POVRŠI U PROSTORU

Površ u prostoru može biti zadata jednačinama :

U parametarskom obliku

$$x = \sigma_1(u, v)$$
$$y = \sigma_2(u, v)$$
$$z = \sigma_3(u, v)$$
$$u_1 \le u \le u_2$$
$$v_1 \le v \le v_2$$

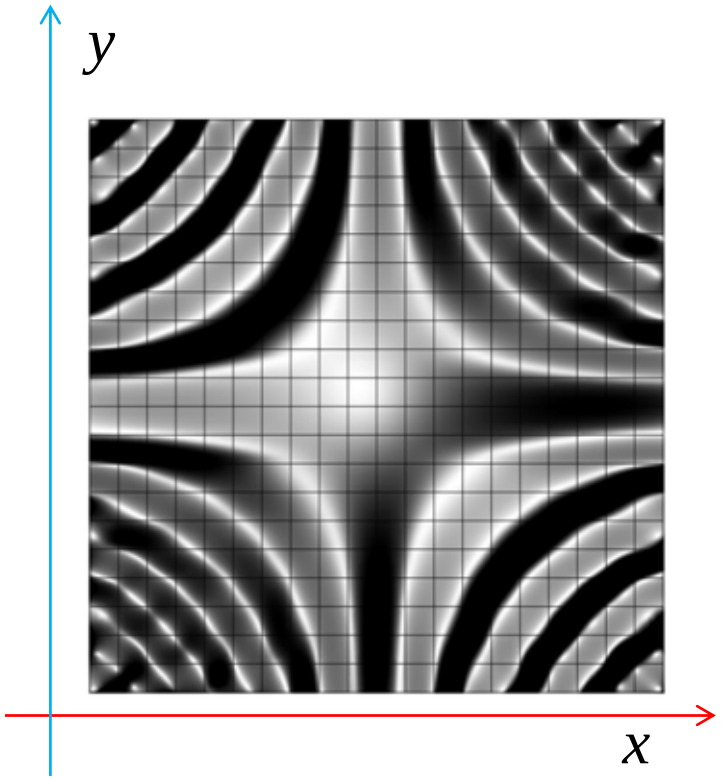
U eksplicitnom obliku

$$z = f(x, y)$$
$$x_1 \le x \le x_2$$
$$y_1 \le y \le y_2$$

U implicitnom obliku

$$F(x, y, z) = 0$$

POVRŠI U PROSTORU

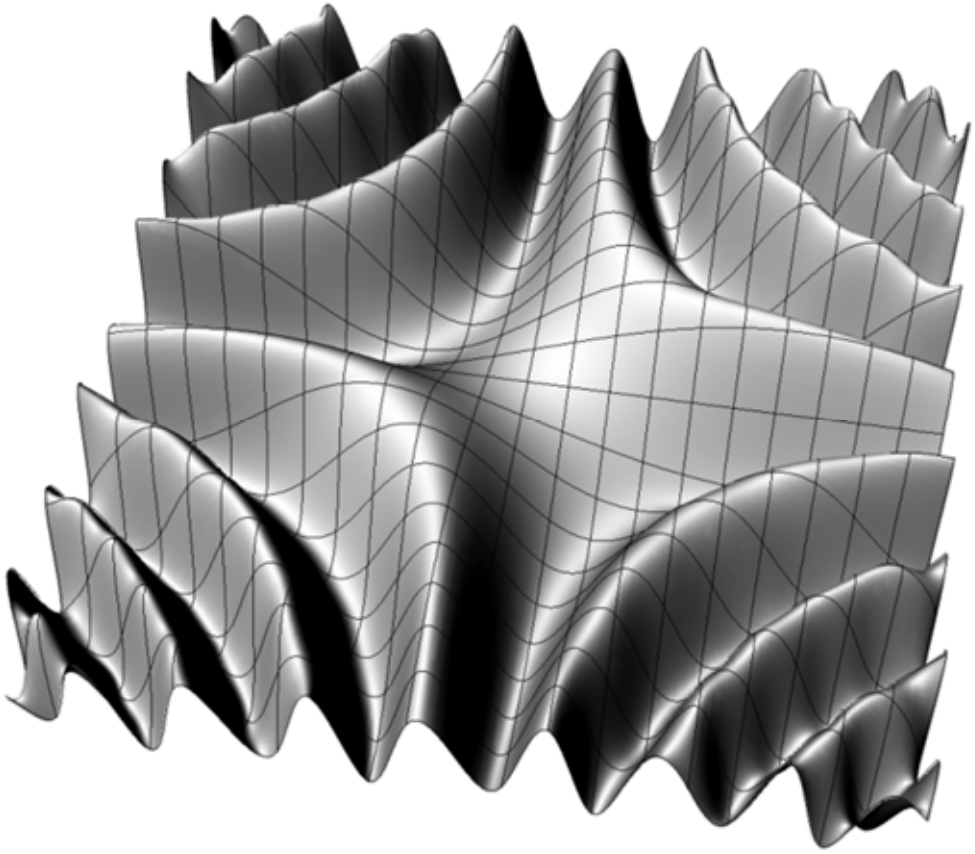


Eksplicitni oblik - primer

$$z = \sin(x \cdot y)$$

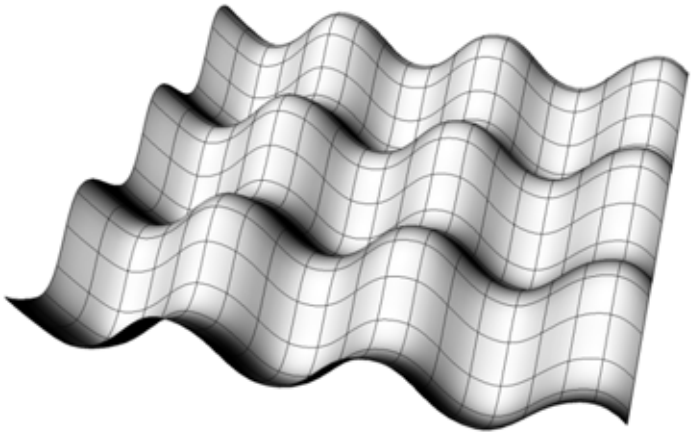
$$-10 \le x \le 10$$

$$-10 \le y \le 10$$

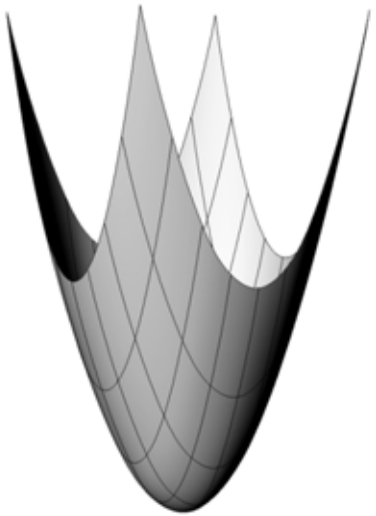


POVRŠI U PROSTORU

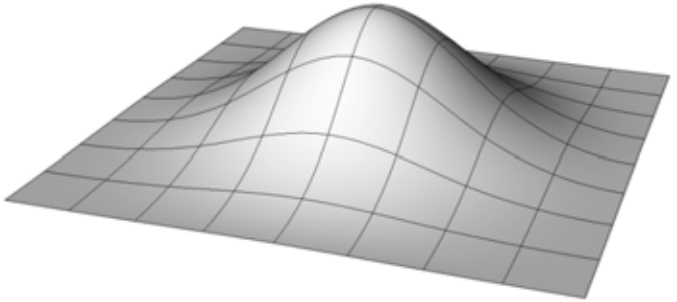
Eksplicitni oblik - primeri



$$z = \sin x + \sin y$$
$$-10 \le x \le 10$$
$$-10 \le y \le 10$$



$$z = x^2 + y^2$$
$$-2 \le x \le 2$$
$$-2 \le y \le 2$$



$$z = e^{-(x^2+y^2)}$$
$$-2 \le x \le 2$$
$$-2 \le y \le 2$$

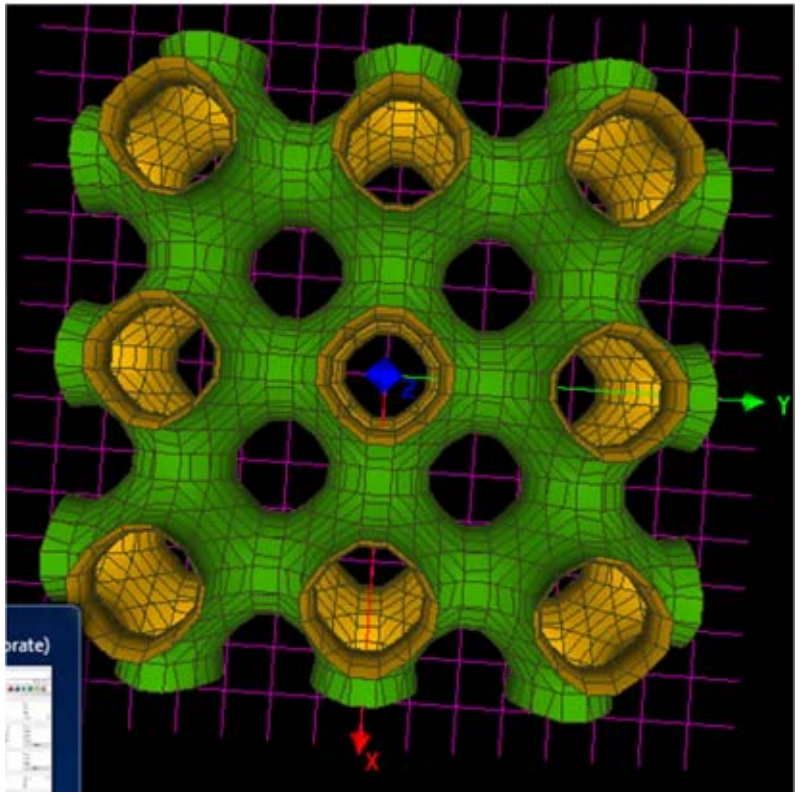
POVRŠI U PROSTORU

Implicitni oblik - primer

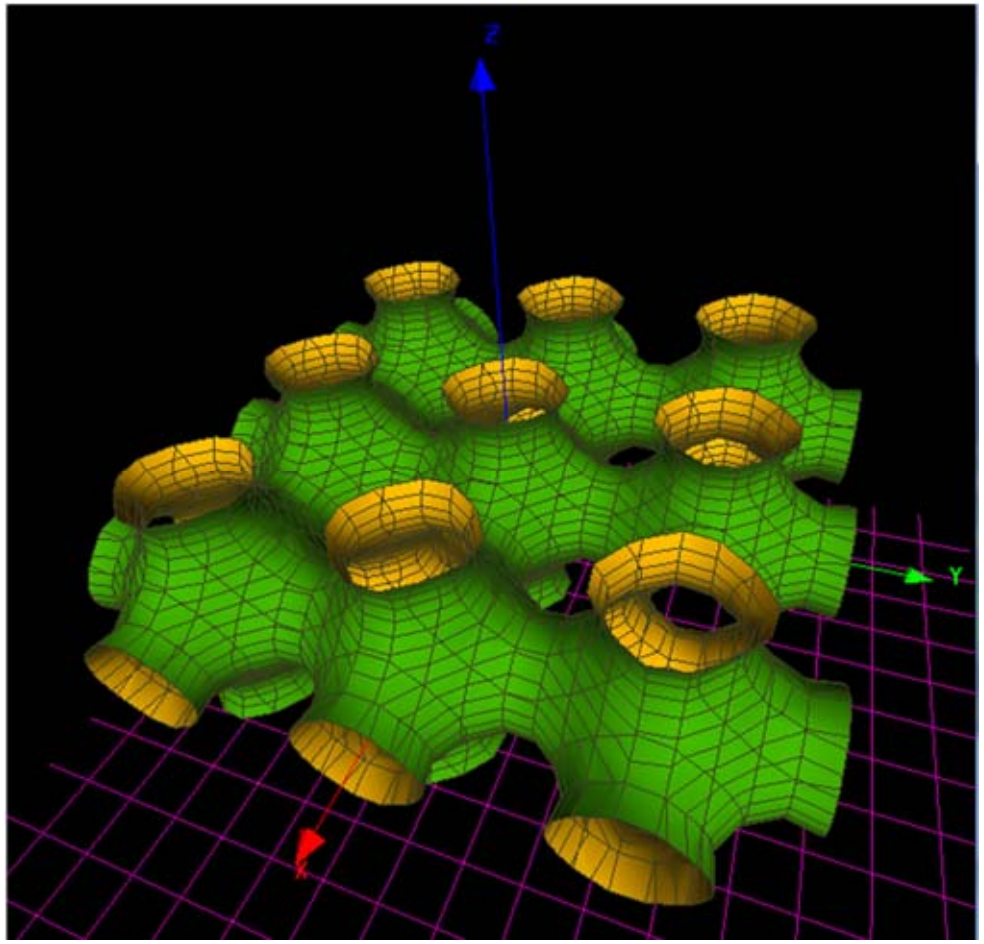
$$cox + \cos y + \cos z = 0$$

$$-10 \le x \le 10$$

$$-10 \le y \le 10$$



Schwartz-ova površ



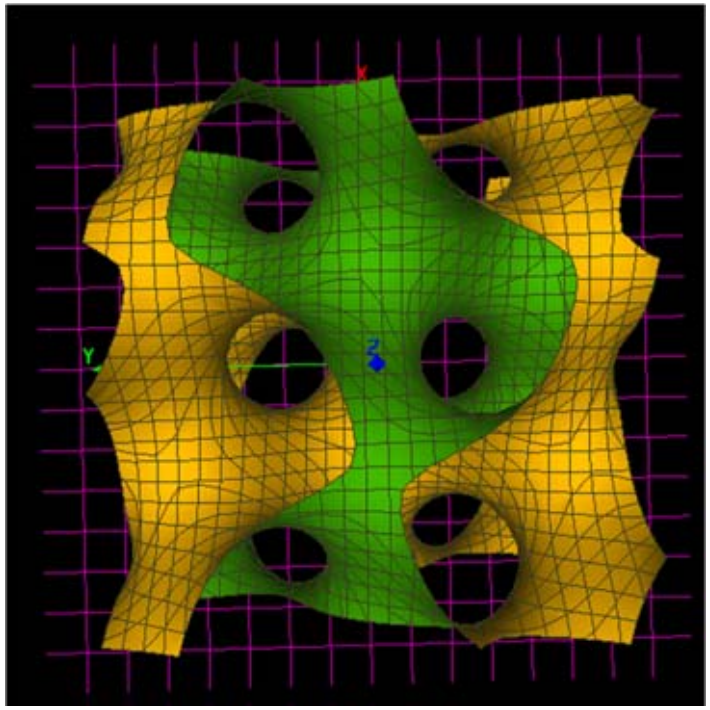
POVRŠI U PROSTORU

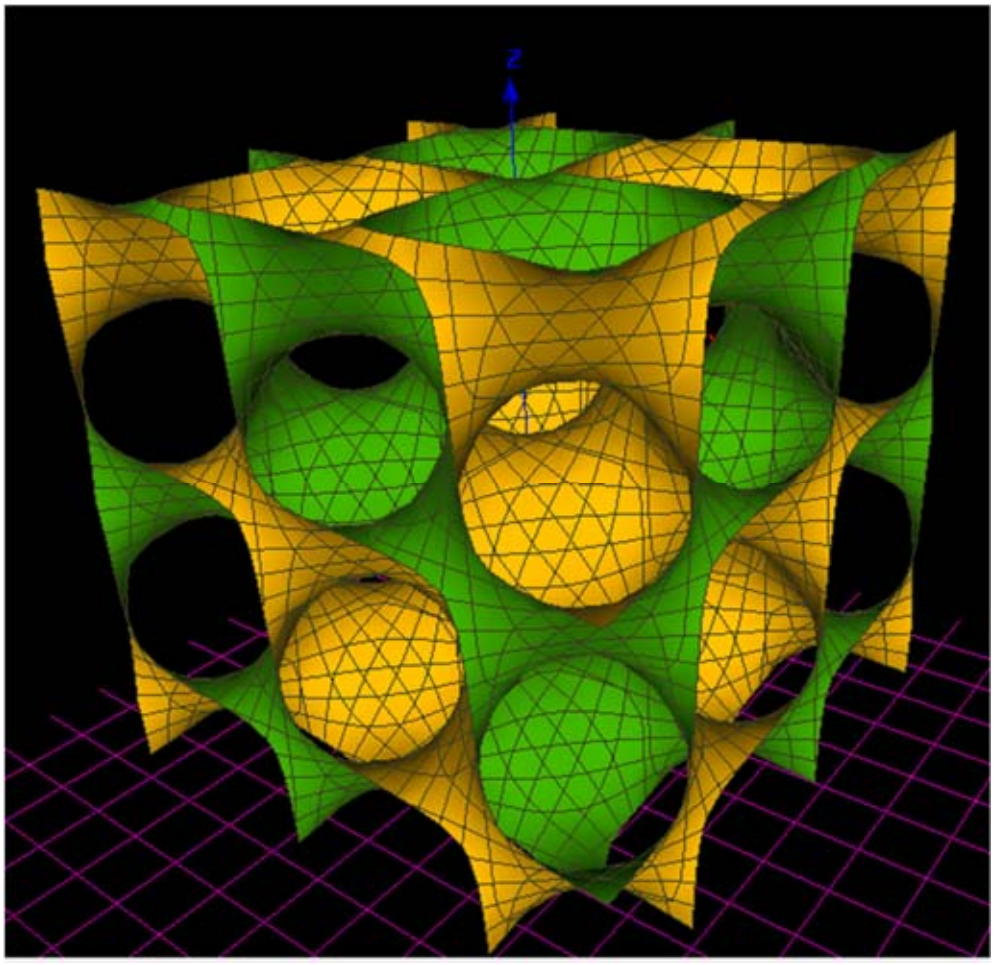
Implicitni oblik - primer

$$cox \cdot \sin y + \cos y \cdot \sin z + \cos z \cdot \sin x = 0$$

$$-4 \le x \le 4$$

$$-4 \le y \le 4$$





Giroid

POVRŠI U PROSTORU

Površi drugog reda

$$Ax^2 + By^2 + Cz^2 + +Dxy + Eyz + Fxz + Mx + Ny + Pz + Q = 0$$

Svodjenjem na kanonski oblik dobijaju se:

Elipsoid I kao poseban slučaj sfera

Jednograni hiperboloid

Dvograni hiperboloid

Eliptički paraboloid

Hiperbolički paraboloid

POVRŠI U PROSTORU

Površi drugog reda

$$Ax^2 + By^2 + Cz^2 + +Dxy + Eyz + Fxz + Mx + Ny + Pz + Q = 0$$

Preseci sa ravnima paralelnim sa *yz*-ravni: $x = p_i$
su krive drugog reda

$$\begin{cases} Ap_i^{\,2} + By^2 + Cz^2 + Dp_i y + Eyz + Fp_i z + Mp_i + Ny + Pz + Q = 0 \\ x = p_i \end{cases}$$

POVRŠI U PROSTORU

Površi drugog reda

$$Ax^2 + By^2 + Cz^2 + +Dxy + Eyz + Fxz + Mx + Ny + Pz + Q = 0$$

Preseci sa ravnima paralelnim sa *xz*-ravni: $y = q_i$
su krive drugog reda

$$\begin{cases} Ax^2 + Bq_i^{\,2} + Cz^2 + Dxq_i + Eq_i z + Fxz + Mx + Nq_i + Pz + Q = 0 \\ y = q_i \end{cases}$$

POVRŠI U PROSTORU

Površi drugog reda

$$Ax^2 + By^2 + Cz^2 + +Dxy + Eyz + Fxz + Mx + Ny + Pz + Q = 0$$

Preseci sa ravnima paralelnim sa *xy*-ravni: $z = r_i$
su krive drugog reda

$$\begin{cases} Ax^2 + By^2 + Cr_i^2 + Dxy + Eyr_i + Fxr_i + Mx + Ny + \mathrm{Pr}_i + Q = 0 \\ z = r_i \end{cases}$$

POVRŠI U PROSTORU

Površi drugog reda - elipsoid

$$Ax^2 + By^2 + Cz^2 + +Dxy + Eyz + Fxz + Mx + Ny + Pz + Q = 0$$

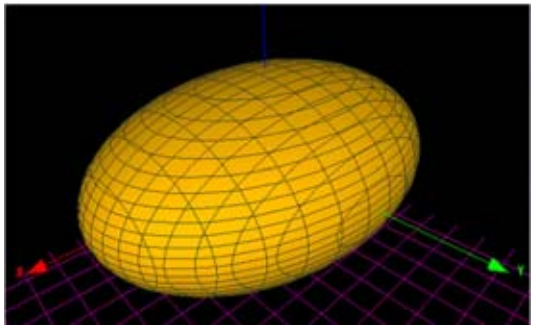
Elipsoid: $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

$-a \le x \le a$

$-b \le y \le b$

$-c \le z \le c$

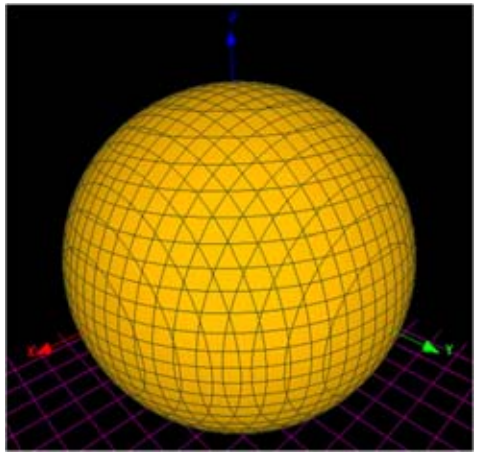
a, b, c - poluose elipsoida



Poseban slučaj $a = b = c = r$

Sfera: $x^2 + y^2 + z^2 = r^2$

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2$$



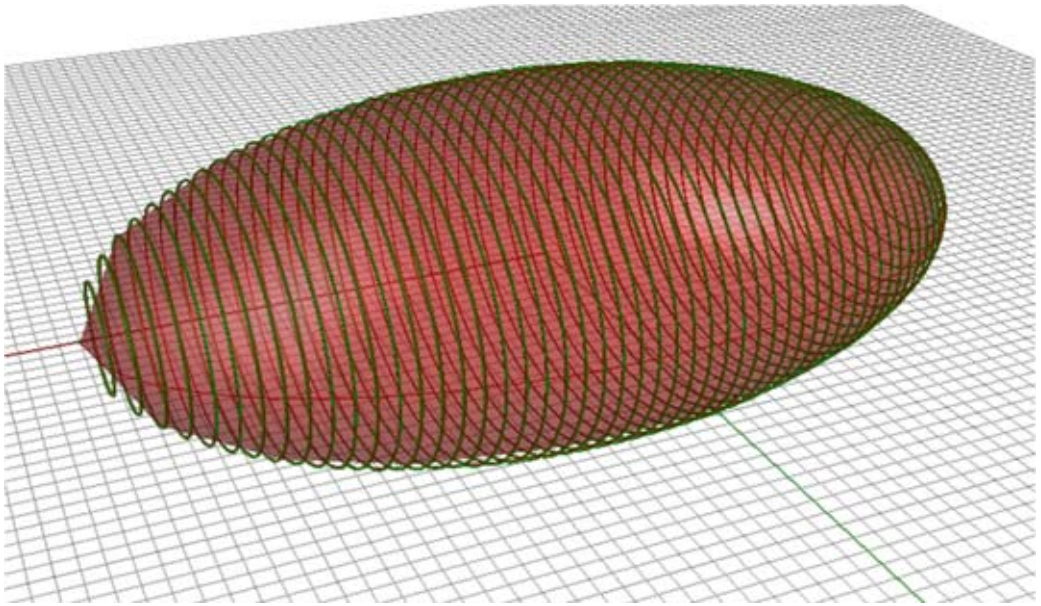
POVRŠI U PROSTORU

Površi drugog reda - elipsoid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

$$-a \le x \le a$$
$$-b \le y \le b$$
$$-c \le z \le c$$

$$x = p_i$$



$$\frac{p_i^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

$$\frac{y^2}{b^2}+\frac{z^2}{c^2}=1-\frac{p_i^2}{a^2}$$

$$\frac{y^2}{b^2\left(1-\frac{p_i^2}{a^2}\right)}+\frac{z^2}{c^2\left(1-\frac{p_i^2}{a^2}\right)}=1$$

$$x = p_i$$

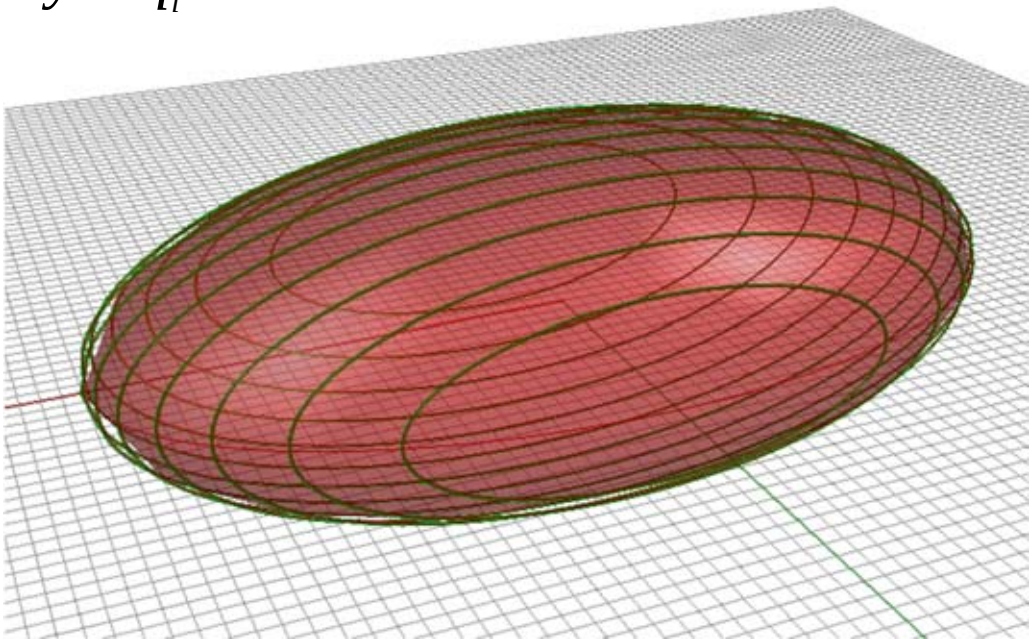
POVRŠI U PROSTORU

Površi drugog reda - elipsoid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

$$-a \le x \le a$$
$$-b \le y \le b$$
$$-c \le z \le c$$

$$y = q_i$$



$$\frac{x^2}{a^2}+\frac{q_i^{\,2}}{b^2}+\frac{z^2}{c^2}=1$$

$$\frac{x^2}{a^2}+\frac{z^2}{c^2}=1-\frac{q_i^{\,2}}{b^2}$$

$$\frac{x^2}{a^2\left(1-\frac{q_i^{\,2}}{b^2}\right)}+\frac{z^2}{c^2\left(1-\frac{q_i^{\,2}}{b^2}\right)}=1$$

$$y = q_i$$

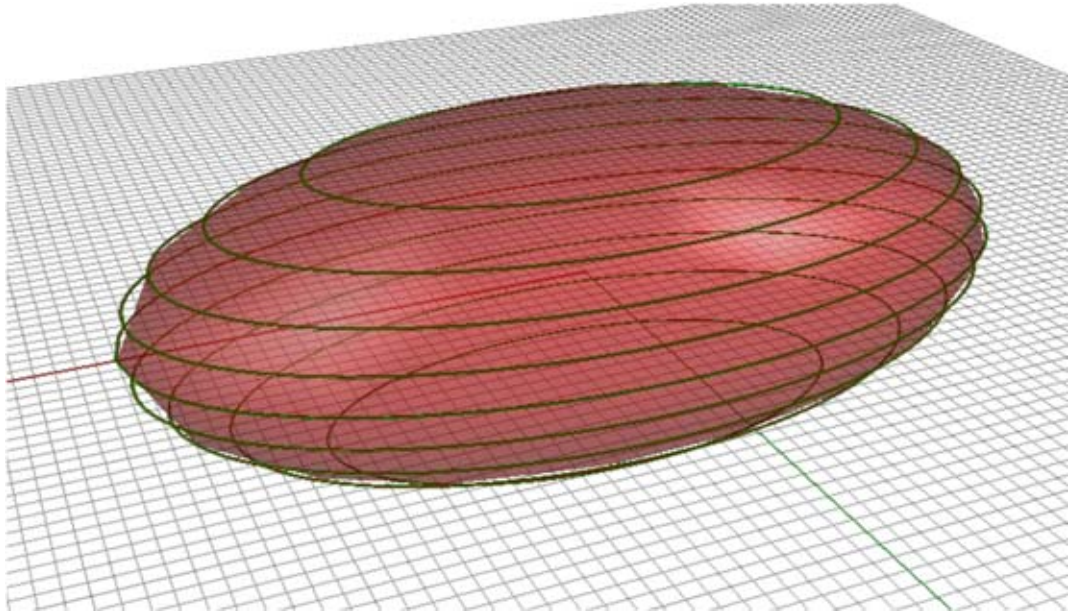
POVRŠI U PROSTORU

Površi drugog reda - elipsoid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

$$-a \le x \le a$$
$$-b \le y \le b$$
$$-c \le z \le c$$

$$z = r_i$$



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{r_i^2}{c^2}=1$$

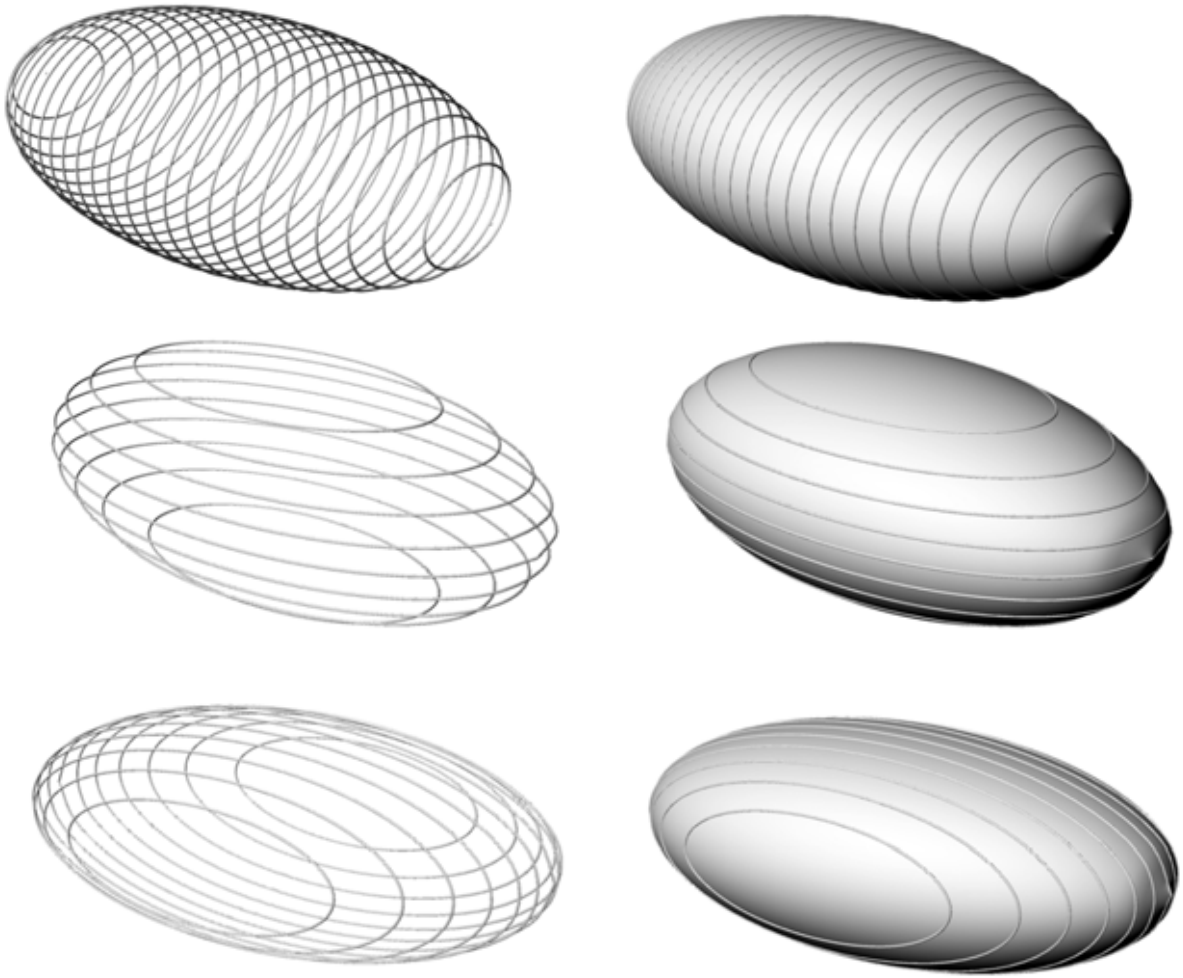
$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1-\frac{r_i^2}{c^2}$$

$$\frac{x^2}{a^2\left(1-\frac{r_i^2}{c^2}\right)}+\frac{y^2}{b^2\left(1-\frac{r_i^2}{c^2}\right)}=1$$

$$z = r_i$$

POVRŠI U PROSTORU

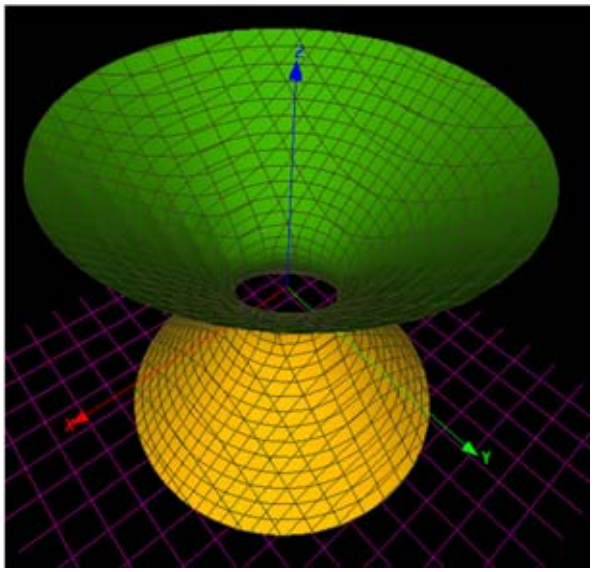
Površi drugog reda - elipsoid



POVRŠI U PROSTORU

Površi drugog reda - jednograni hiperboloid

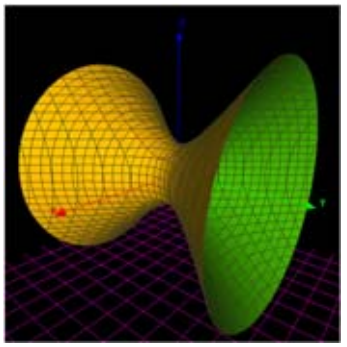
Jednograni hiperboloid: $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$



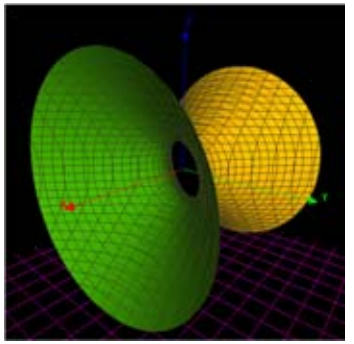
a, b, c - poluose hiperboloida

c – imaginarna poluosa

Oz - imaginarna osa jednogranog hiperboloida i on sa njom nema zajedničkih tačaka



$\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ *Oy* - imaginarna osa



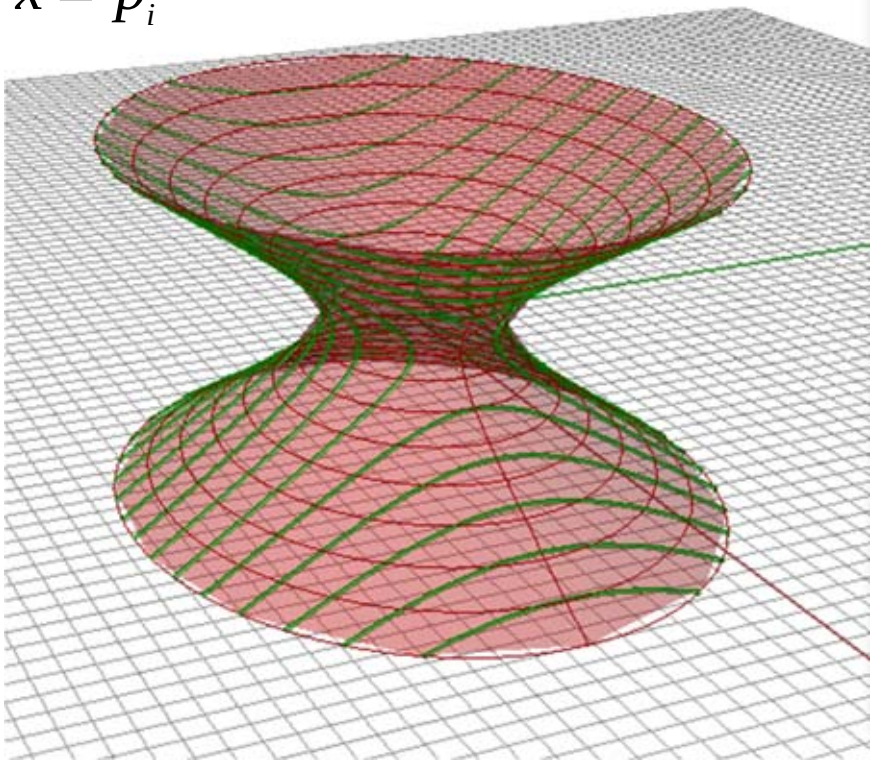
$-\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ *Ox* - imaginarna osa

POVRŠI U PROSTORU

Površi drugog reda - jednograni hiperboloid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$$

$$x = p_i$$



$$\frac{p_i^{\,2}}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$$

$$\frac{y^2}{b^2}-\frac{z^2}{c^2}=1-\frac{p_i^{\,2}}{a^2}$$

$$\frac{y^2}{b^2\left(1-\frac{p_i^{\,2}}{a^2}\right)}-\frac{z^2}{c^2\left(1-\frac{p_i^{\,2}}{a^2}\right)}=1$$

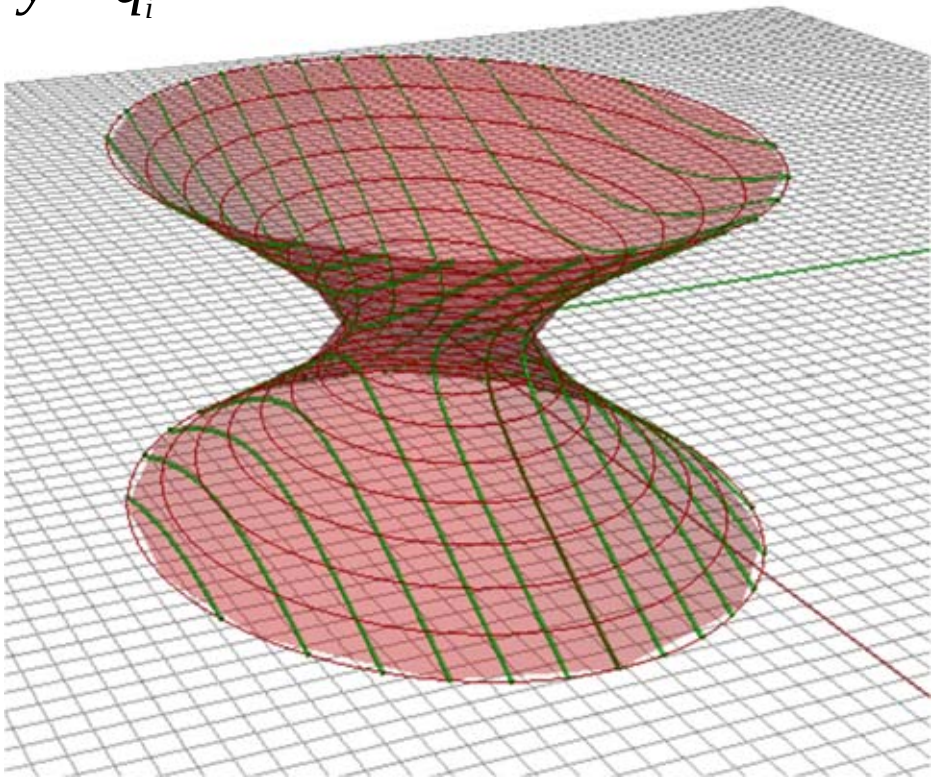
$$x = p_i$$

POVRŠI U PROSTORU

Površi drugog reda - jednograni hiperboloid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$$

$$y=q_i$$



$$\frac{x^2}{a^2}+\frac{q_i^{\,2}}{b^2}-\frac{z^2}{c^2}=1$$

$$\frac{x^2}{a^2}-\frac{z^2}{c^2}=1-\frac{q_i^{\,2}}{b^2}$$

$$\frac{x^2}{a^2\left(1-\frac{q_i^{\,2}}{b^2}\right)}-\frac{z^2}{c^2\left(1-\frac{q_i^{\,2}}{b^2}\right)}=1$$

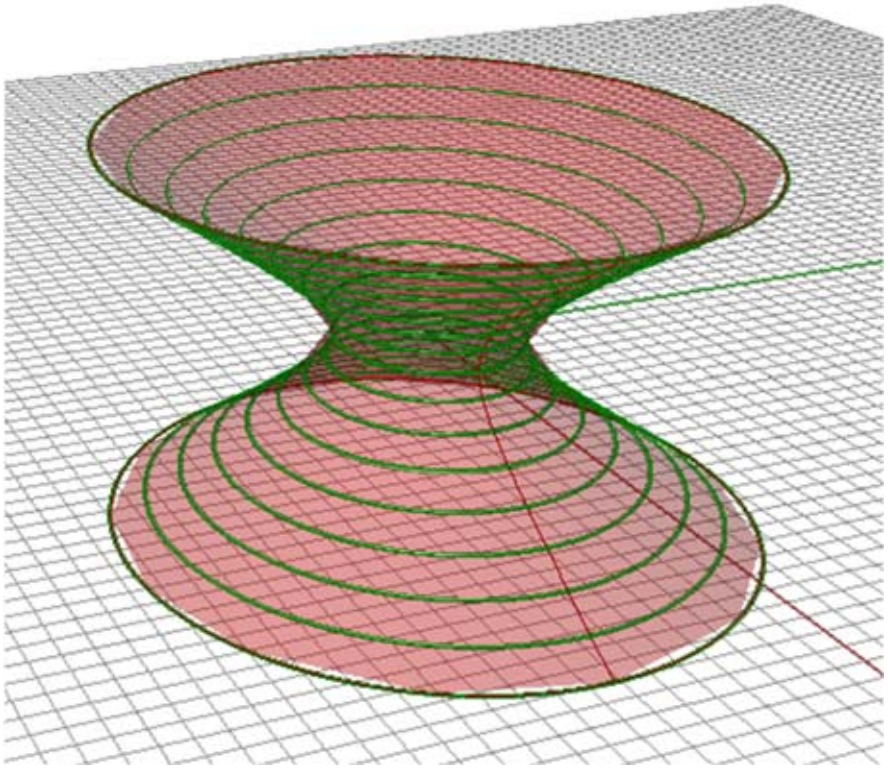
$$y=q_i$$

POVRŠI U PROSTORU

Površi drugog reda - jednograni hiperboloid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$$

$$z=r_i$$



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{r_i^2}{c^2}=1$$

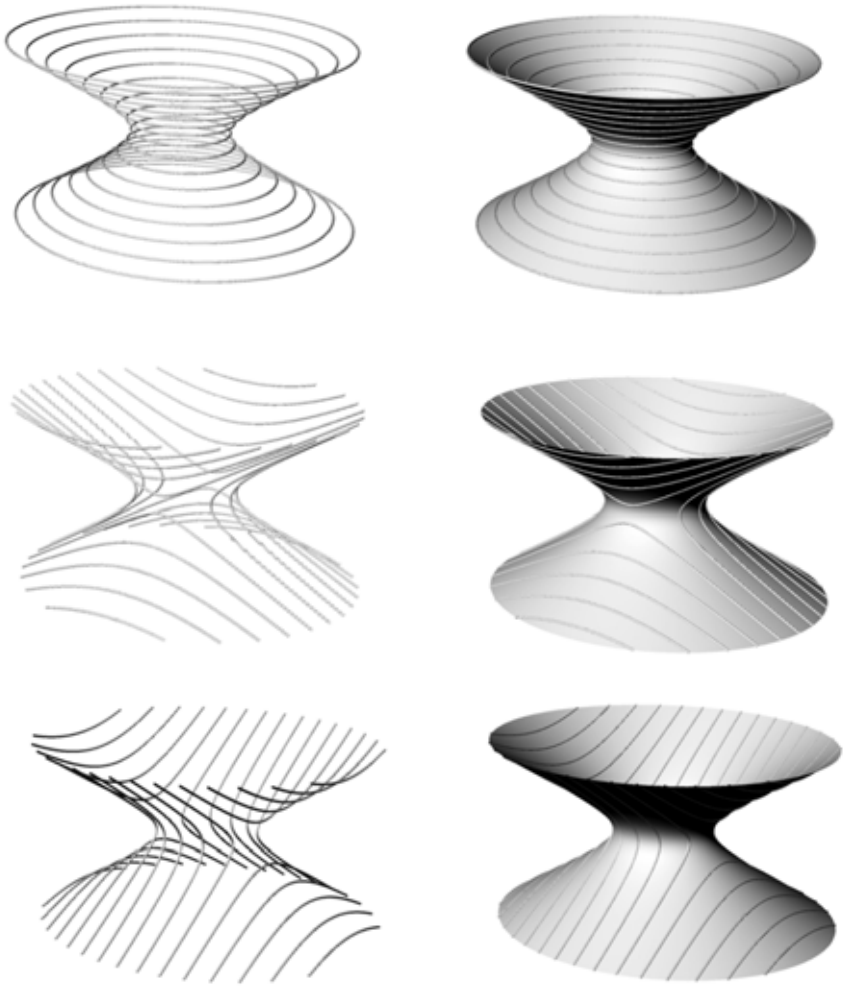
$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1+\frac{r_i^2}{c^2}$$

$$\frac{x^2}{a^2\left(1+\frac{r_i^2}{c^2}\right)}+\frac{y^2}{b^2\left(1+\frac{r_i^2}{c^2}\right)}=1$$

$$z=r_i$$

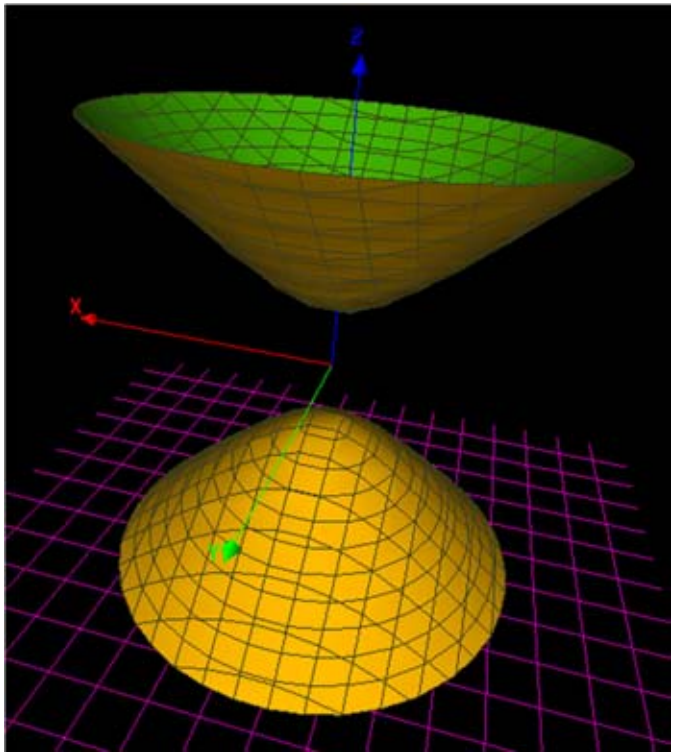
POVRŠI U PROSTORU

Površi drugog reda - jednograni hiperboloid



POVRŠI U PROSTORU

Površi drugog reda - dvograni hiperboloid



Dvograni hiperboloid: $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1$

a, b, c - poluose hiperboloida

a, b - imaginarne poluose

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=-1$$

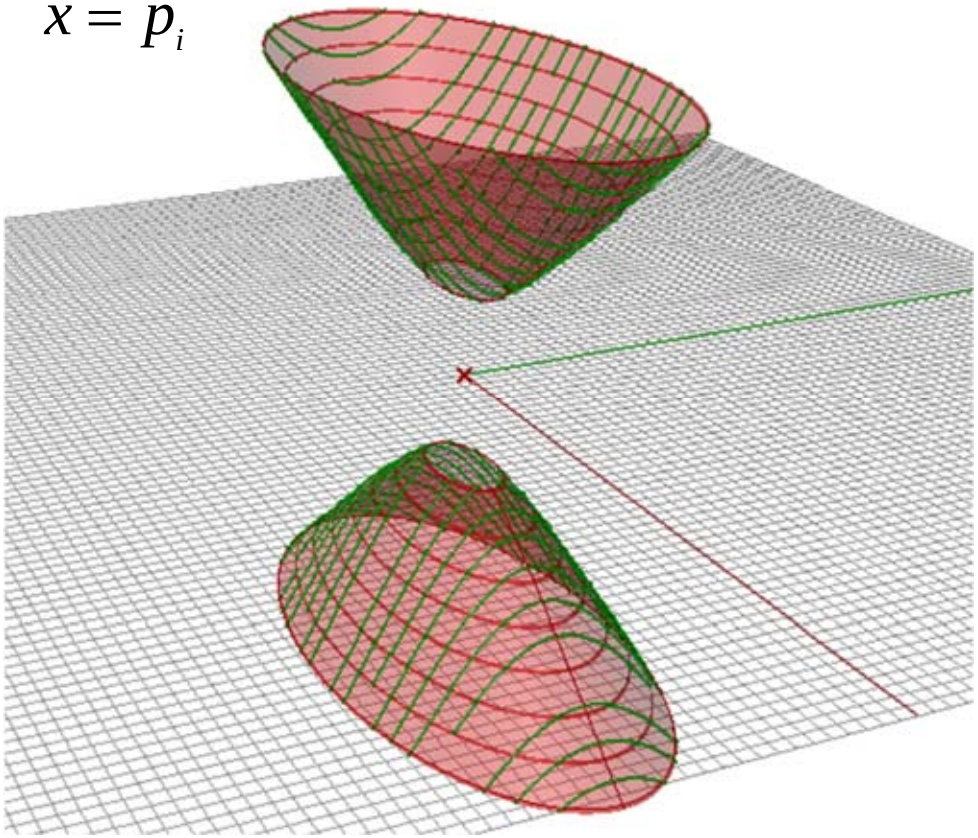
$$-\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=-1$$

POVRŠI U PROSTORU

Površi drugog reda - dvograni hiperboloid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1$$

$$x = p_i$$



$$\frac{p_i^{\,2}}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1$$

$$\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1-\frac{p_i^{\,2}}{a^2}$$

$$\frac{z^2}{c^2}-\frac{y^2}{b^2}=1+\frac{p_i^{\,2}}{a^2}$$

$$\frac{z^2}{c^2\left(1+\frac{p_i^{\,2}}{a^2}\right)}-\frac{y^2}{b^2\left(1+\frac{p_i^{\,2}}{a^2}\right)}=1$$

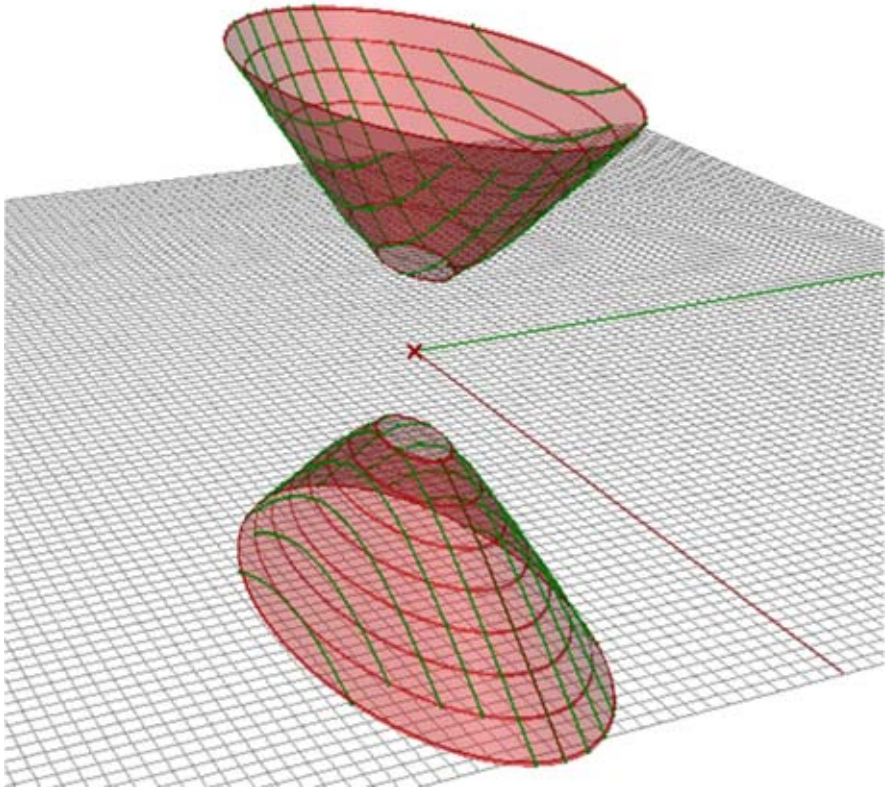
$$x = p_i$$

POVRŠI U PROSTORU

Površi drugog reda - dvograni hiperboloid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1$$

$$y=q_i$$



$$\frac{x^2}{a^2}+\frac{q_i^{\,2}}{b^2}-\frac{z^2}{c^2}=-1$$

$$\frac{x^2}{a^2}-\frac{z^2}{c^2}=-1-\frac{q_i^{\,2}}{b^2}$$

$$\frac{z^2}{c^2}-\frac{x^2}{a^2}=1+\frac{q_i^{\,2}}{b^2}$$

$$\frac{z^2}{c^2\left(1+\frac{q_i^{\,2}}{b^2}\right)}-\frac{x^2}{a^2\left(1+\frac{q_i^{\,2}}{b^2}\right)}=1+\frac{q_i^{\,2}}{b^2}$$

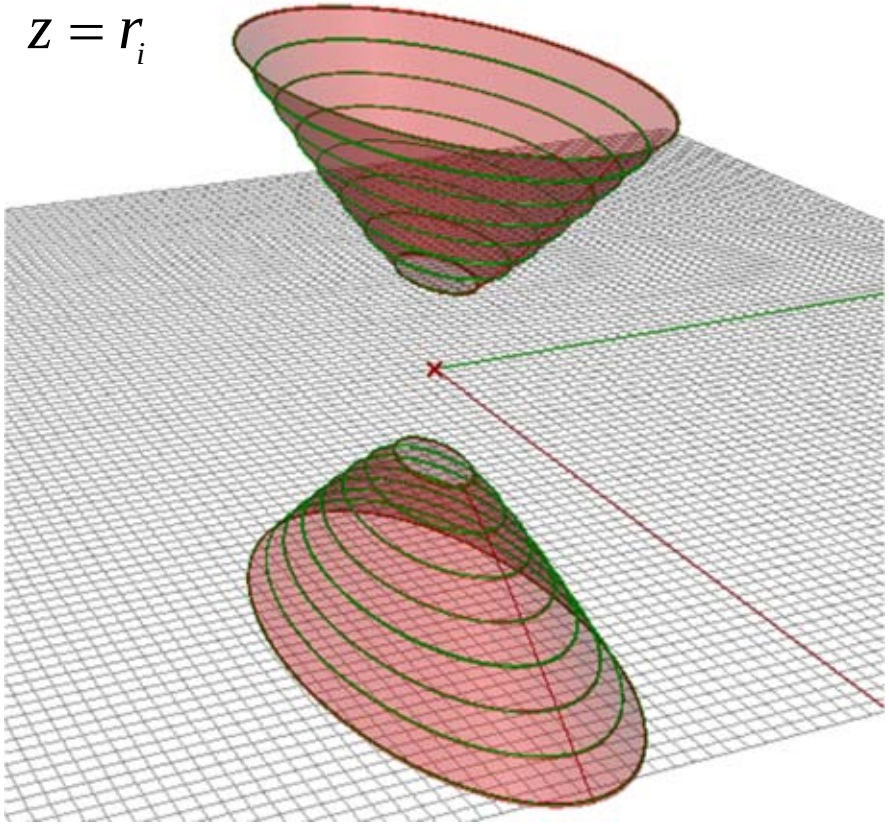
$$y=q_i$$

POVRŠI U PROSTORU

Površi drugog reda - dvograni hiperboloid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1$$

$$z=r_i$$



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{r_i^2}{c^2}=-1$$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=-1+\frac{r_i^2}{c^2}$$

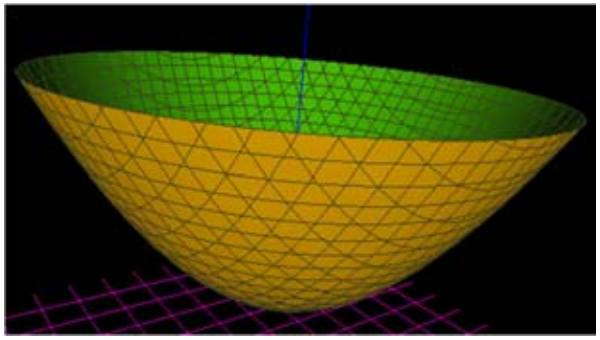
$$\frac{x^2}{a^2\left(-1+\frac{r_i^2}{c^2}\right)}+\frac{y^2}{b^2\left(-1+\frac{r_i^2}{c^2}\right)}=1$$

$$z=r_i$$

POVRŠI U PROSTORU

Površi drugog reda - eliptički paraboloid

Eliptički paraboloid: $\frac{x^2}{p} + \frac{y^2}{q} = 2z$



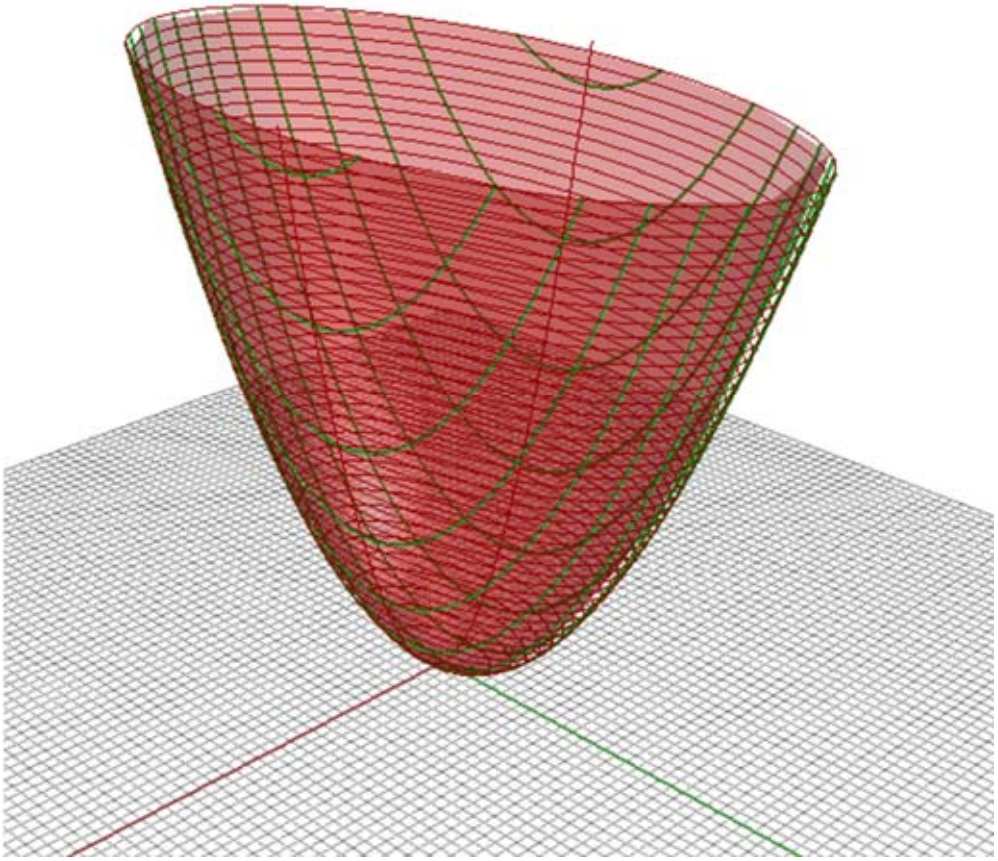
$\frac{y^2}{p} + \frac{z^2}{q} = 2x$

$\frac{x^2}{p} + \frac{z^2}{q} = 2y$

POVRŠI U PROSTORU

Površi drugog reda - eliptički paraboloid

$$\frac{x^2}{p}+\frac{y^2}{q}=2z \qquad x=p_i$$



$$\frac{p_i^{\,2}}{p}+\frac{y^2}{q}=2z$$

$$\frac{y^2}{q}=2z-\frac{p_i^{\,2}}{p}$$

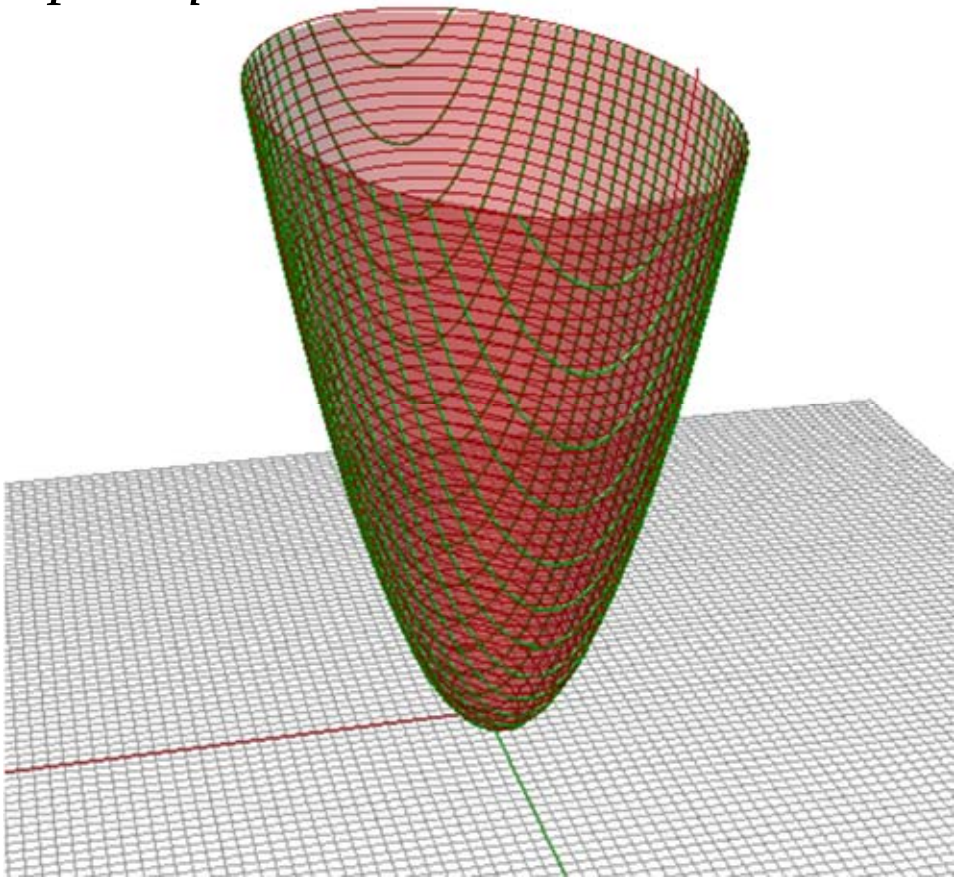
$$y^2=2q\left(z-\frac{p_i^{\,2}}{2p}\right)$$

$$x=p_i$$

POVRŠI U PROSTORU

Površi drugog reda - eliptički paraboloid

$$\frac{x^2}{p}+\frac{y^2}{q}=2z \qquad y=q_i$$



$$\frac{x^2}{p}+\frac{q_i^{\,2}}{q}=2z$$

$$\frac{x^2}{p}=2z-\frac{q_i^{\,2}}{q}$$

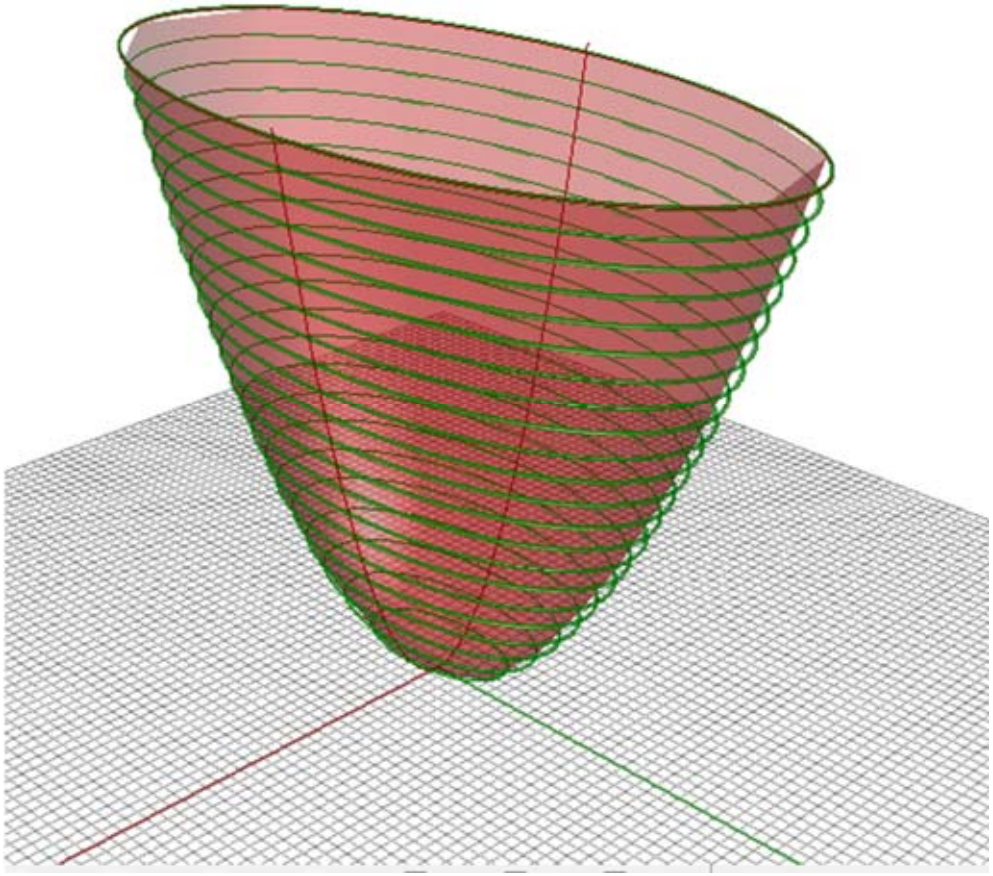
$$x^2=2p\left(z-\frac{q_i^{\,2}}{2q}\right)$$

$$y=q_i$$

POVRŠI U PROSTORU

Površi drugog reda - eliptički paraboloid

$$\frac{x^2}{p}+\frac{y^2}{q}=2z \qquad z=r_i$$



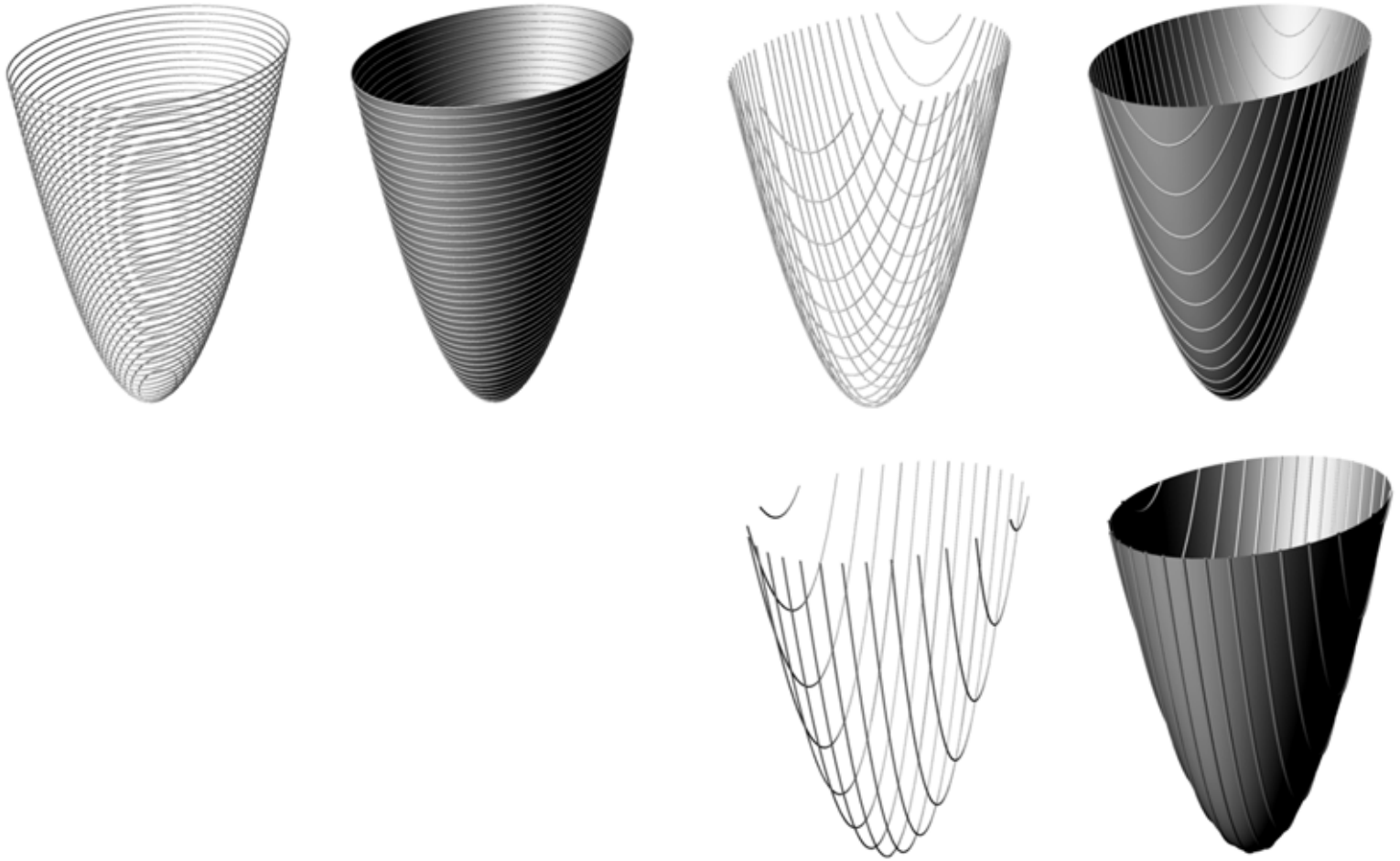
$$\frac{x^2}{p}+\frac{y^2}{q}=2r_i$$

$$\frac{x^2}{2pr_i}+\frac{y^2}{2qr_i}=1$$

$$z=r_i$$

POVRŠI U PROSTORU

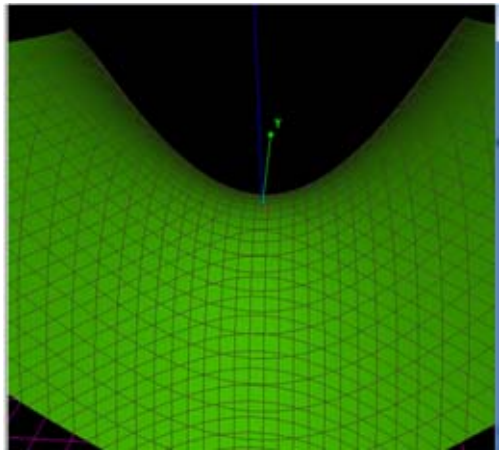
Površi drugog reda - eliptički paraboloid



POVRŠI U PROSTORU

Površi drugog reda - hiperbolički paraboloid

Hiperbolički paraboloid: $\frac{x^2}{p} - \frac{y^2}{q} = 2z$



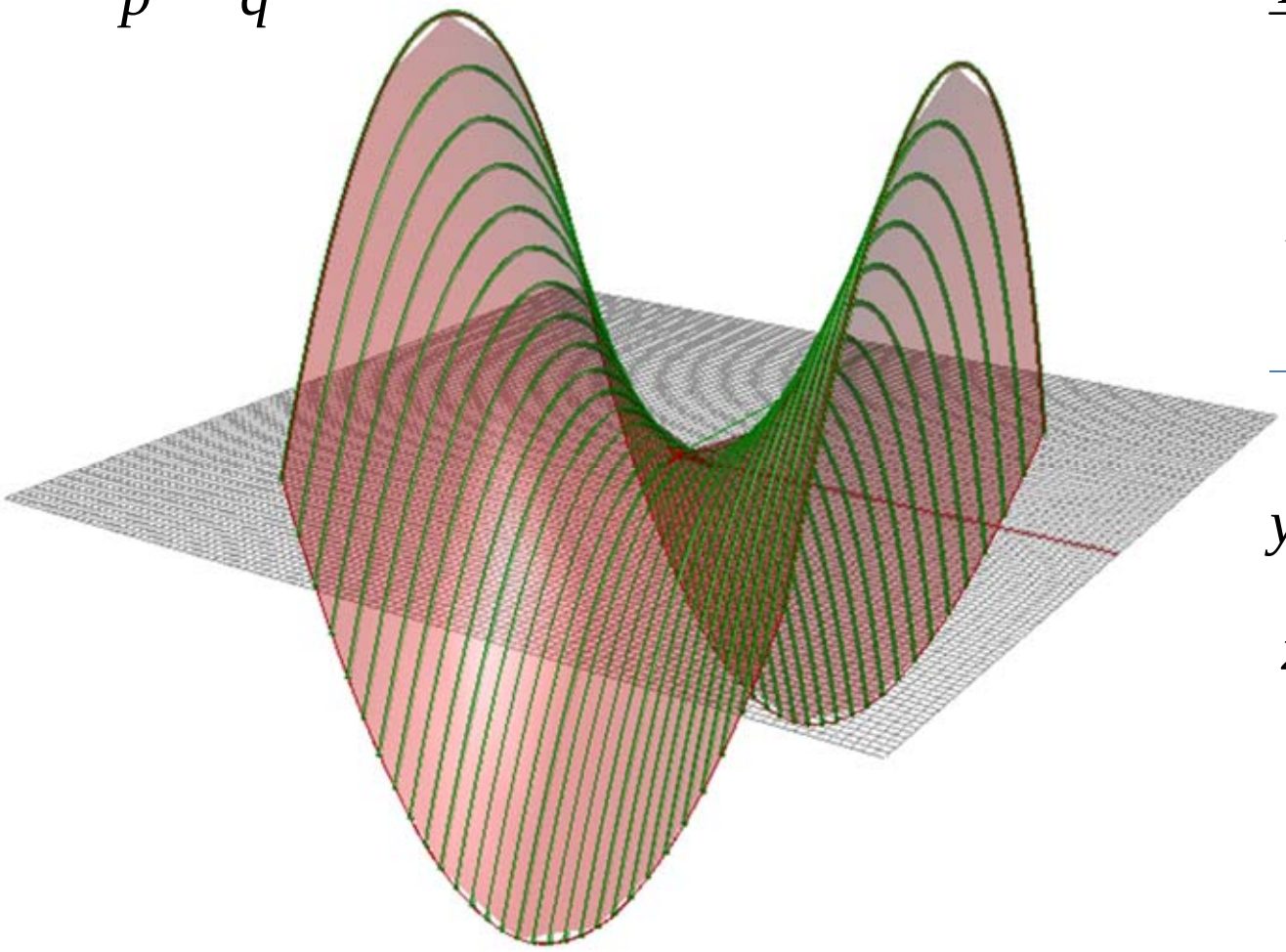
$\frac{y^2}{p} - \frac{z^2}{q} = 2x$

$\frac{x^2}{p} - \frac{z^2}{q} = 2y$

POVRŠI U PROSTORU

Površi drugog reda - hiperbolički paraboloid

$$\frac{x^2}{p} - \frac{y^2}{q} = 2z \qquad x = p_i$$



$$\frac{p_i^{\,2}}{p} - \frac{y^2}{q} = 2z$$

$$\frac{y^2}{q} = -2z + \frac{p_i^{\,2}}{p}$$

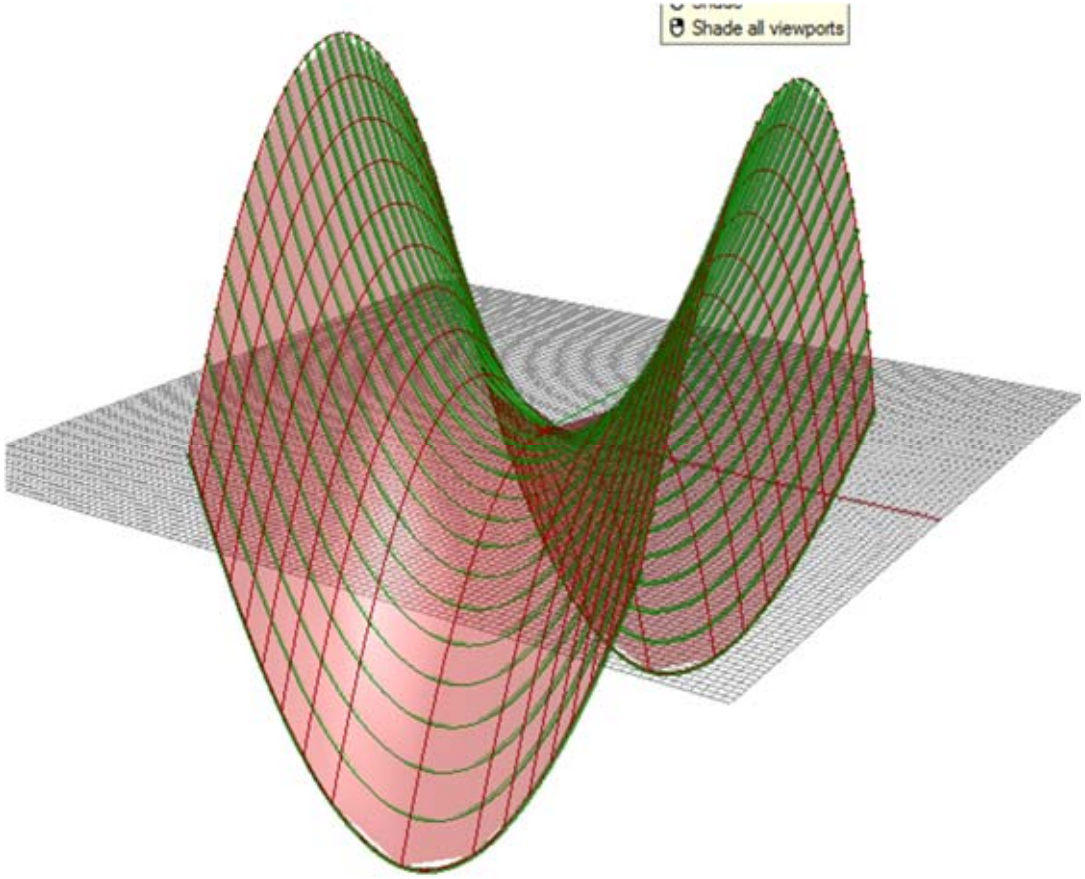
$$y^2 = -2q\left(z - \frac{p_i^{\,2}}{2p}\right)$$

$$x = p_i$$

POVRŠI U PROSTORU

Površi drugog reda - hiperbolički paraboloid

$$\frac{x^2}{p} - \frac{y^2}{q} = 2z \qquad y = q_i$$



$$\frac{x^2}{p} - \frac{q_i^2}{q} = 2z$$

$$\frac{x^2}{p} = 2z + \frac{q_i^2}{q}$$

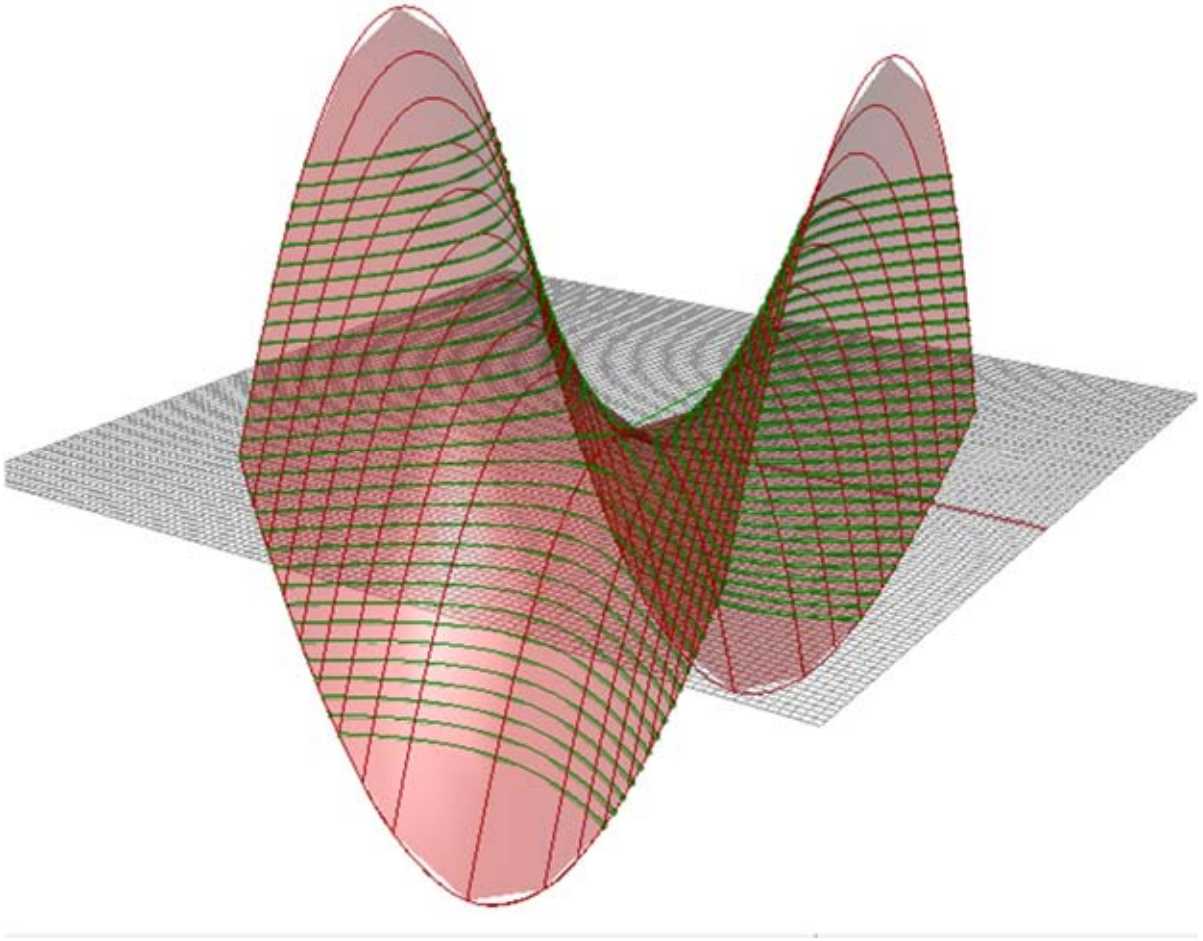
$$x^2 = 2p\left(z + \frac{q_i^2}{2q}\right)$$

$$y = q_i$$

POVRŠI U PROSTORU

Površi drugog reda - hiperbolički paraboloid

$$\frac{x^2}{p} - \frac{y^2}{q} = 2z \qquad z = r_i$$



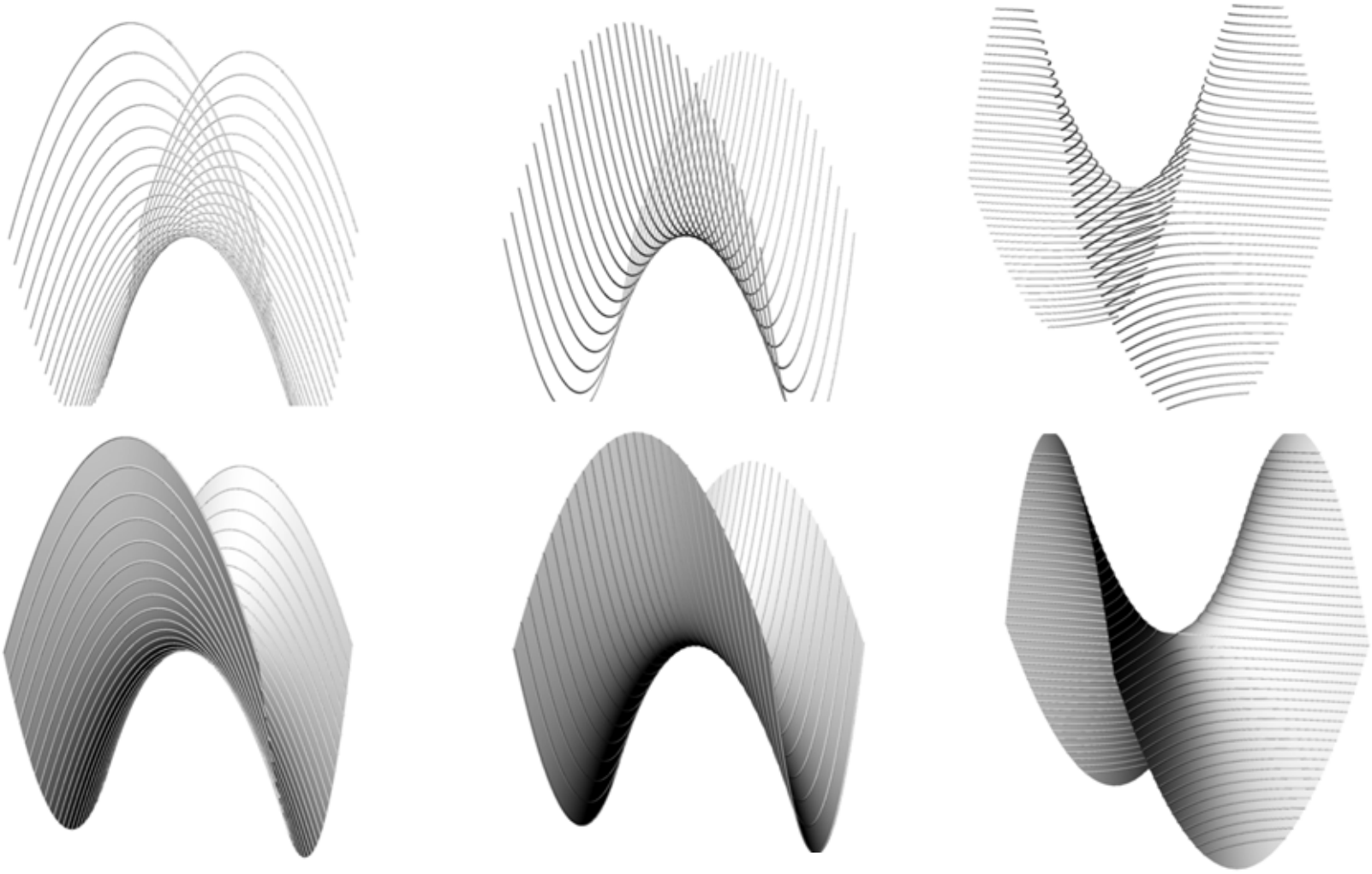
$$\frac{x^2}{p} - \frac{y^2}{q} = 2r_i$$

$$\frac{x^2}{2pr_i} - \frac{y^2}{2qr_i} = 1_i$$

$$z = r_i$$

POVRŠI U PROSTORU

Površi drugog reda - hiperbolički paraboloid



POVRŠI U PROSTORU

Površi drugog reda - hiperbolički paraboloid