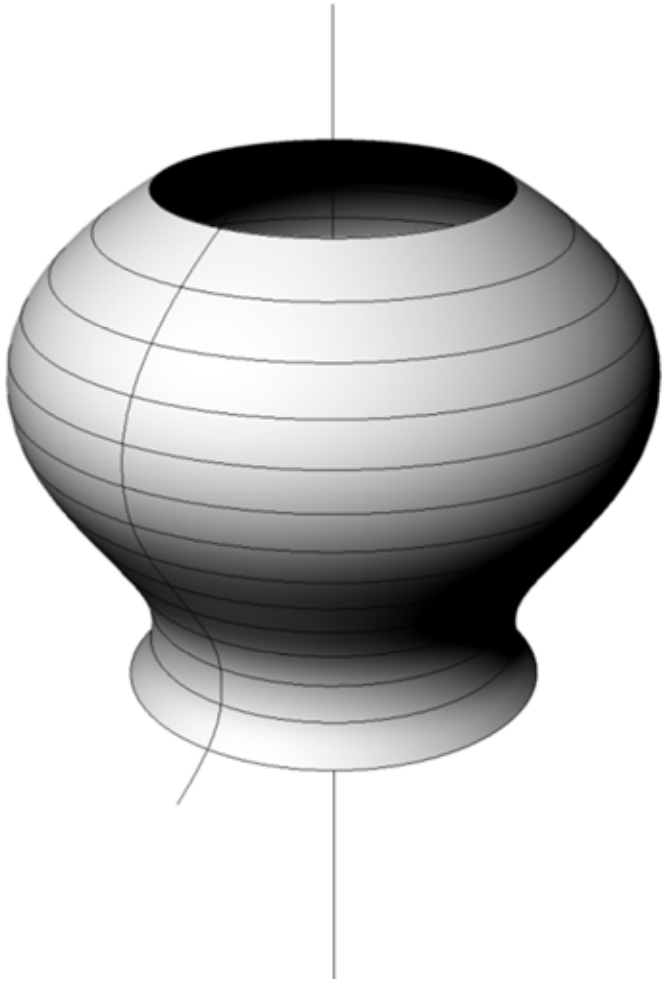
POVRŠI U PROSTORU

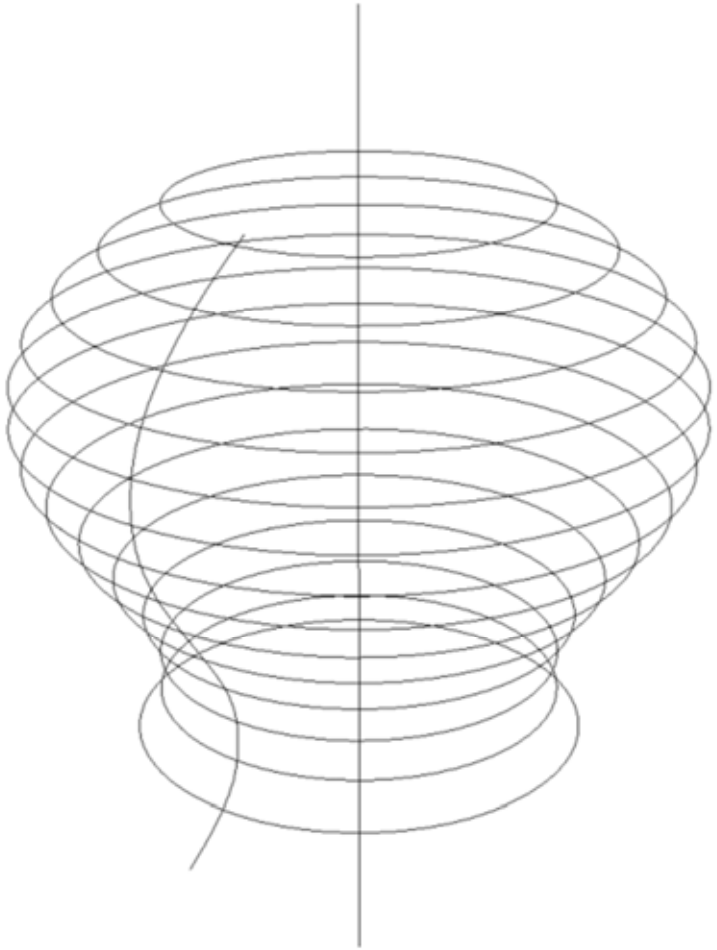
Površ kao trag pokretne krive – hiperbolički paraboloid



POVRŠI U PROSTORU

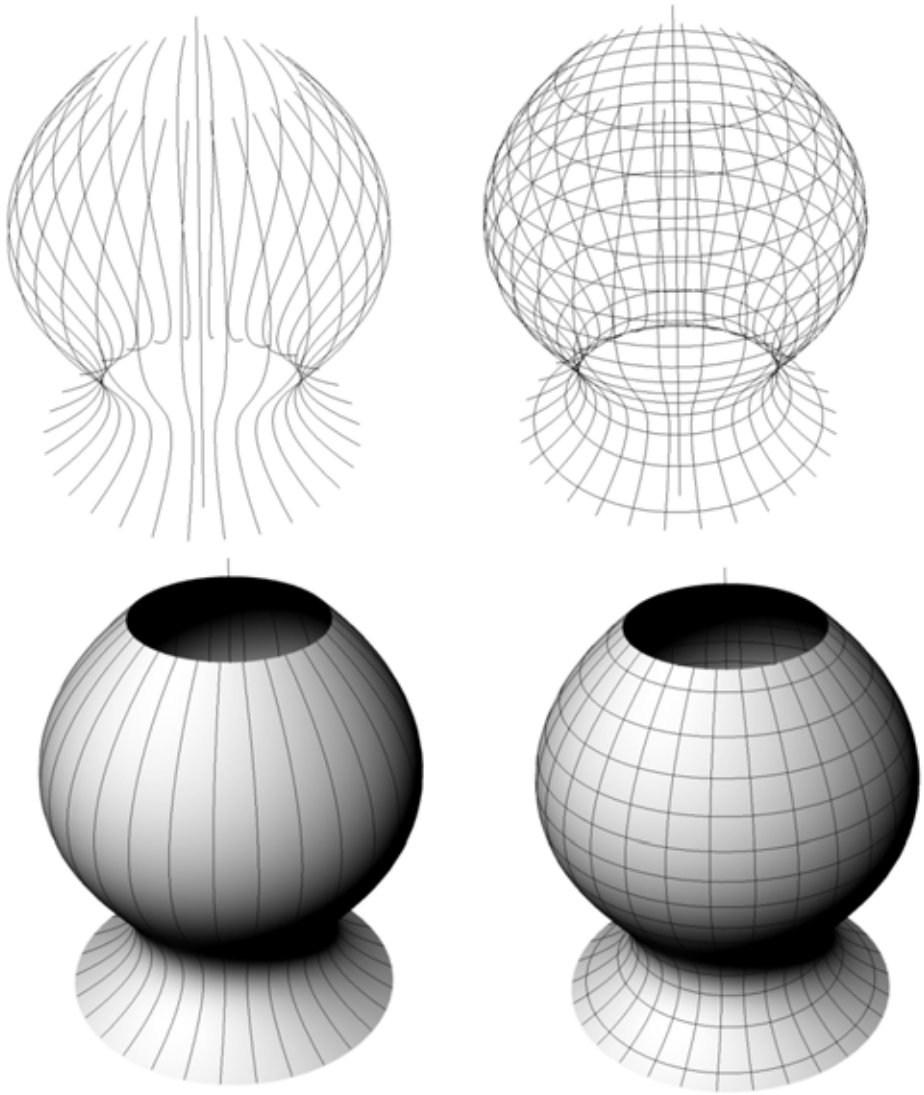
Rotacione površi





POVRŠI U PROSTORU

Rotacione površi



POVRŠI U PROSTORU

Rotacione površi

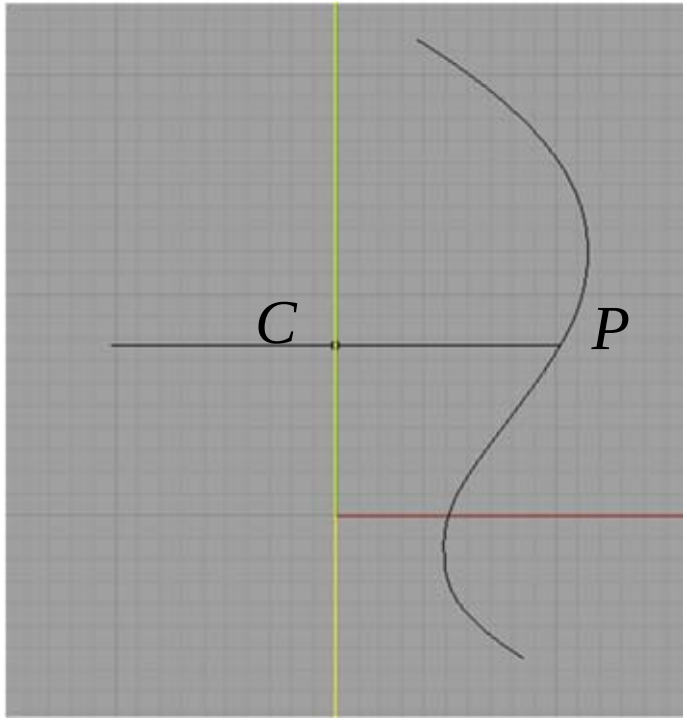
Rotacija oko *z* - ose

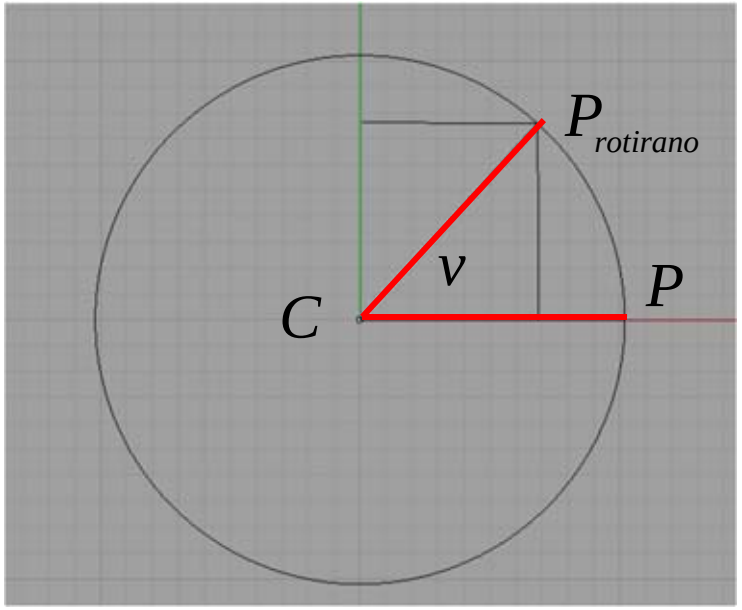
Jednačina krive u *xz* – ravni:

$$x = \alpha_1(u),\ y = 0,\ \ z = \alpha_3(u)$$

$$P(\alpha_1(u), 0, \alpha_3(u))$$

$$C(0,0,\alpha_3(t))$$





$$x = \alpha_1(u)\cos v,\ y = \alpha_1(u)\sin v,\ \ z = \alpha_3(u)$$

POVRŠI U PROSTORU

Rotacione površi

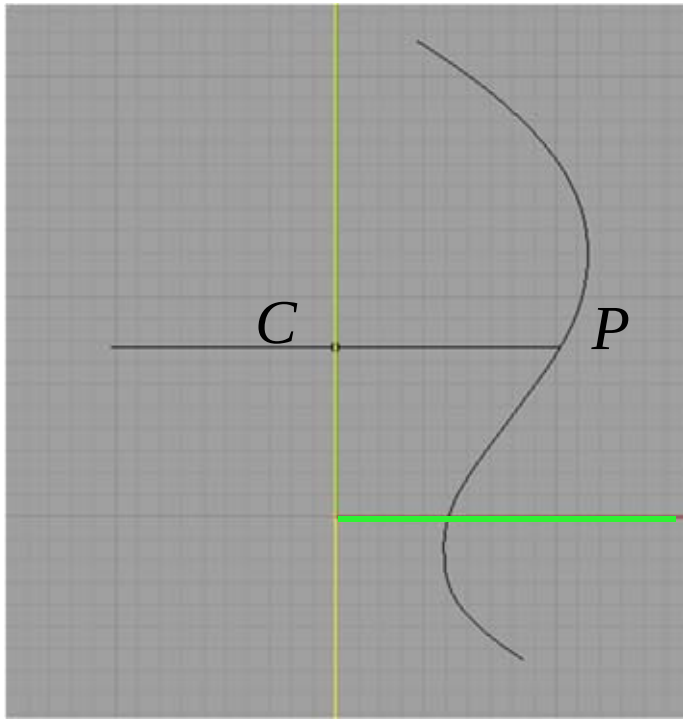
Rotacija oko *z* - ose

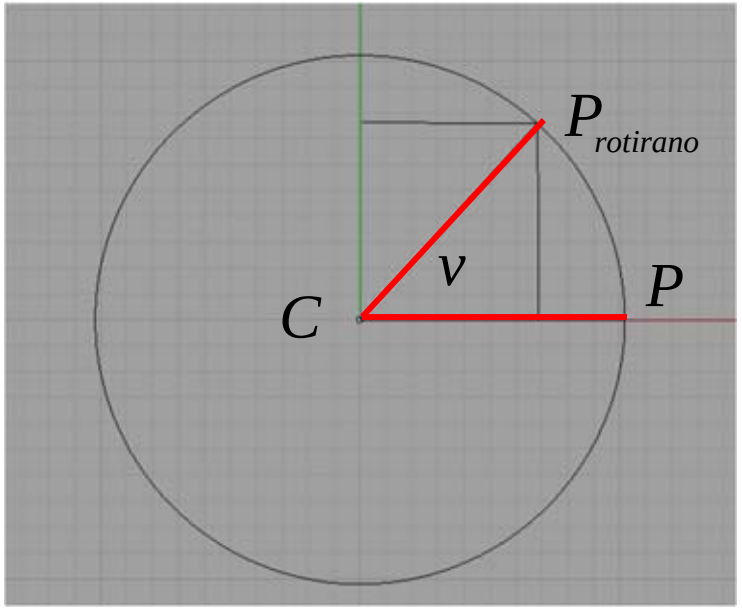
Jednačina krive u *yz* – ravni:

$$x = 0,\ y = \alpha_2(u),\ z = \alpha_3(u)$$

$$P(0, \alpha_2(u), \alpha_3(u))$$

$$C(0,0,\alpha_3(t))$$





$$x = \alpha_2(u)\cos v,\ y = \alpha_2(u)\sin v,\ z = \alpha_3(u)$$

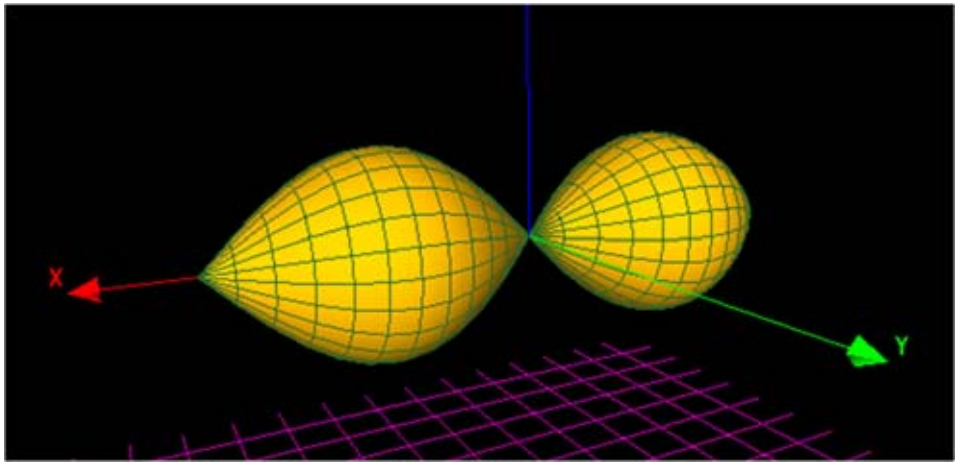
POVRŠI U PROSTORU

Rotacione površi - primeri

$$x = u,\ y = \sin u,\ z = 0$$

$$0 \le u \le 2\pi$$

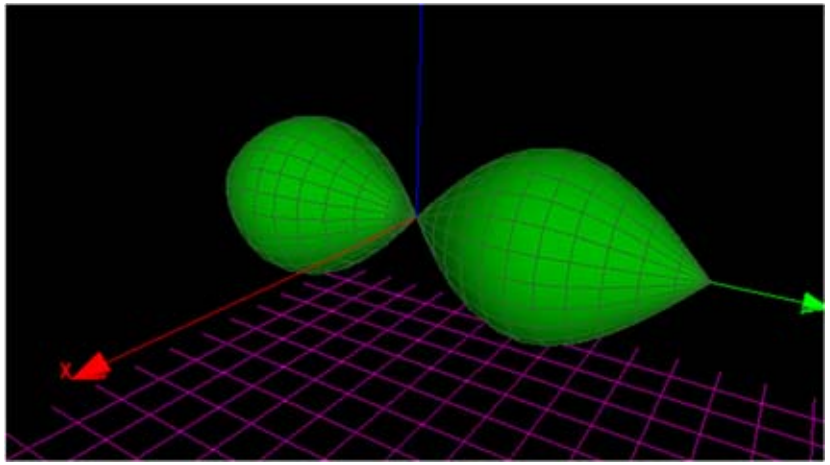
Rotacija oko *x* - ose



$$x = \sin u,\ y = u,\ z = 0$$

$$0 \le u \le 2\pi$$

Rotacija oko *y* - ose



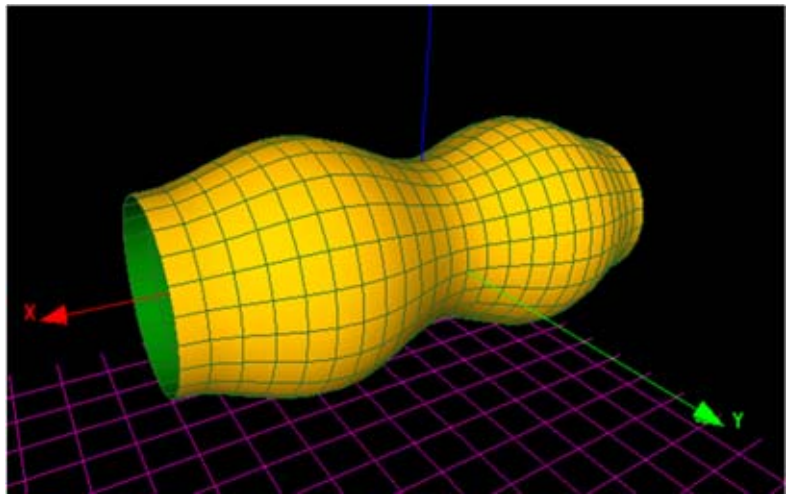
POVRŠI U PROSTORU

Rotacione površi - primeri

$$x = u,\ y = \sin u,\ z = 1$$

$$0 \le u \le 2\pi$$

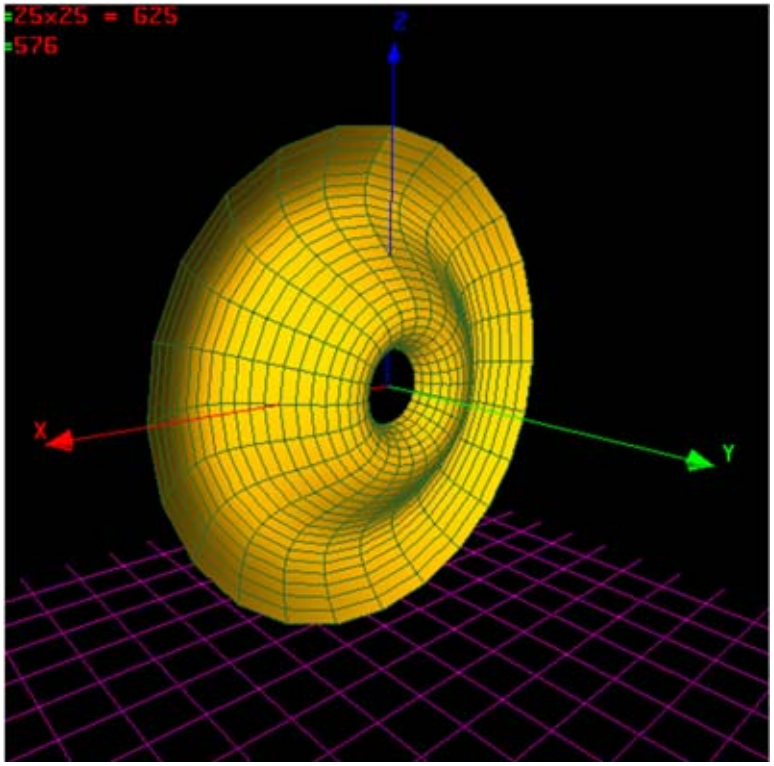
Rotacija oko *x* - ose



$$x = u,\ y = \sin u,\ z = 1$$

$$0 \le u \le 2\pi$$

Rotacija oko *y* - ose



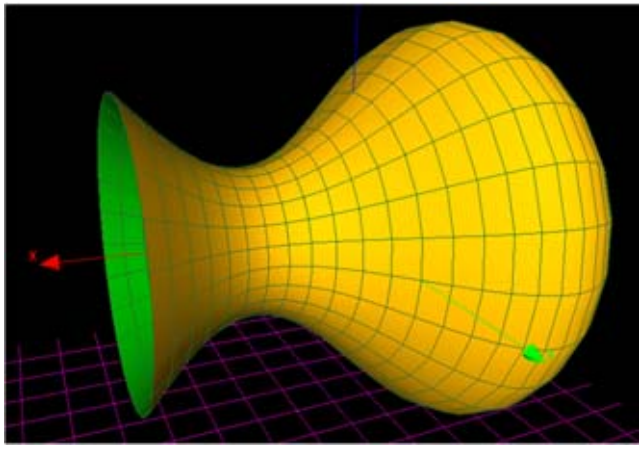
POVRŠI U PROSTORU

Rotacione površi - primeri

$$x = u,\ y = \cos u + 2,\ z = 0$$

$$0 \le u \le 2\pi$$

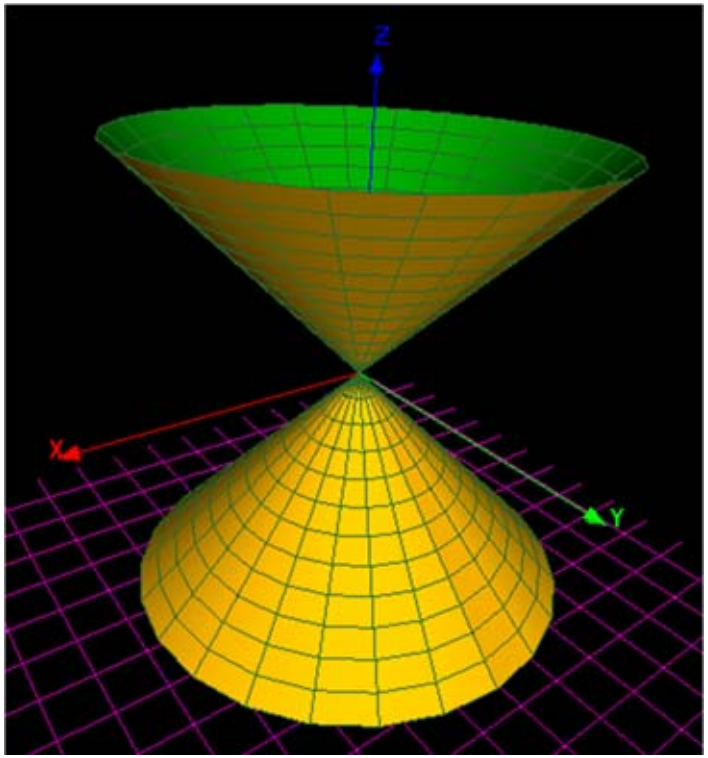
Rotacija oko *x* - ose



$$x = u,\ y = 0,\ z = u$$

$$-2\pi \le u \le 2\pi$$

Rotacija oko *z* - ose



POVRŠI U PROSTORU

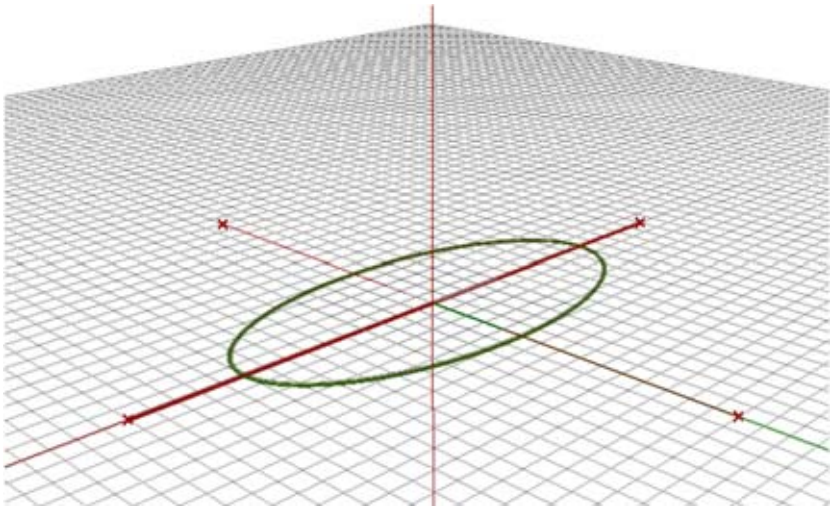
Rotacione površi – primeri – rotacioni elipsoid

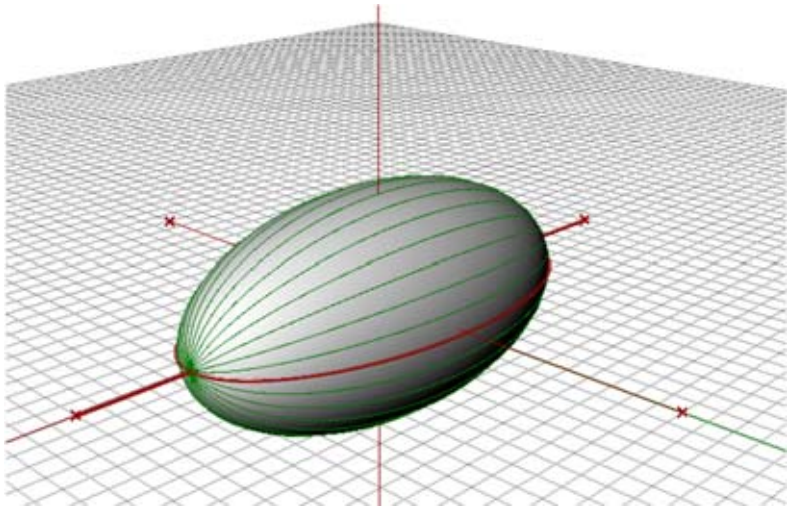
Rotacija oko x-ose

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

$$x=10\cos u,\ y=5\sin u,\ z=0$$

$$0\le u\le 2\pi$$





$$a=10,\ b=c=5 \quad \Rightarrow \frac{x^2}{10^2}+\frac{y^2}{5^2}+\frac{z^2}{5^2}=1 \quad \Rightarrow \frac{x^2}{100}+\frac{y^2}{25}+\frac{z^2}{25}=1$$

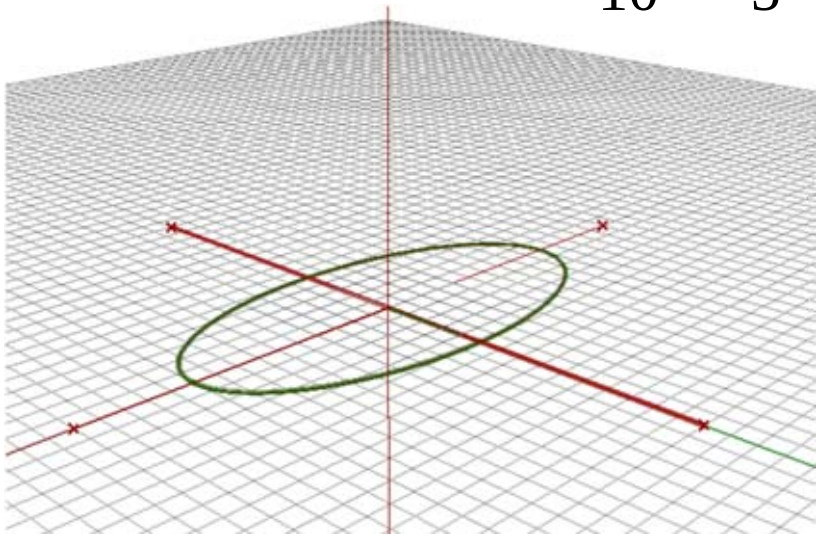
POVRŠI U PROSTORU

Rotacione površi – primeri – rotacioni elipsoid

$$x = 10\cos u,\ y = 5\sin u,\ z = 0$$

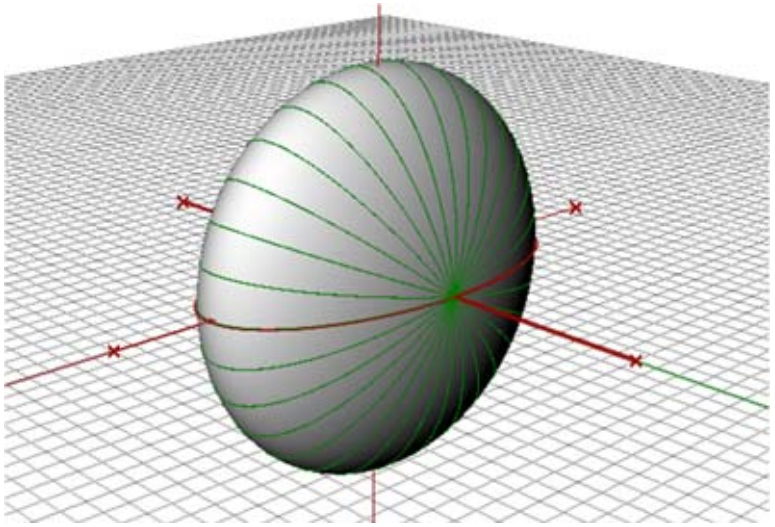
$$0 \le u \le 2\pi$$

$$\frac{x^2}{10^2} + \frac{y^2}{5^2} = 1,\ z = 0$$



Rotacija oko y-ose

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$



$$a = c = 10,\ b = 5 \quad \Rightarrow \frac{x^2}{10^2} + \frac{y^2}{5^2} + \frac{z^2}{10^2} = 1 \quad \Rightarrow \frac{x^2}{100} + \frac{y^2}{25} + \frac{z^2}{100} = 1$$

POVRŠI U PROSTORU

$$x = 15\sec u,\ y = 0,\ z = 5\tan u$$

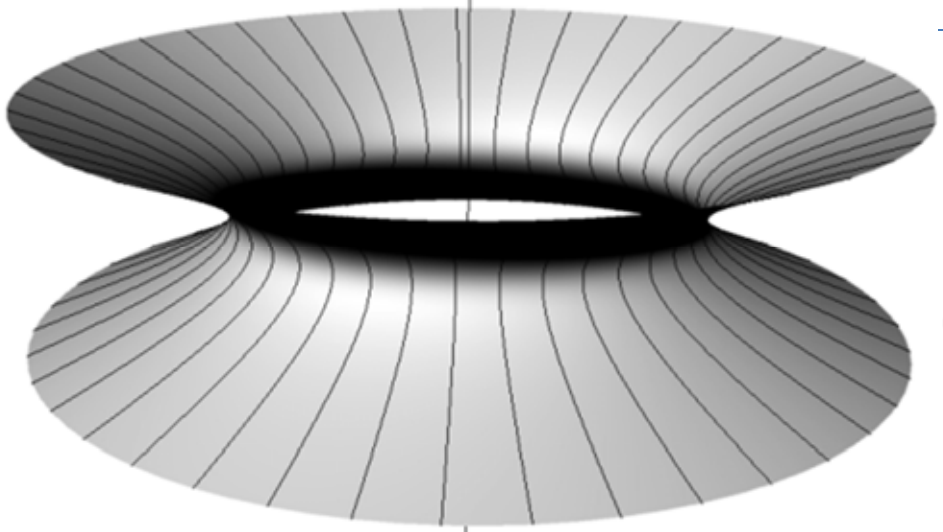
$$-1 \le u \le 1$$

Rotacione površi – primeri

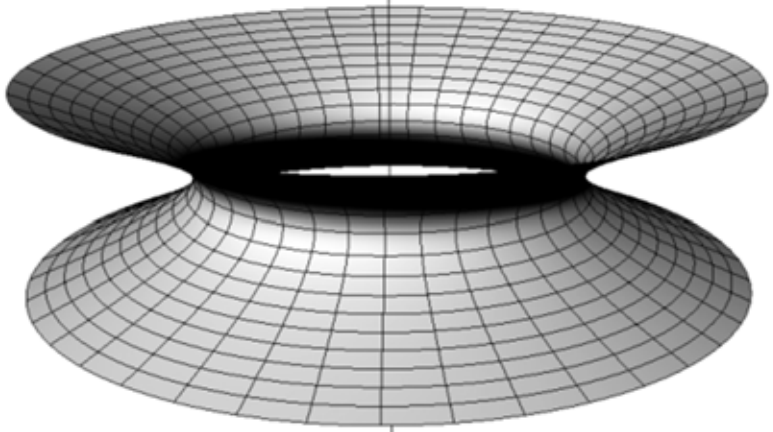
Rotacioni jednograni hiperboloid

$$\frac{x^2}{15^2} - \frac{z^2}{5^2} = 1,\ y = 0$$

Rotacija oko *z* - ose



$$a = b = 15,\quad c = 5$$



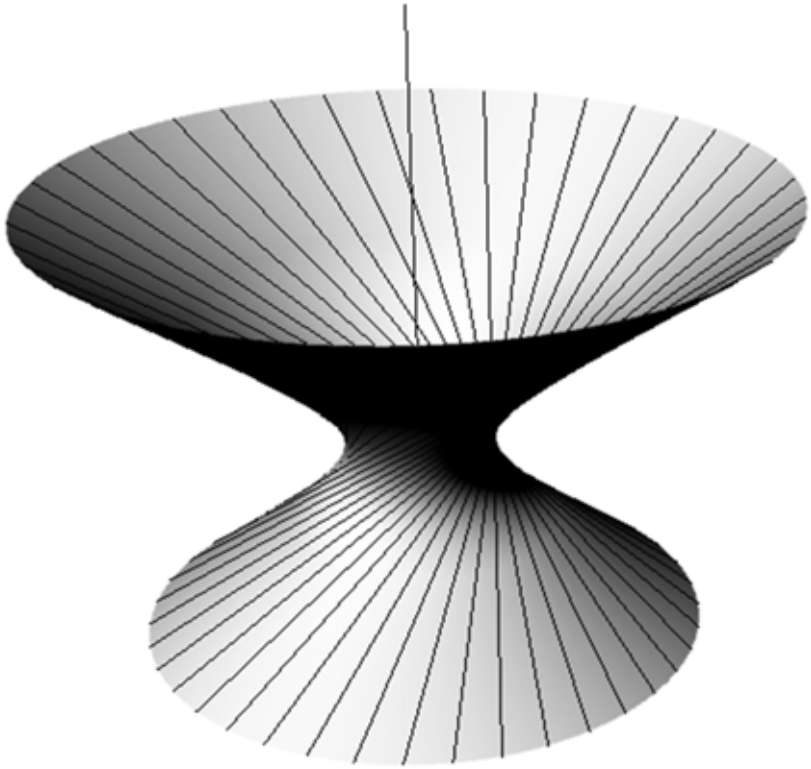
Jednačina rotacionog jednogranog hiperboloida

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1 \Rightarrow \frac{x^2}{15^2} + \frac{y^2}{15^2} - \frac{z^2}{5^2} = 1 \Rightarrow \frac{x^2}{225} + \frac{y^2}{225} - \frac{z^2}{25} = 1$$

POVRŠI U PROSTORU

Rotacione površi – primeri

Rotacioni jednograni hiperboloid

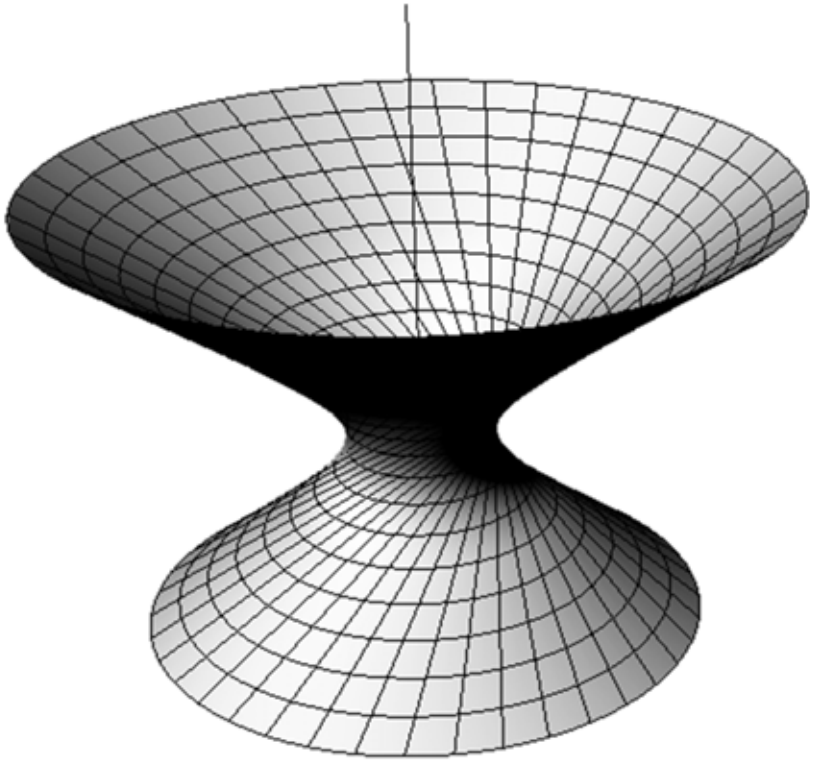


$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1 \Rightarrow \frac{x^2}{a^2}+\frac{y^2}{a^2}-\frac{z^2}{c^2}=1$$

$$x=u+18,\ y=u+2,\ z=u$$

$$-40\le u\le 30$$

Rotacija oko *z* - ose



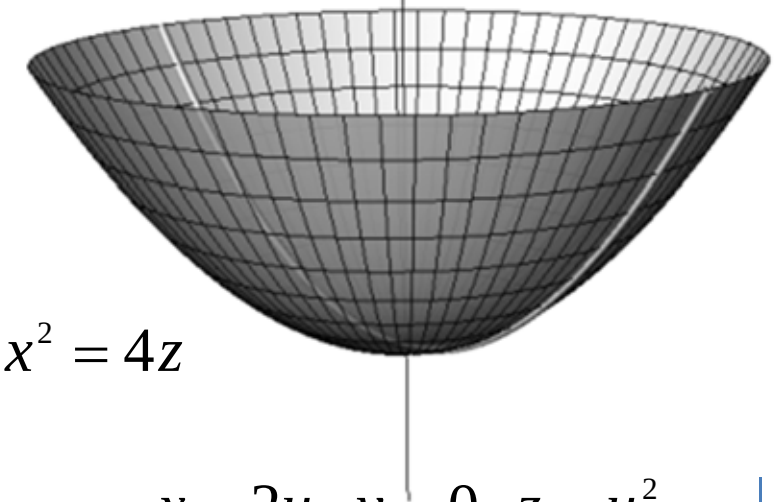
$a=b$ Najkraće rastojanje izmedju mimoilaznih pravih: ose i prave koja rotira

POVRŠI U PROSTORU

Rotacione površi – primeri

Rotacioni paraboloid

$$\frac{x^2}{p}+\frac{y^2}{q}=2z$$



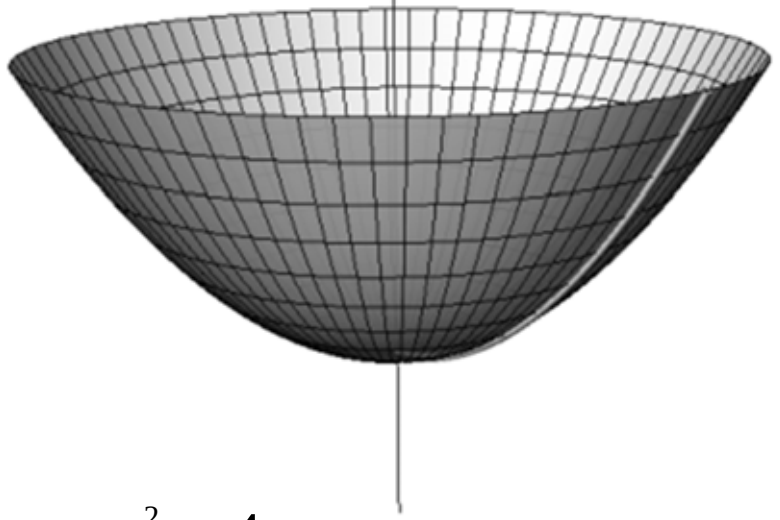
$$x^2=4z$$

$$x=2u,\ y=0,\ z=u^2$$

$$-5\le u\le 5 \quad 0\le u\le 5$$

Rotacija oko *z* - ose

Dvostruka povrs



$$x^2=4z$$

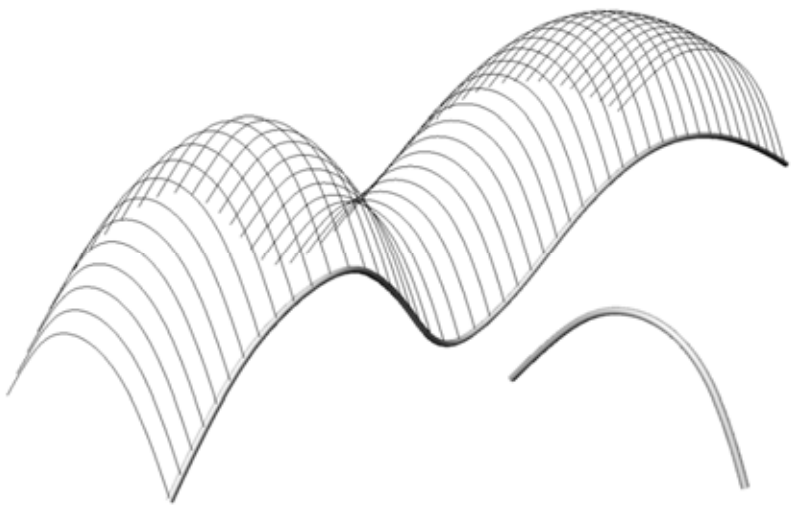
$$x^2=2pz$$

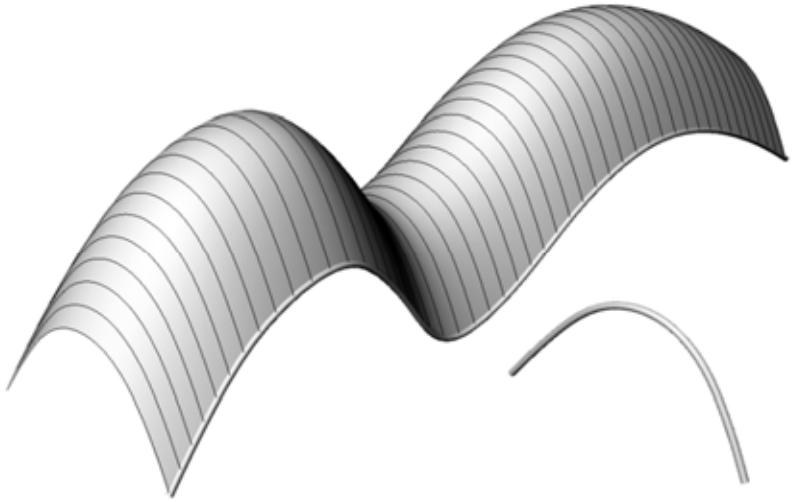
$$p=q=2$$

$$\frac{x^2}{2}+\frac{y^2}{2}=2z$$

POVRŠI U PROSTORU

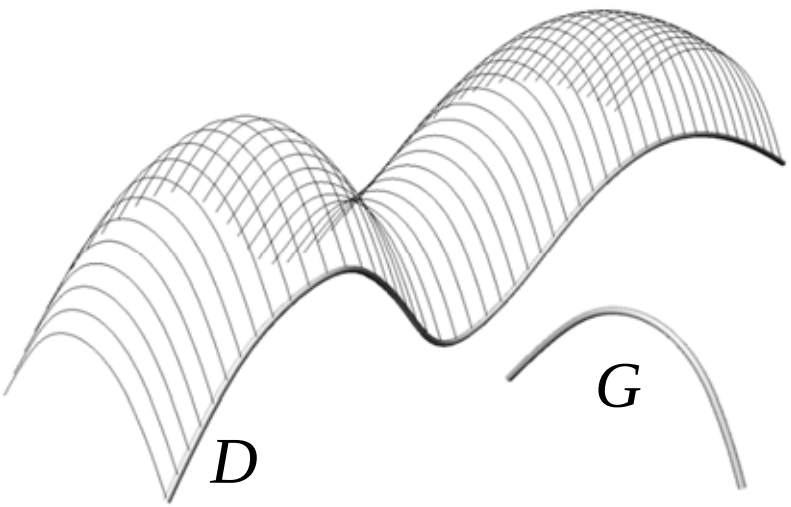
Translatorne površi

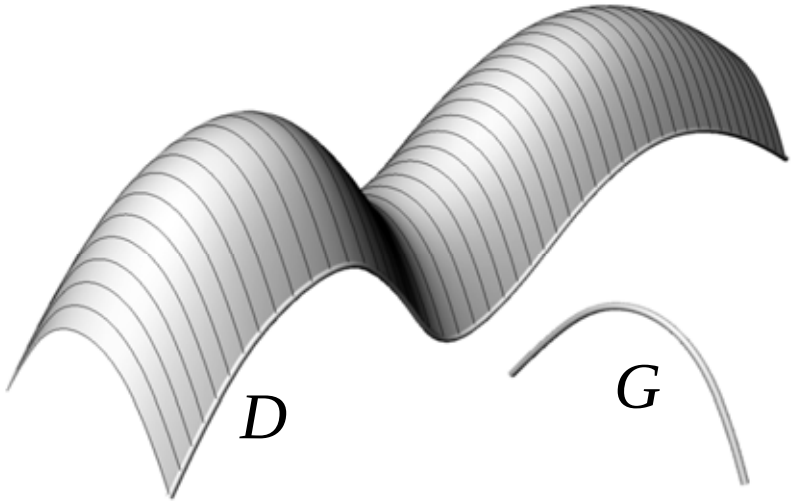




POVRŠI U PROSTORU

Translatorne površi

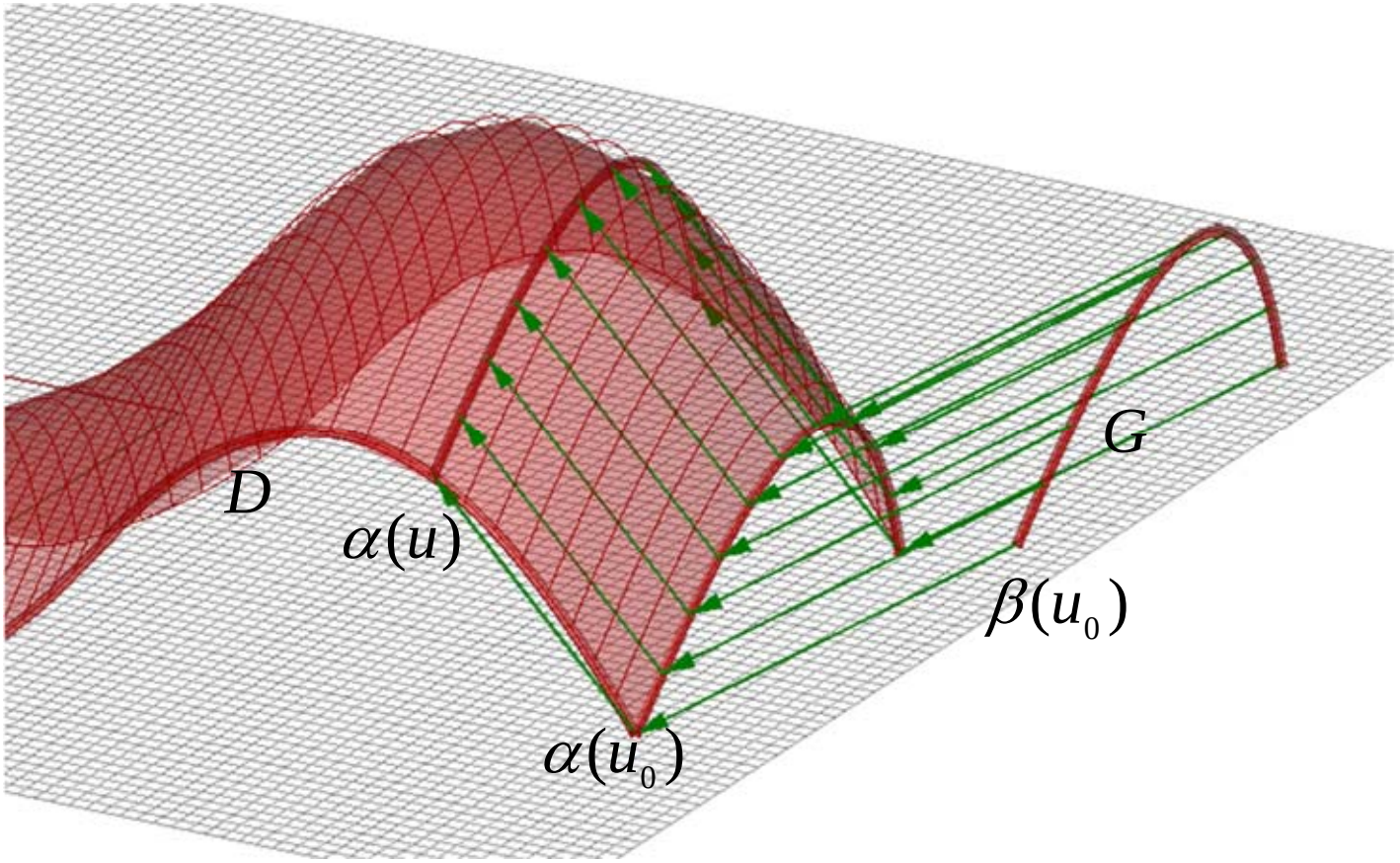




Translatornim kretanjem generatrise $\boldsymbol{G}$ duž direktrise $\boldsymbol{D}$ nastaje translatorna površ.

POVRŠI U PROSTORU

Translatorne površi

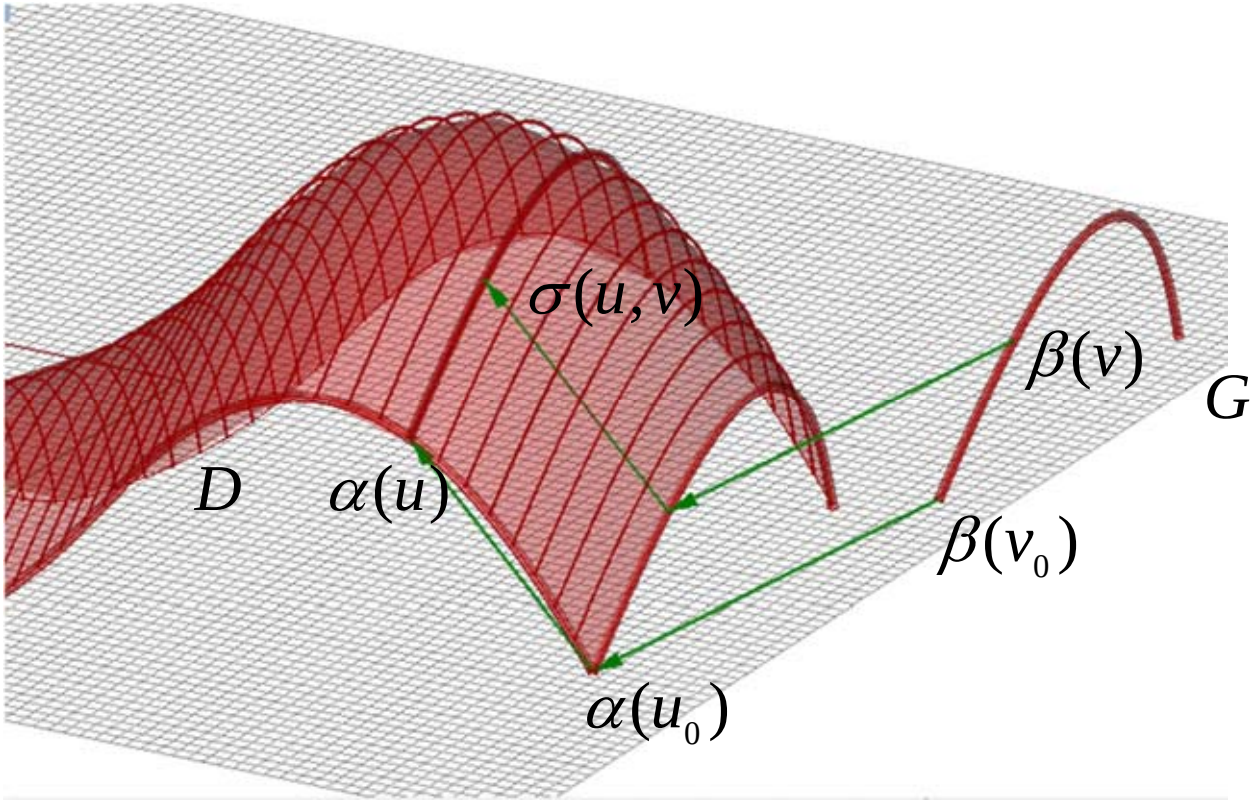


POVRŠI U PROSTORU

Translatorne površi

$$D:\ \alpha = \alpha(u),\ u_0 \le u \le u_1$$

$$G:\ \beta = \beta(v),\ v_0 \le v \le v_1$$



$$\sigma:\ \sigma = \sigma(u,v) = \beta(v) + \alpha(u_0) - \beta(v_0) + \alpha(u) - \alpha(u_0)$$

$$\sigma:\ \sigma = \sigma(u,v) = \alpha(u) + \beta(v) - \beta(v_0)$$

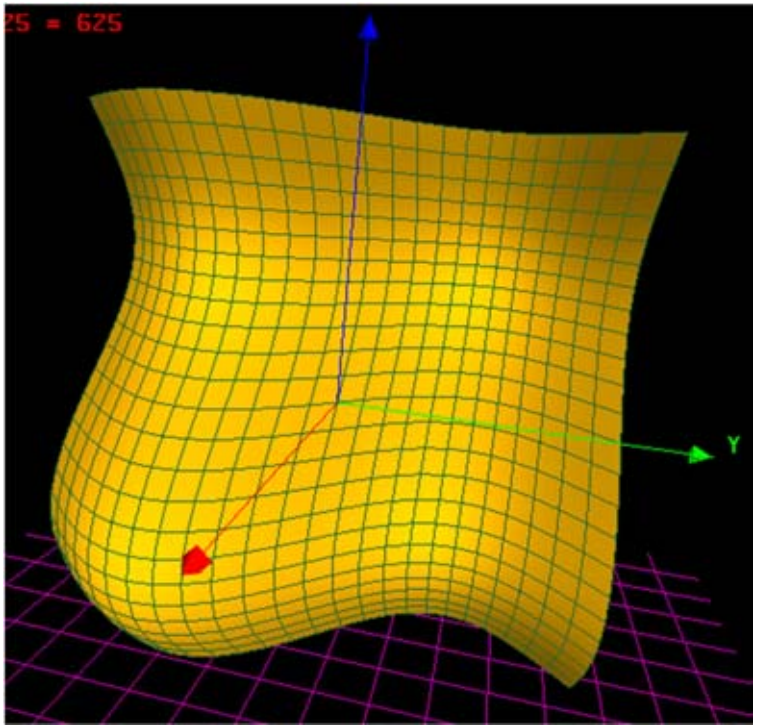
POVRŠI U PROSTORU

Translatorne površi - primeri

$$\sigma:\ \sigma = \sigma(u,v) = \alpha(u) + \beta(v) - \beta(v_0)$$

$$D:\ \ x = \sin u,\ y = u,\ z = 0$$
$$0 \le u \le 2\pi$$

$$G:\ \ x = \sin v,\ y = 0,\ z = v$$
$$0 \le v \le 2\pi$$

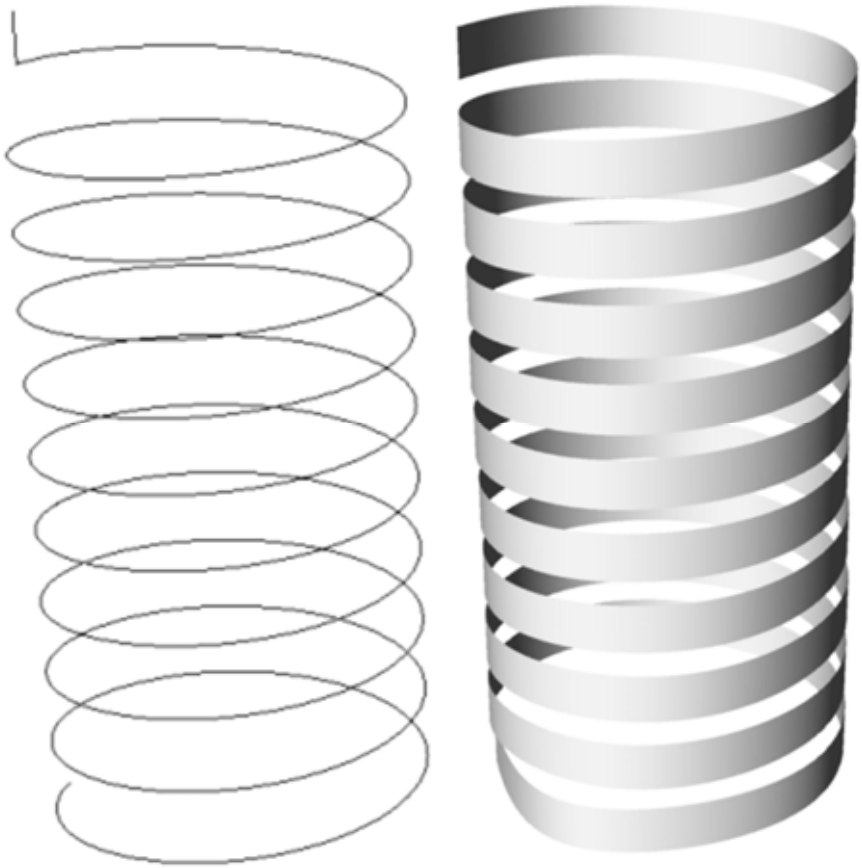


$$\sigma = \sigma(u,v) = (\sin u + \sin v,\ u,\ v)$$
$$0 \le u \le 2\pi,\ 0 \le v \le 2\pi$$

$$\sigma = \sigma(u,v):$$
$$x_\sigma(u,v) = \sin u + \sin v$$
$$y_\sigma = u$$
$$z_\sigma = v$$

POVRŠI U PROSTORU

Translatorne površi - primeri



$$x = 0,\ y = 0,\ z = v$$

$$x = r\cos u,\ y = r\sin u,\ z = cu$$

$$\sigma = \sigma(u,v) = (r\cos u,\ r\sin u,\ cu + v)$$

$\sigma = \sigma(u,v)$:

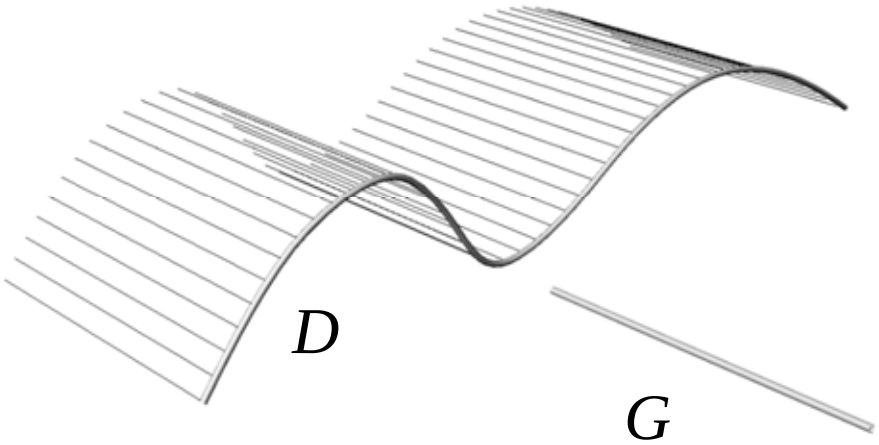
$x = r\cos u$

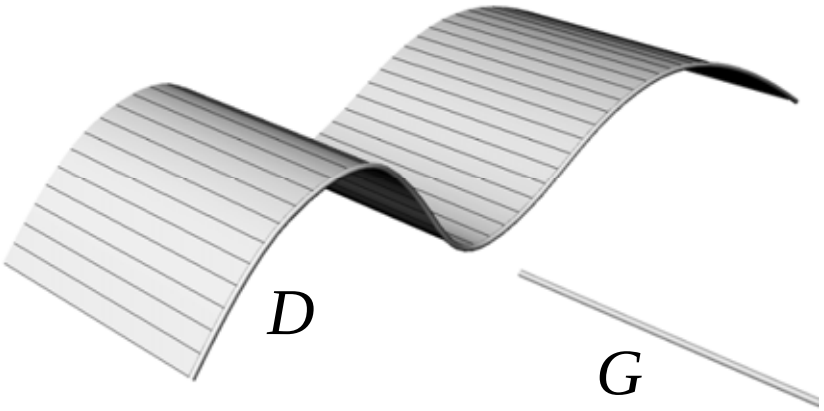
$y = r\sin u$

$z = cu + v$

POVRŠI U PROSTORU

Cilindrične površi

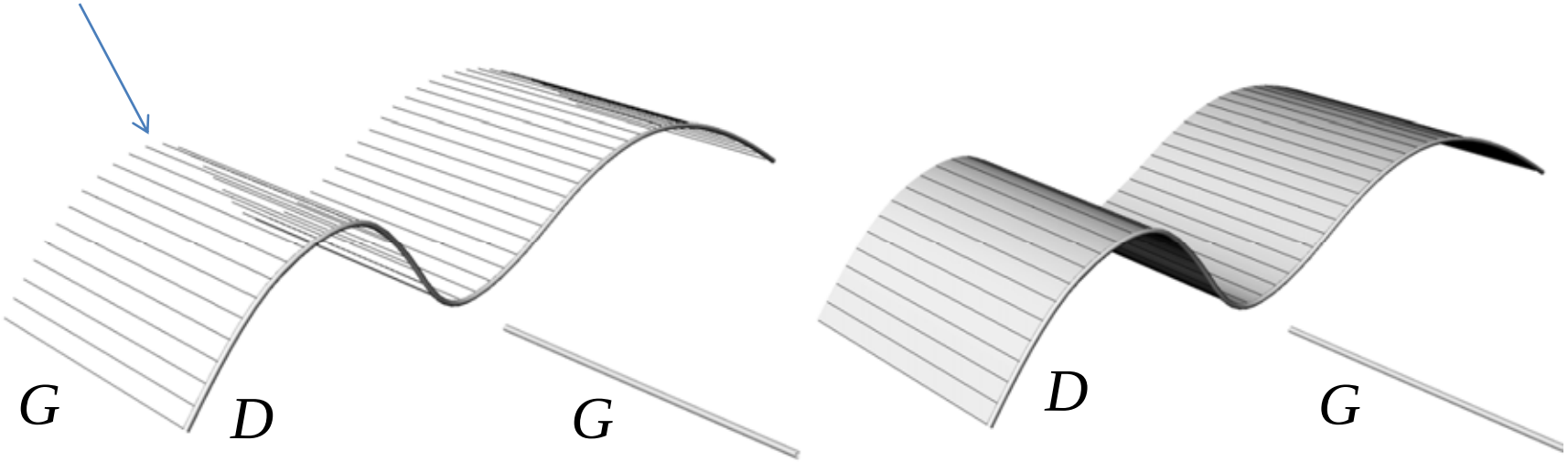




POVRŠI U PROSTORU

Cilindrične površi

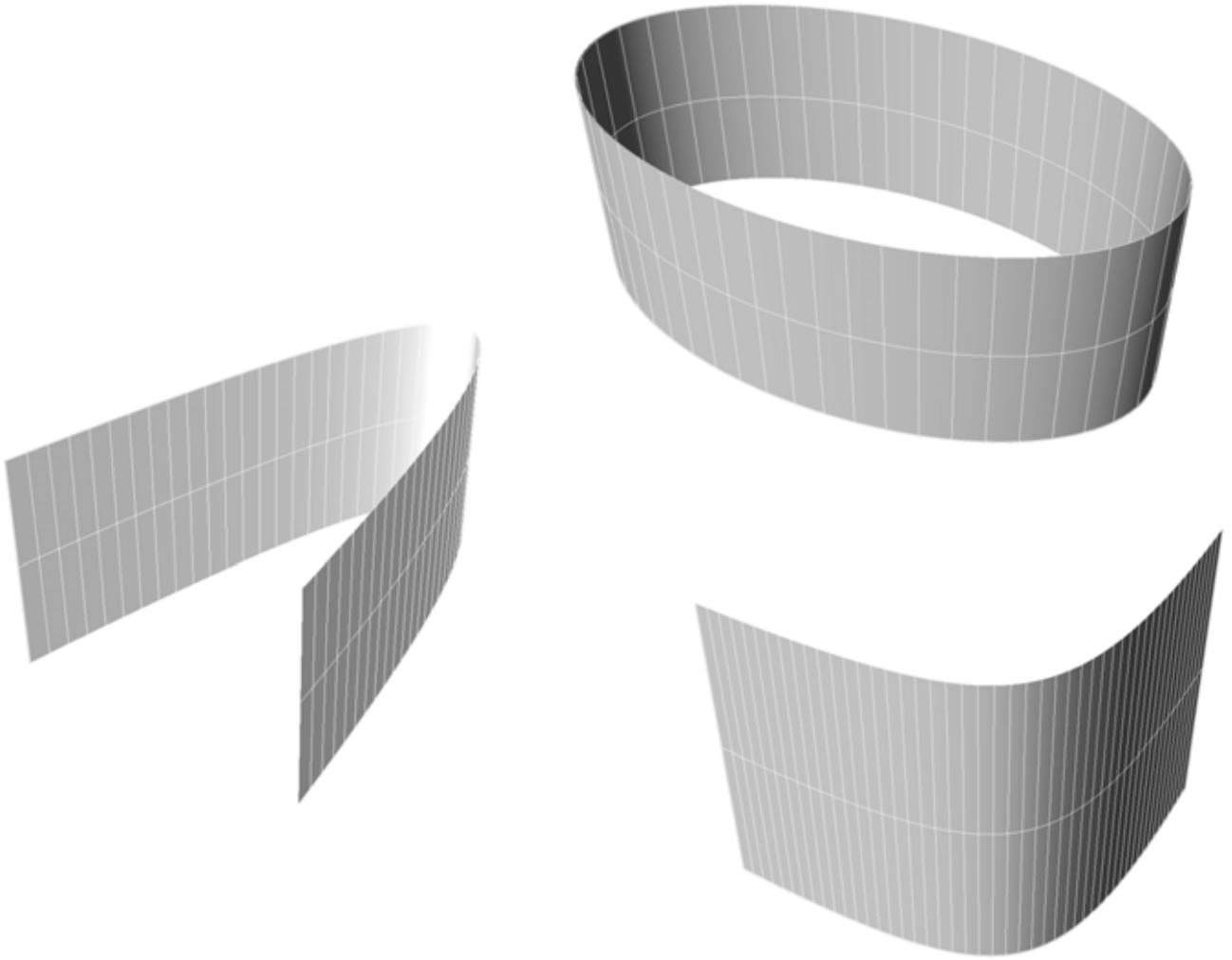
Izvodnice



Translatornim kretanjem prave linije $\boldsymbol{G}$ duž direktrise $\boldsymbol{D}$ nastaje cilindrična površ.

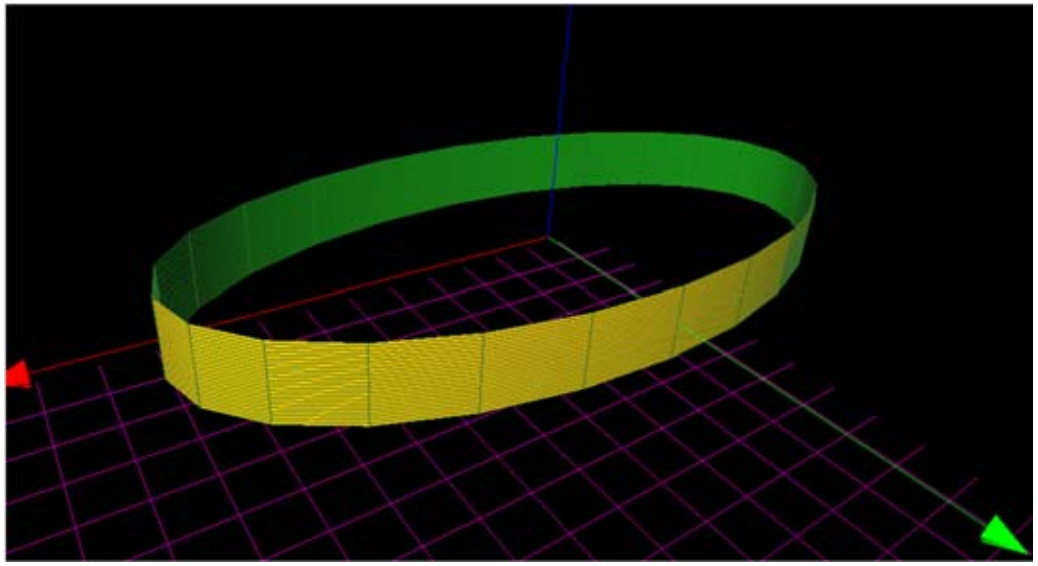
POVRŠI U PROSTORU

Cilindrične površi



POVRŠI U PROSTORU

Cilindrične površi - primeri



Direktrisa: elipsa

$$x = 10\cos u,\ y = 5\sin u,\ z = -1$$
$$0 \le u \le 2\pi$$

Generatrisa:

$$x = 0,\ y = 0,\ z = v$$
$$-1 \le v \le 1$$

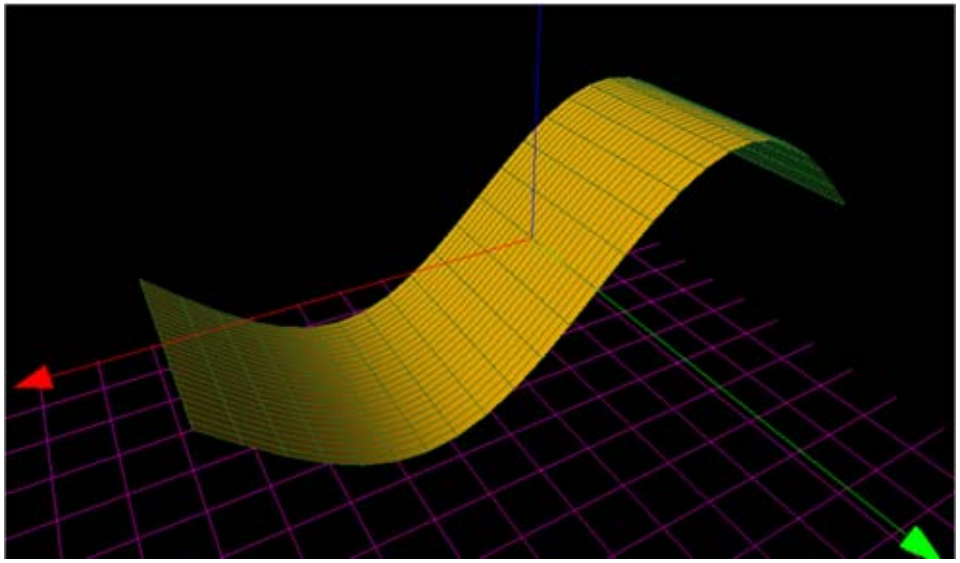
Cilindrična površ:

$$x = 10\cos u,\ y = 5\sin u,\ z = v$$

$$0 \le u \le 2\pi,\ -1 \le v \le 1$$

POVRŠI U PROSTORU

Cilindrične površi - primeri



Direktrisa: kriva

$$x = u,\ y = -1,\ z = \sin u$$

$$-\pi \le u \le \pi$$

Generatrisa:

$$x = 0,\ y = v,\ z = 0$$

$$-1 \le v \le 1$$

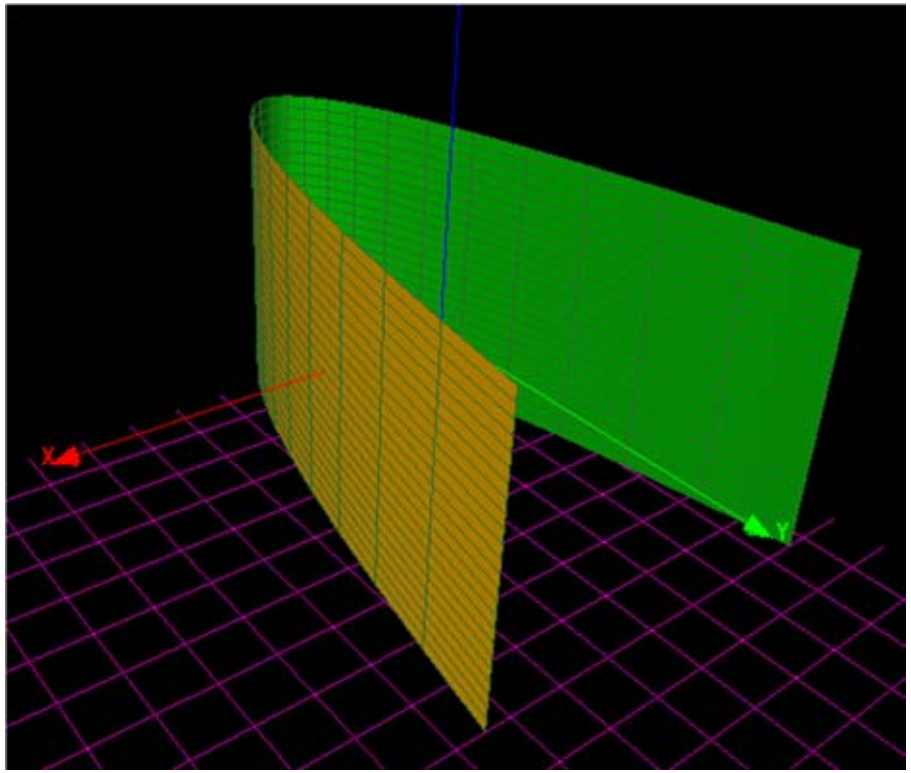
Cilindrična površ:

$$x = u,\ y = v,\ z = \sin u$$

$$-\pi \le u \le \pi,\ -1 \le v \le 1$$

POVRŠI U PROSTORU

Cilindrične površi - primeri



Direktrisa: parabola

$$x = u,\ y = u^2,\ z = -2.5$$

$$-\pi \le u \le \pi$$

Generatrisa:

$$x = 0,\ y = 0,\ z = v$$

$$-2.5 \le v \le 2.5$$

Cilindrična površ:

$$x = u,\ y = u^2,\ z = v$$

$$-\pi \le u \le \pi,\ -2.5 \le v \le 2.5$$

POVRŠI U PROSTORU

Pravoizvodne površi

Površ

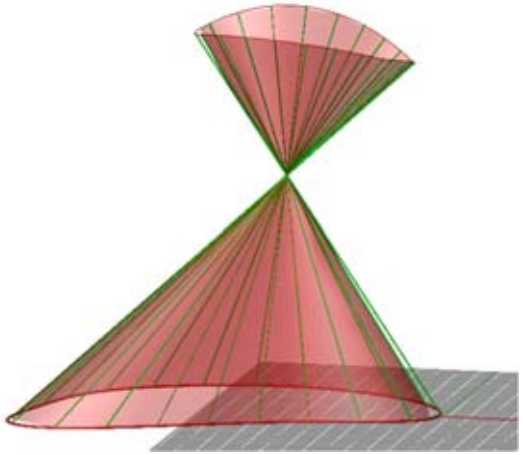
$\sigma = \sigma(u, v)$:

je pravoizvodna ako kroz svaku njenu tačku prolazi prava koja pripada toj površi.

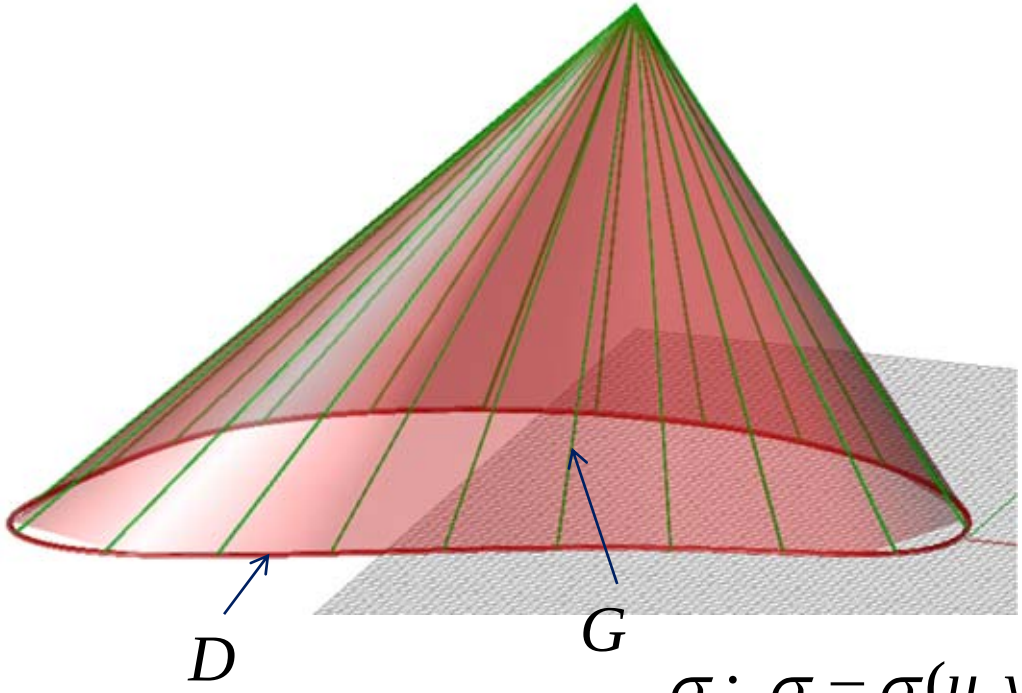
Çilindrične površi su pravoizvodne površi.
Konusne površi su pravoizvodne površi.

POVRŠI U PROSTORU

Pravoizvodne površi - konusne površi



$$C(x_0, y_0, z_0)$$



$$D:\ \alpha = \alpha(u),\ u_0 \le u \le u_1$$

$$\alpha(u) = (\alpha_1(u), \alpha_2(u), \alpha_3(u))$$

G: Prava odredjena sa dve tacke

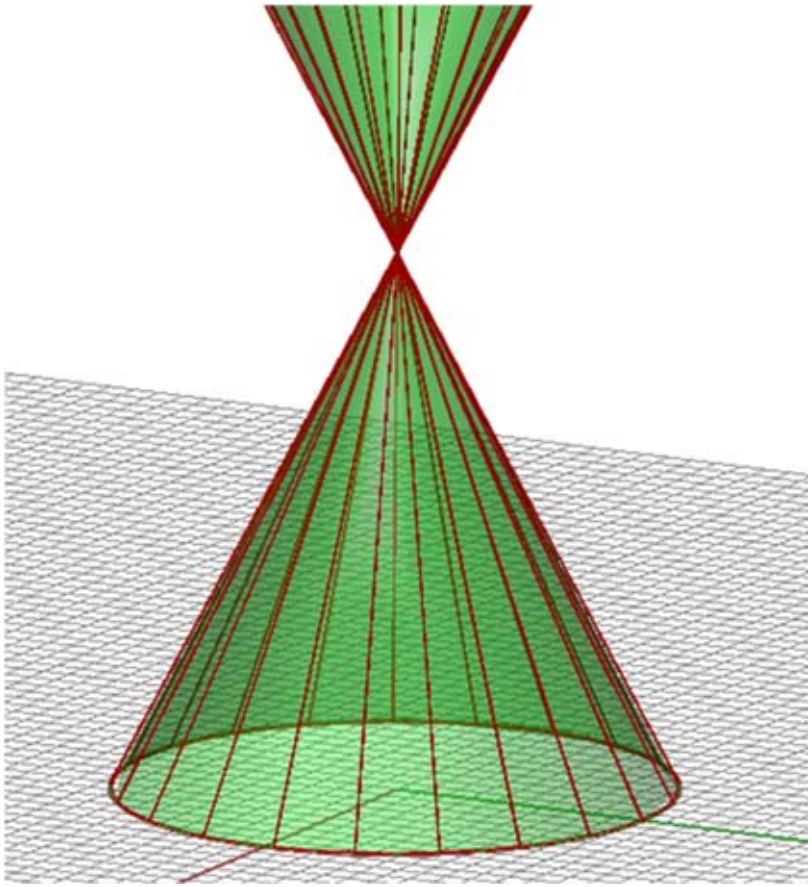
$$\alpha = \alpha(u) \ \text{i} \ C(x_0, y_0, z_0)$$

$$\sigma:\ \sigma = \sigma(u,v) = C + v(\alpha(u) - C),\ v \in R$$

$$x = x_0 + v(\alpha_1(u) - x_0),\ y_0 + v(\alpha_2(u) - y_0),\ z_0 + v(\alpha_3(u) - z_0),\ \ v \in R$$

POVRŠI U PROSTORU

Pravoizvodne površi - konusne površi - primer



$$x = 10\cos u,\ y = 10\sin u,\ z = 0$$

$$C(0,0,20)$$

Konusna površ:

$$x = 10v\cos u$$

$$y = 10v\sin u$$

$$z = -20v$$

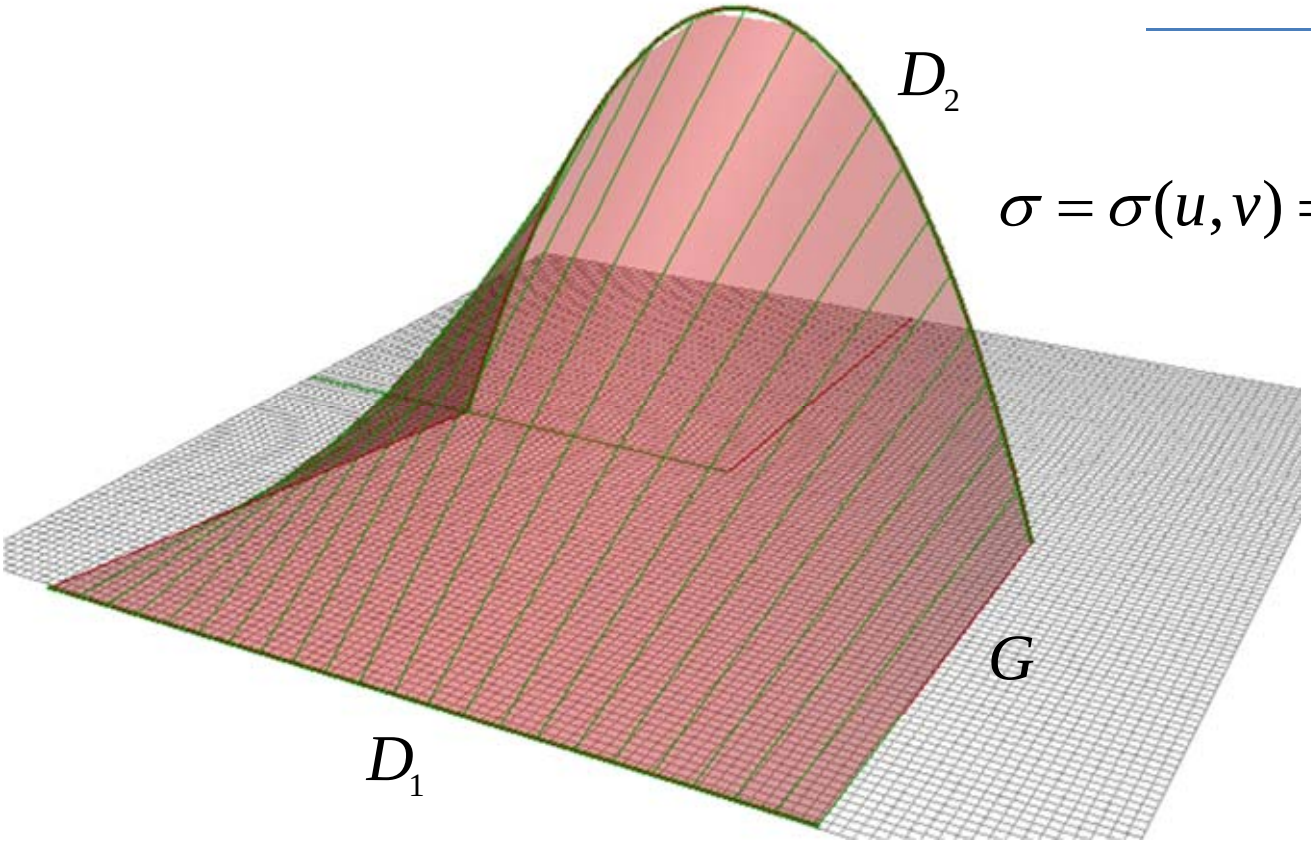
$$v \in R$$

POVRŠI U PROSTORU

Pravoizvodne površi

$$D_1 : \ \alpha(u), \ 0 \le u \le l_1$$

$$D_2 : \ \beta(u), \ 0 \le u \le l_2$$

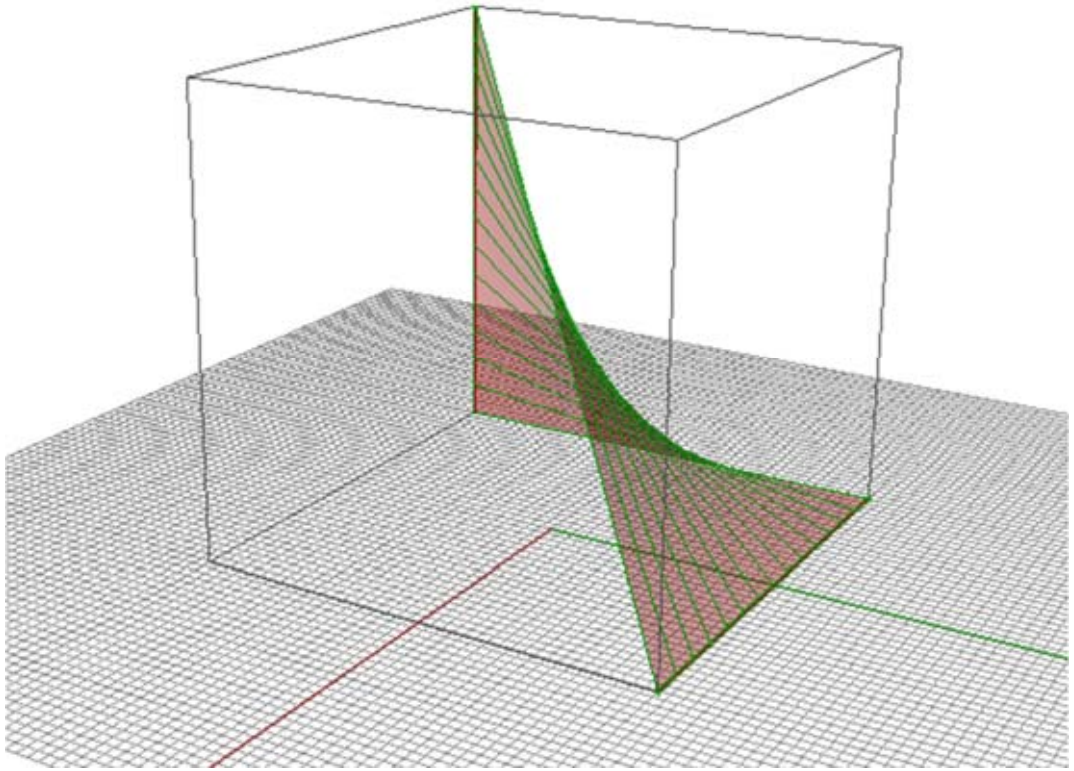


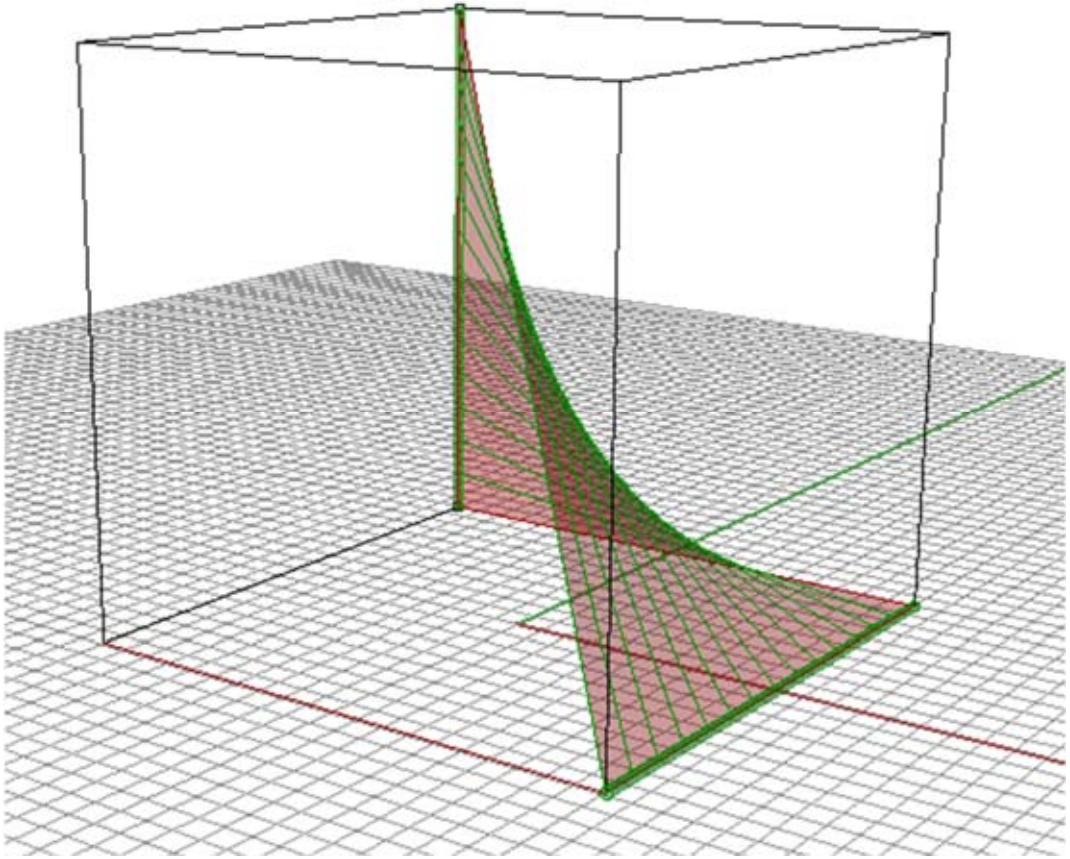
$$\sigma = \sigma(u,v) = \alpha(u) + v \ \beta\left(\frac{l_2}{l_1}u\right)$$

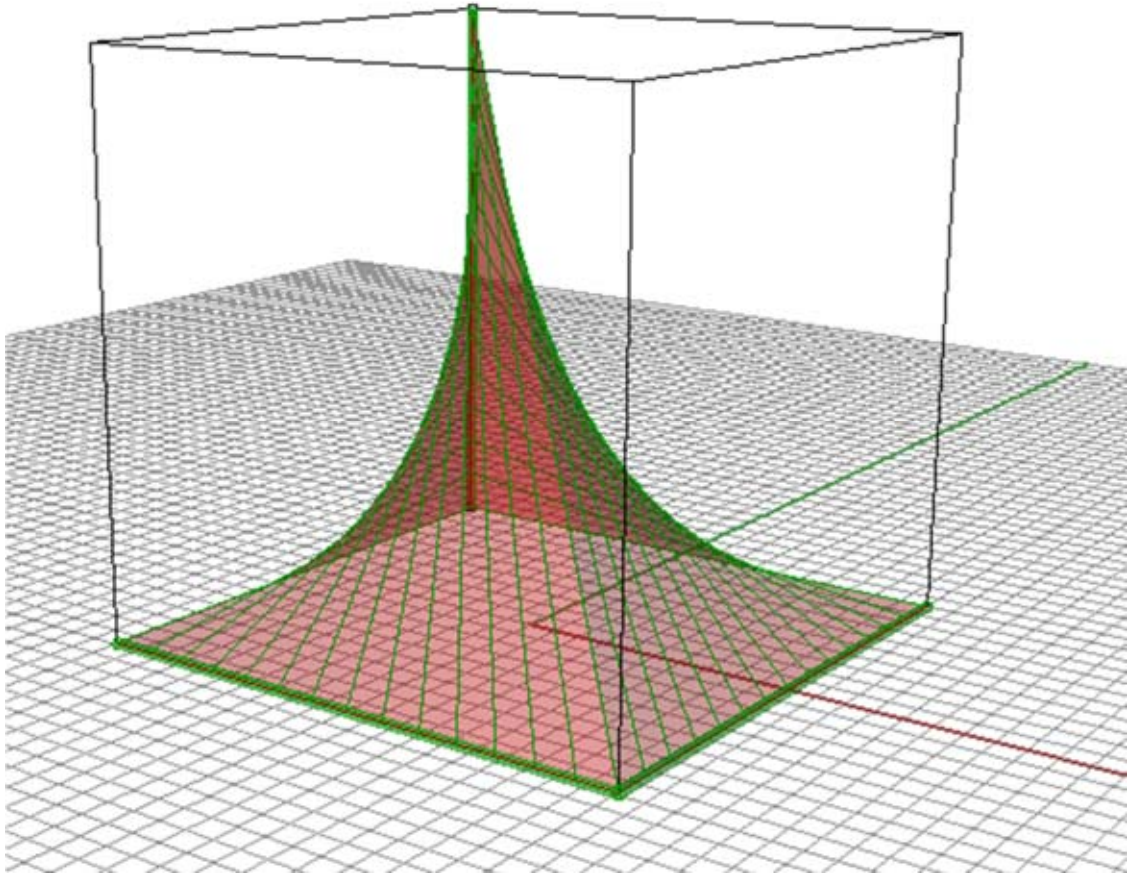
$$0 \le v \le 1$$

POVRŠI U PROSTORU

Pravoizvodne površi

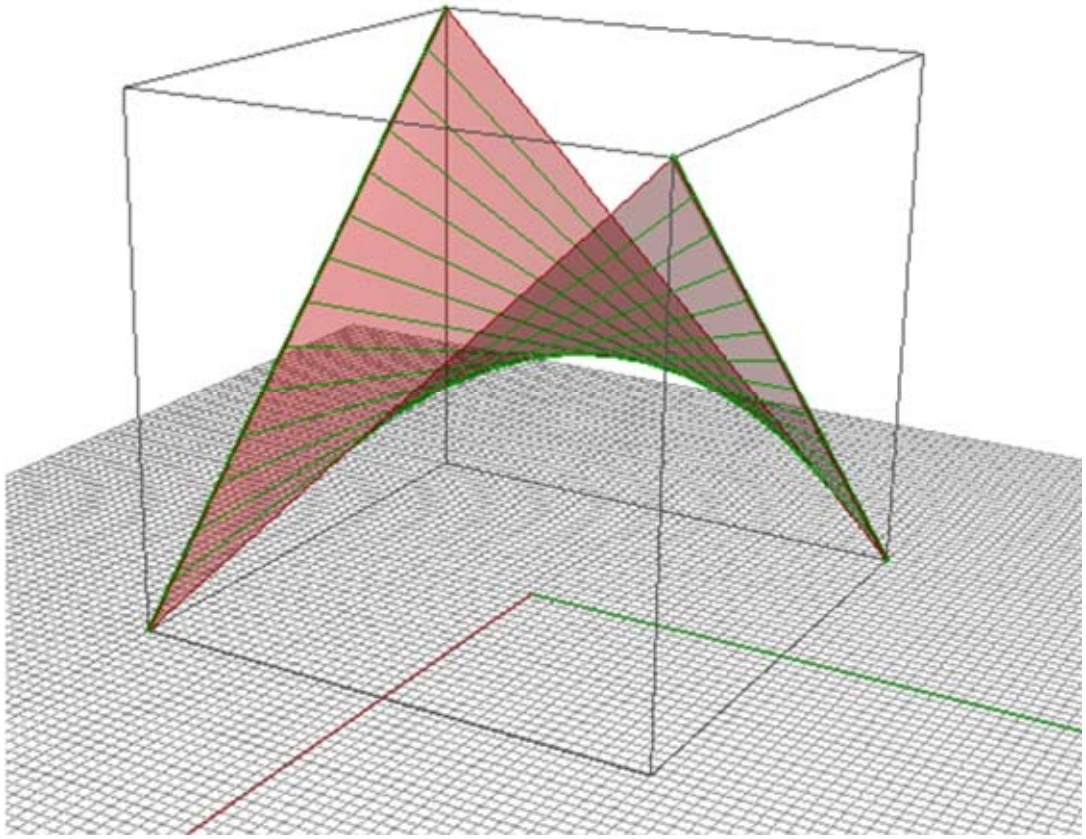


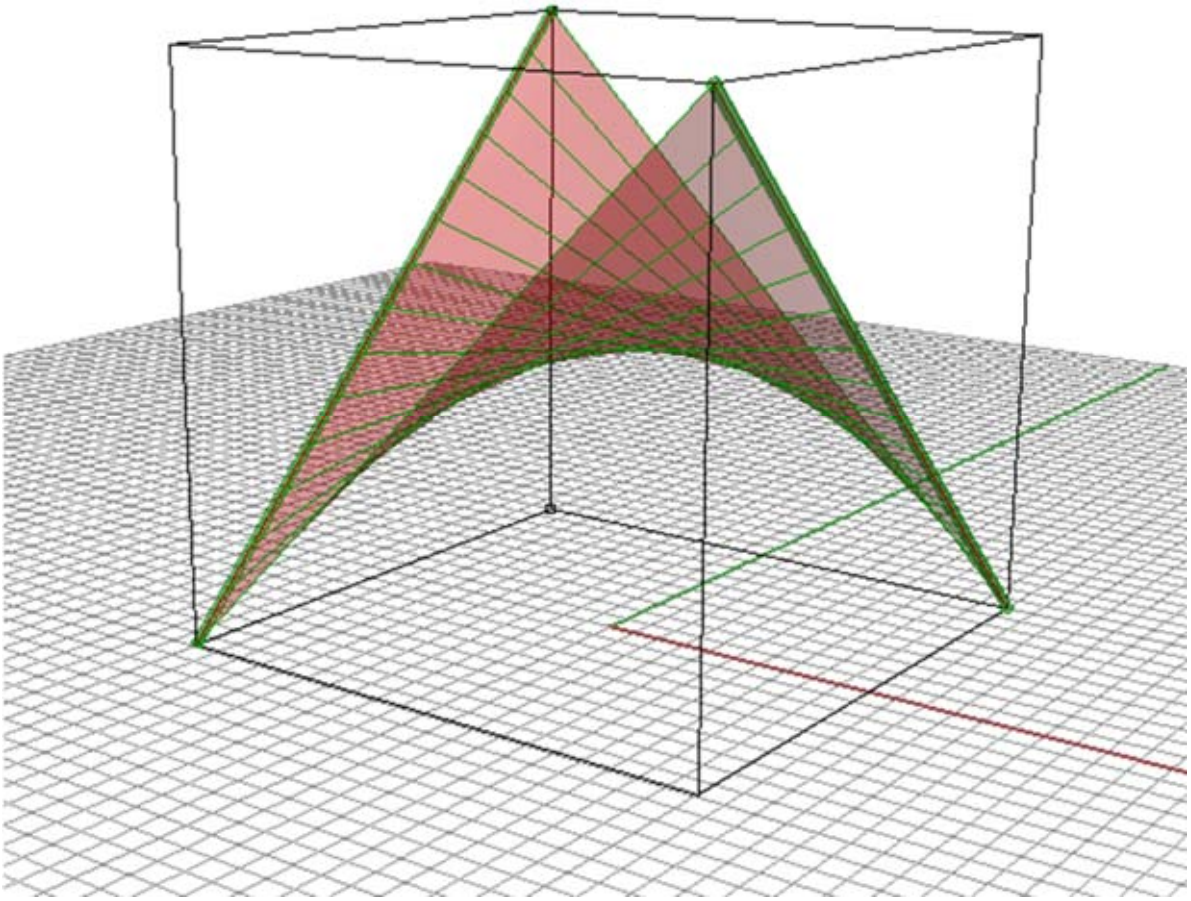




POVRŠI U PROSTORU

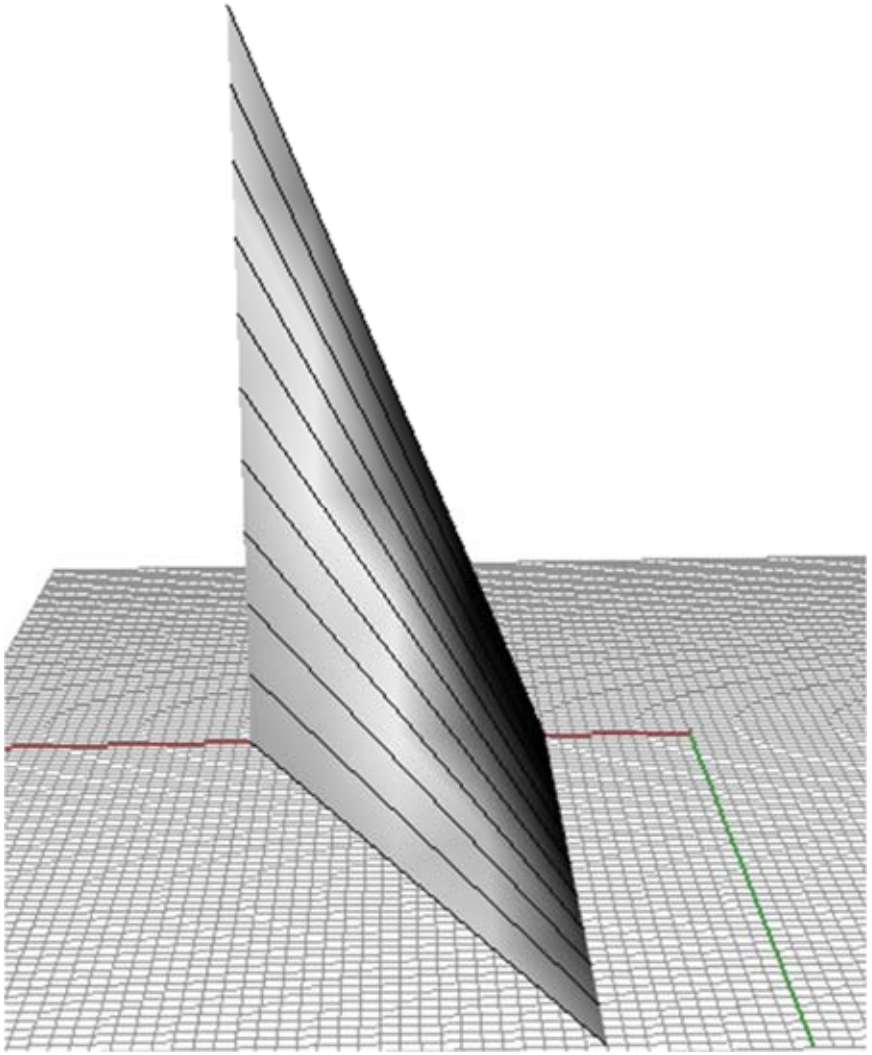
Pravoizvodne površi

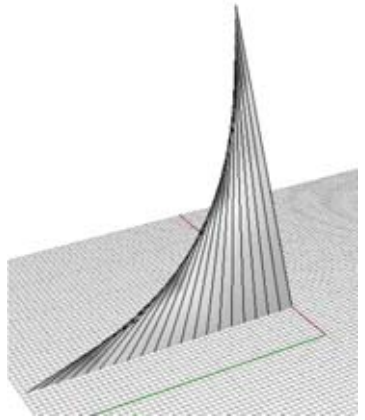




POVRŠI U PROSTORU

Pravoizvodne površi





$$\alpha = \alpha(u):$$

$$x = 10,\ y = u,\ z = 0$$

$$0 \le u \le 49$$

$$\beta = \beta(u):$$

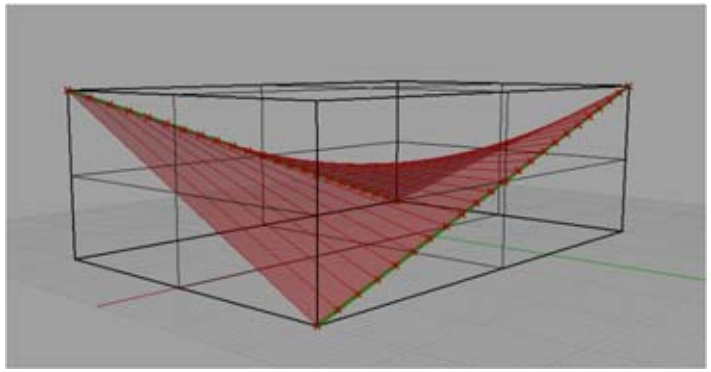
$$x = 30,\ y = 0,\ z = u$$

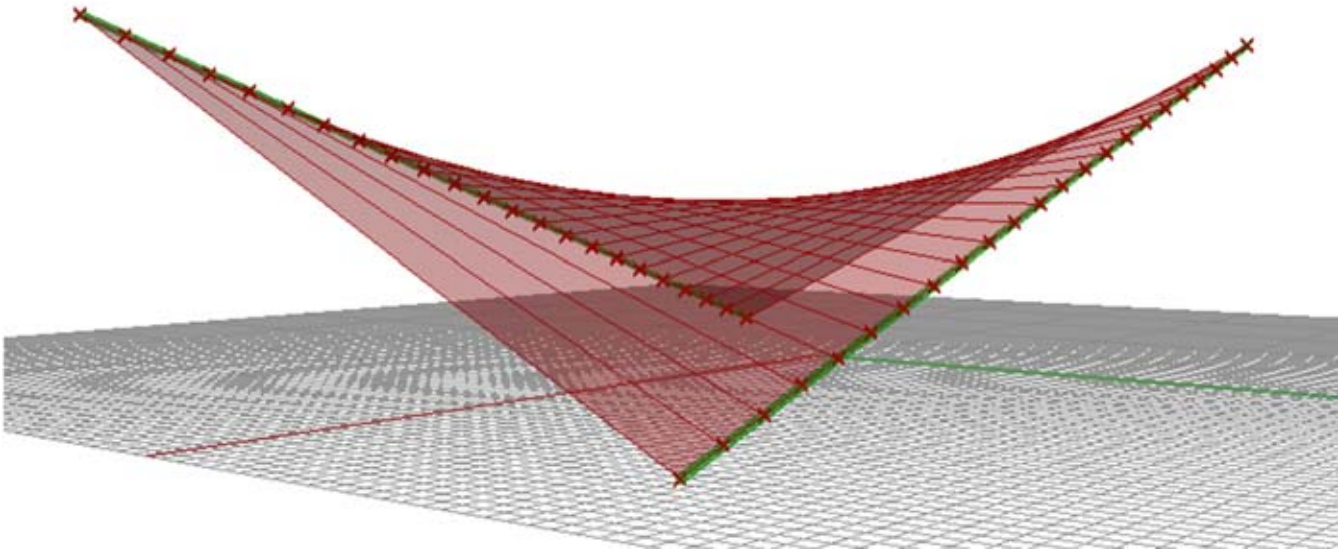
$$0 \le u \le 49$$

$$\sigma = \sigma(u,v):$$

$$\sigma(u,v) = \alpha(u) + v(\beta(u) - \alpha(u))$$

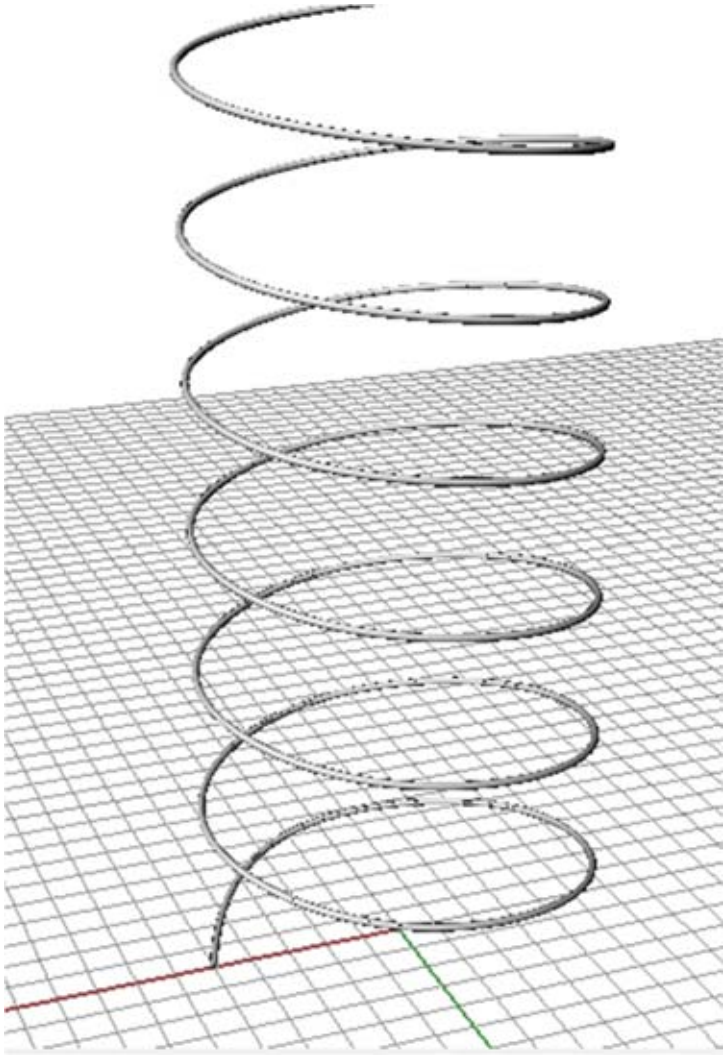
$$0 \le u \le 49,\ 0 \le v \le 1$$





POVRŠI U PROSTORU

Pravoizvodne površi – tangencijalne površi

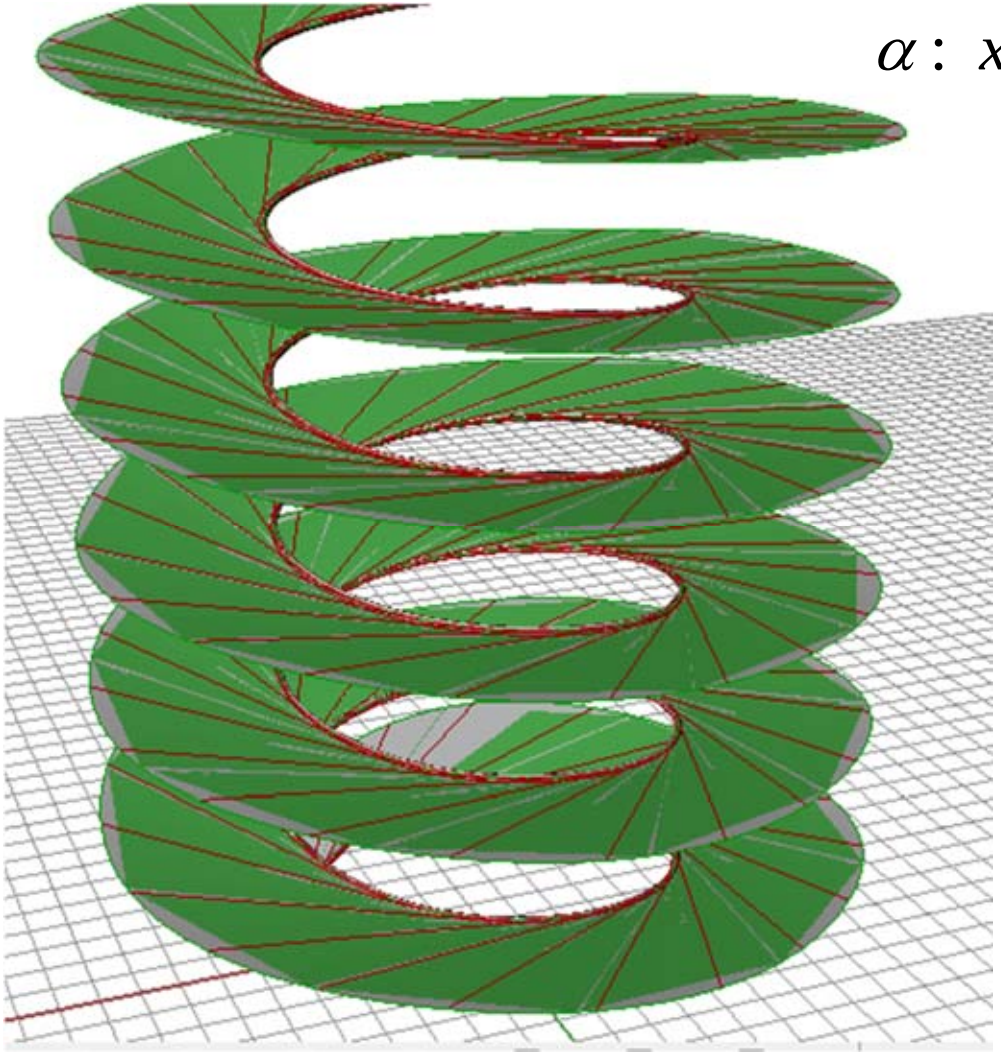


$$x = 5\cos u,\ y = 5\sin u,\ z = 0.5u$$

$$0 \le u \le 20\pi$$

POVRŠI U PROSTORU

Pravoizvodne površi – tangencijalne površi



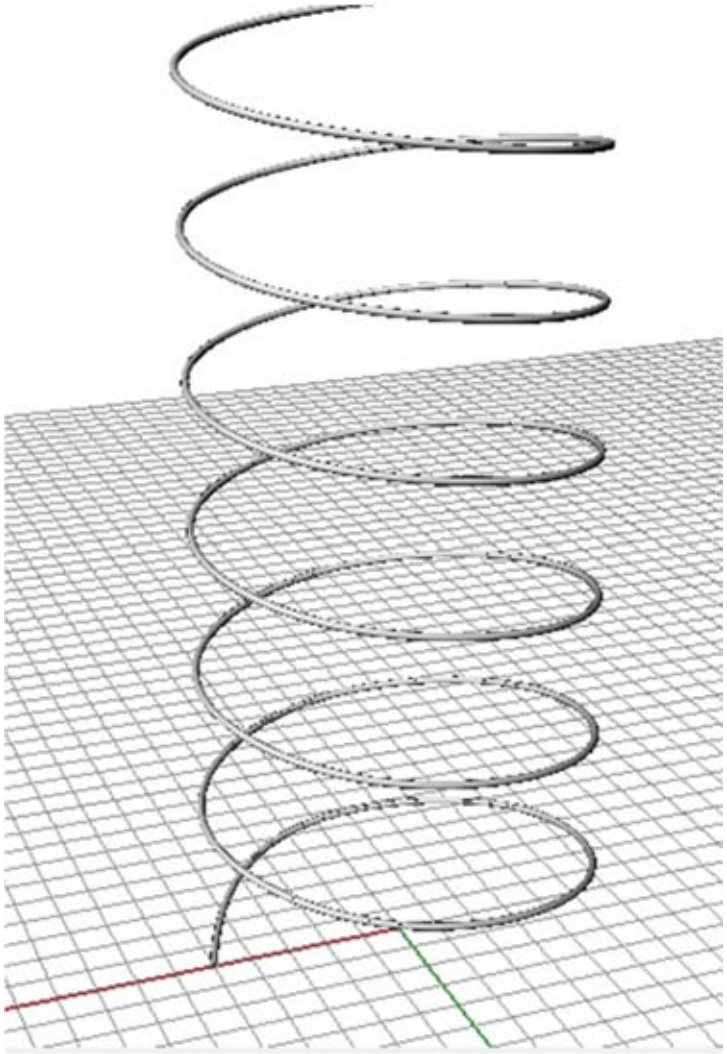
$$\alpha:\ x = 5\cos u,\ y = 5\sin u,\ z = 0.5u$$
$$0 \le u \le 20\pi$$

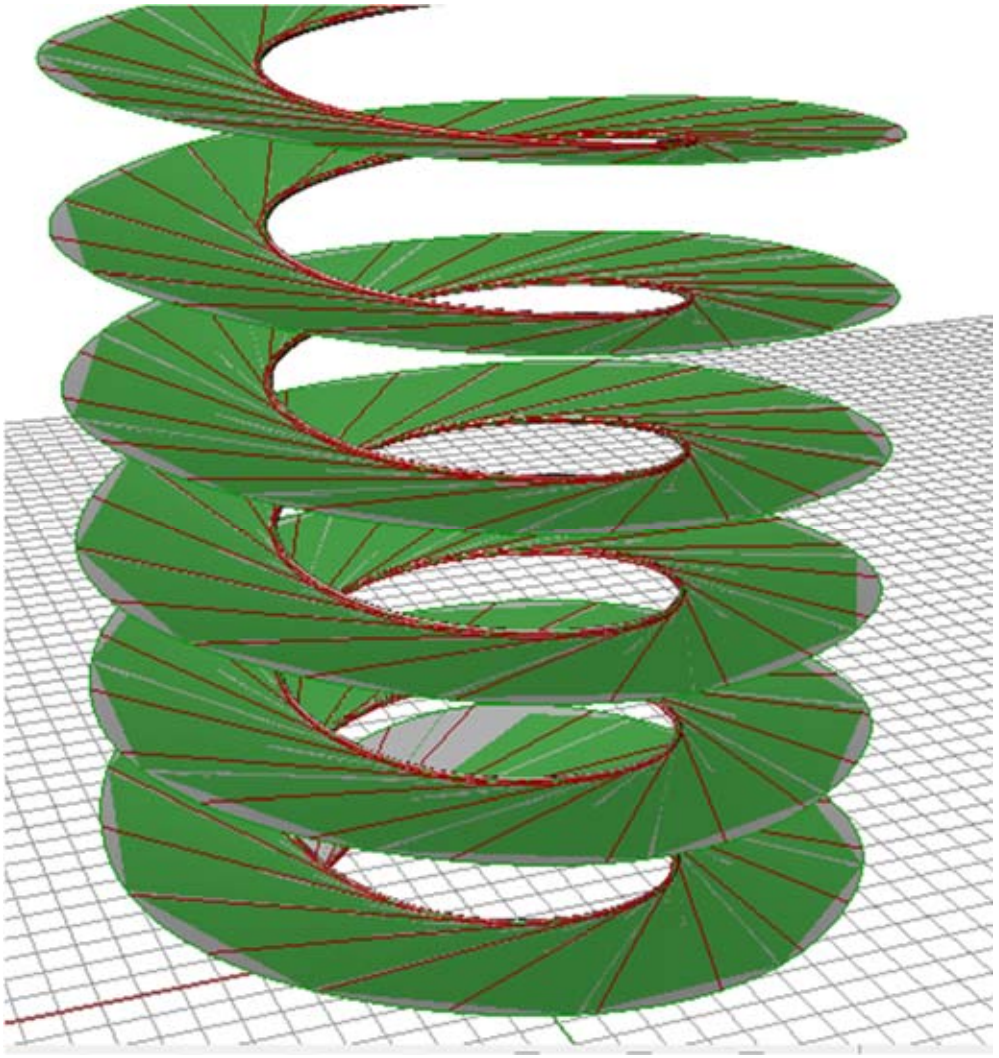
$$\sigma = \sigma(u,v) = \alpha(u) + v\alpha'(u)$$
$$a \le u \le b,\quad c \le v \le d$$

$$\sigma:$$
$$x = 5\cos u - 5v\sin u$$
$$y = 5\sin u + 5v\cos u$$
$$z = 0.5u + 0.5v$$
$$0 \le u \le 20\pi,\quad 0 \le v \le 8$$

POVRŠI U PROSTORU

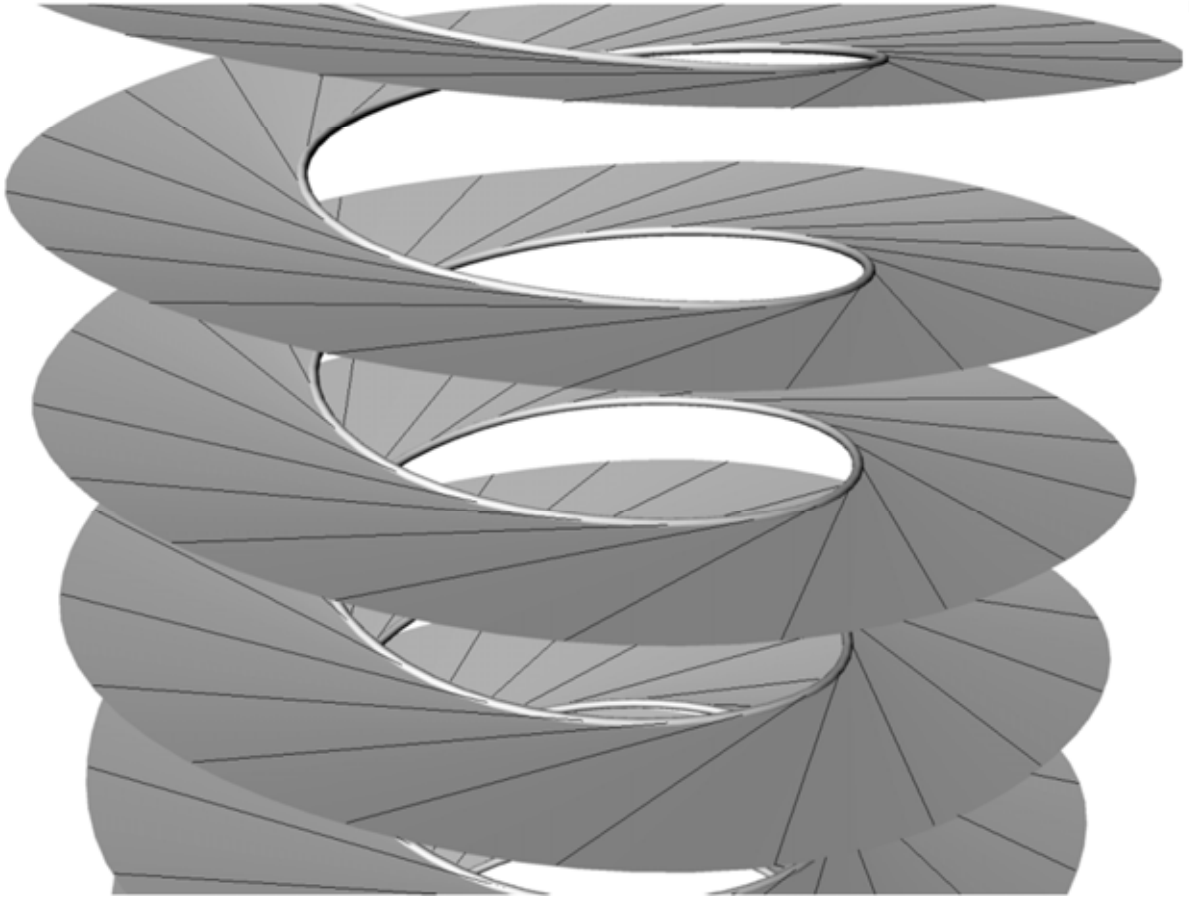
Pravoizvodne površi – tangencijalne površi

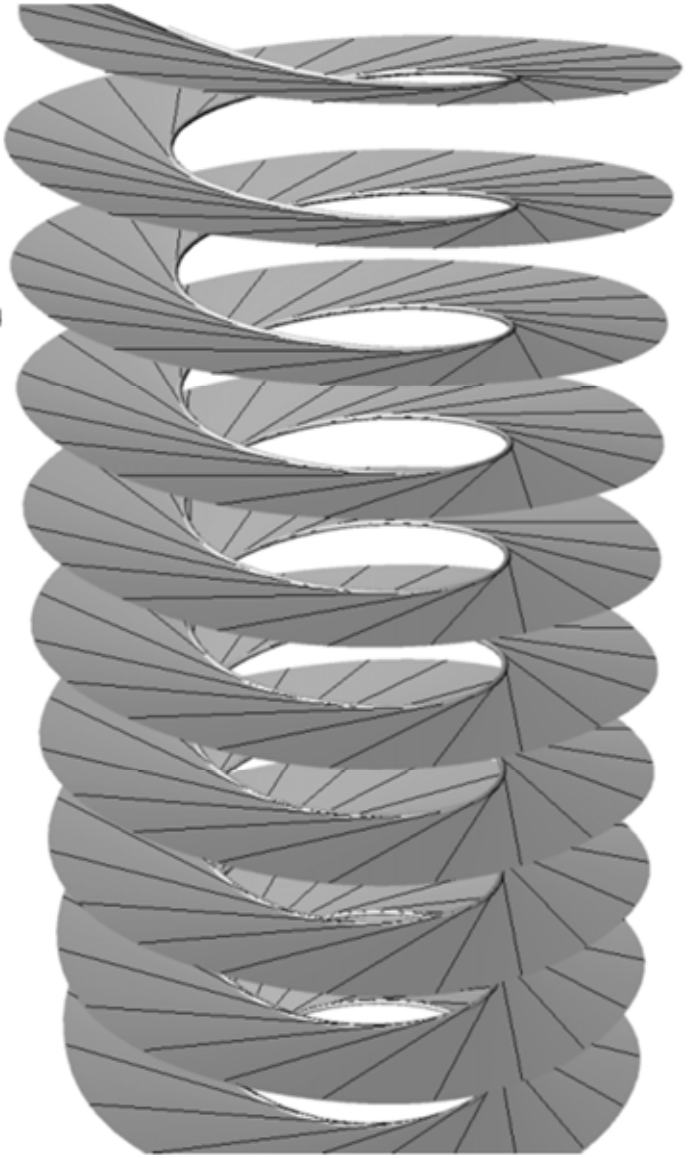




POVRŠI U PROSTORU

Pravoizvodne površi – tangencijalne površi



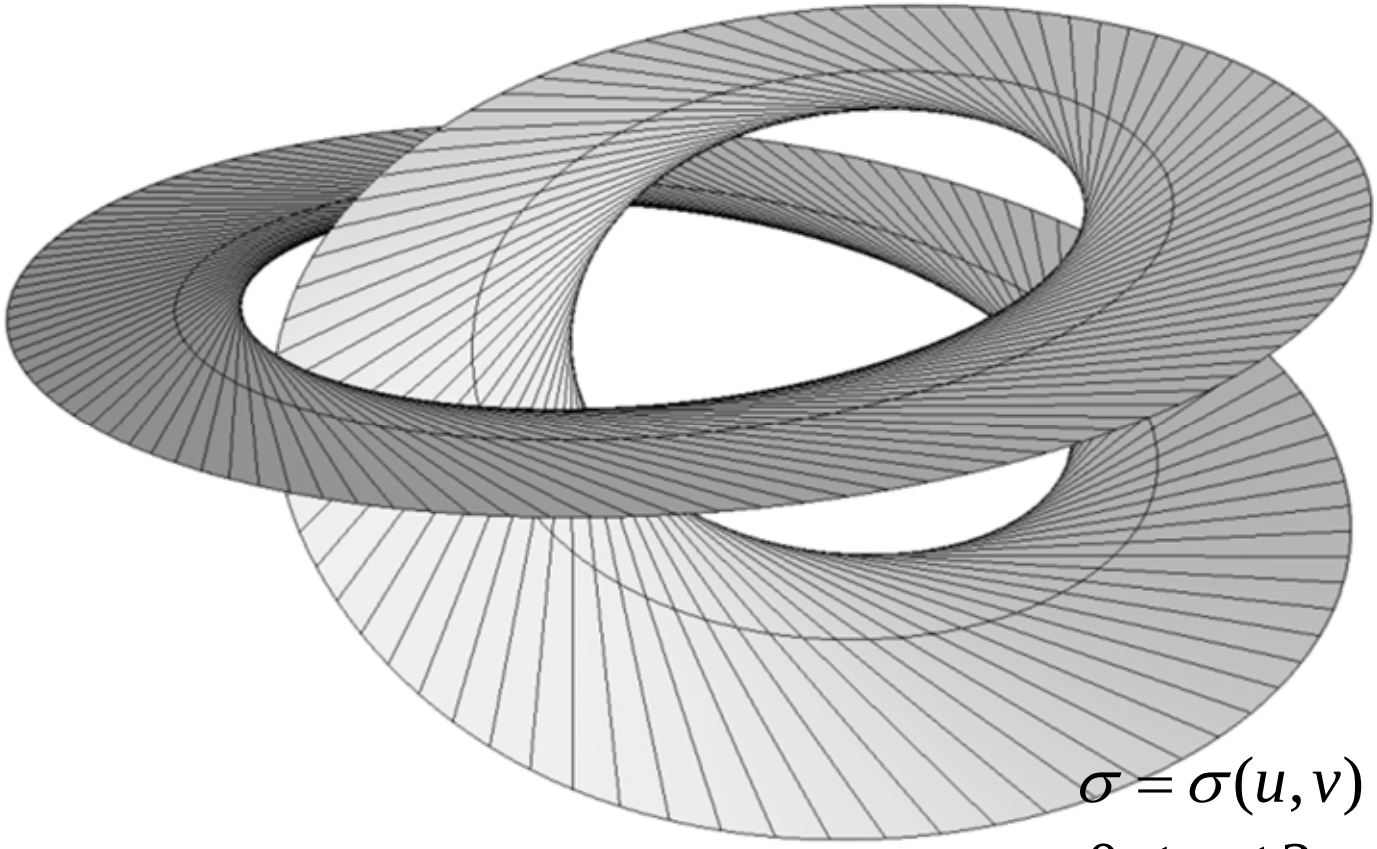


POVRŠI U PROSTORU

$$\alpha(u) = (x(u), y(u), z(u))$$

Pravoizvodne površi – tangencijalne površi

$$\alpha : x(u) = 5\left(2 + \cos\frac{qu}{p}\right)\cos u,\ y(u) = 5\left(2 + \cos\frac{qu}{p}\right)\sin u,\ z = 5\sin\frac{qu}{p}$$



$$p = 2,\ q = 1$$

$$0 \le u \le 2\pi$$

$$\sigma = \sigma(u, v) = \alpha(u) + v\alpha'(u)$$

$$0 \le u \le 2\pi,\quad 0 \le v \le 12$$