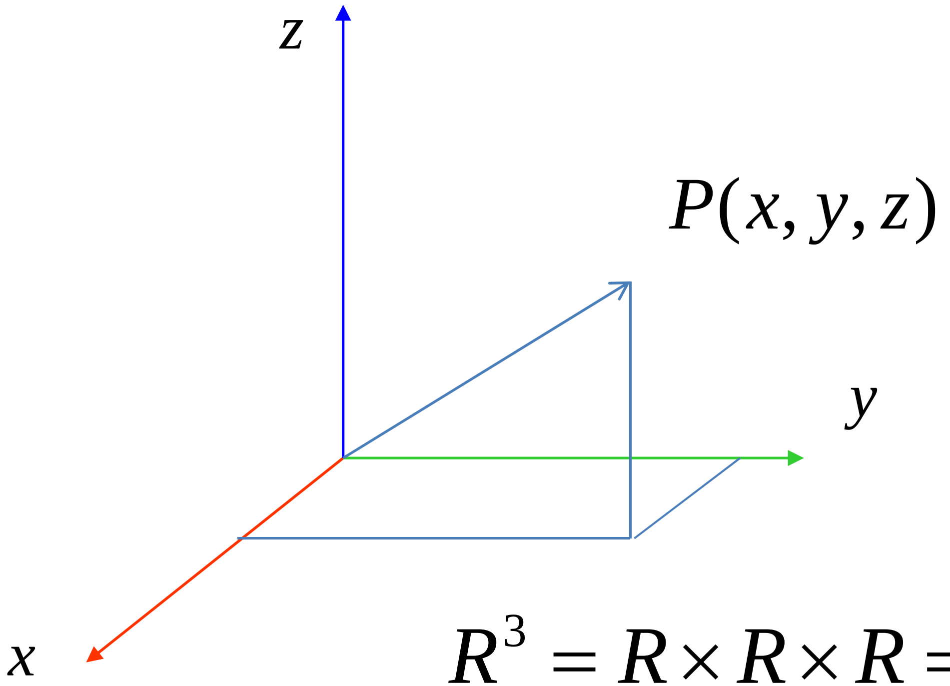
KOORDINATNI SISTEMI U PROSTORU

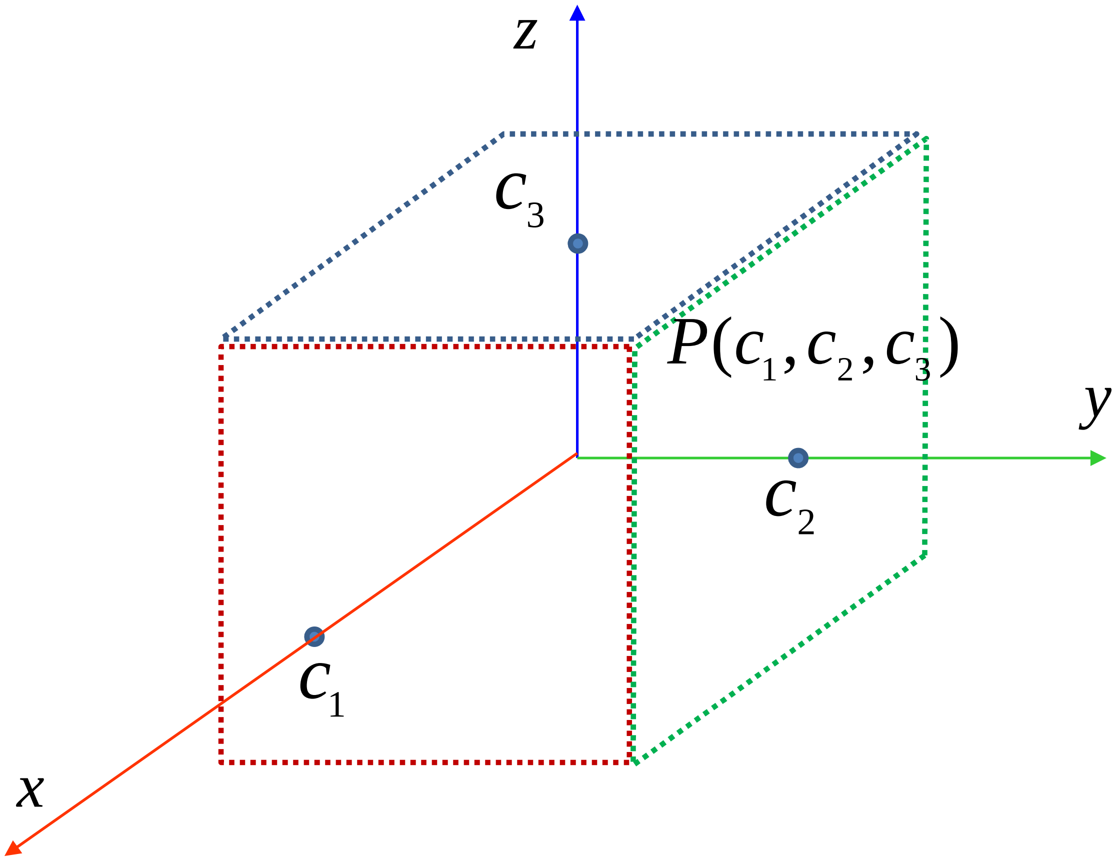
Dekartov pravougli koordinatni sistem



$$R^3 = R \times R \times R = \{(x, y, z) \mid \; x, y, z \in R\}$$

KOORDINATNI SISTEMI U PROSTORU

Dekartov pravougli koordinatni sistem



$P(x, y, z)$

$x = const$ $\quad x = c_1$

ravan

$y = const$ $\quad y = c_2$

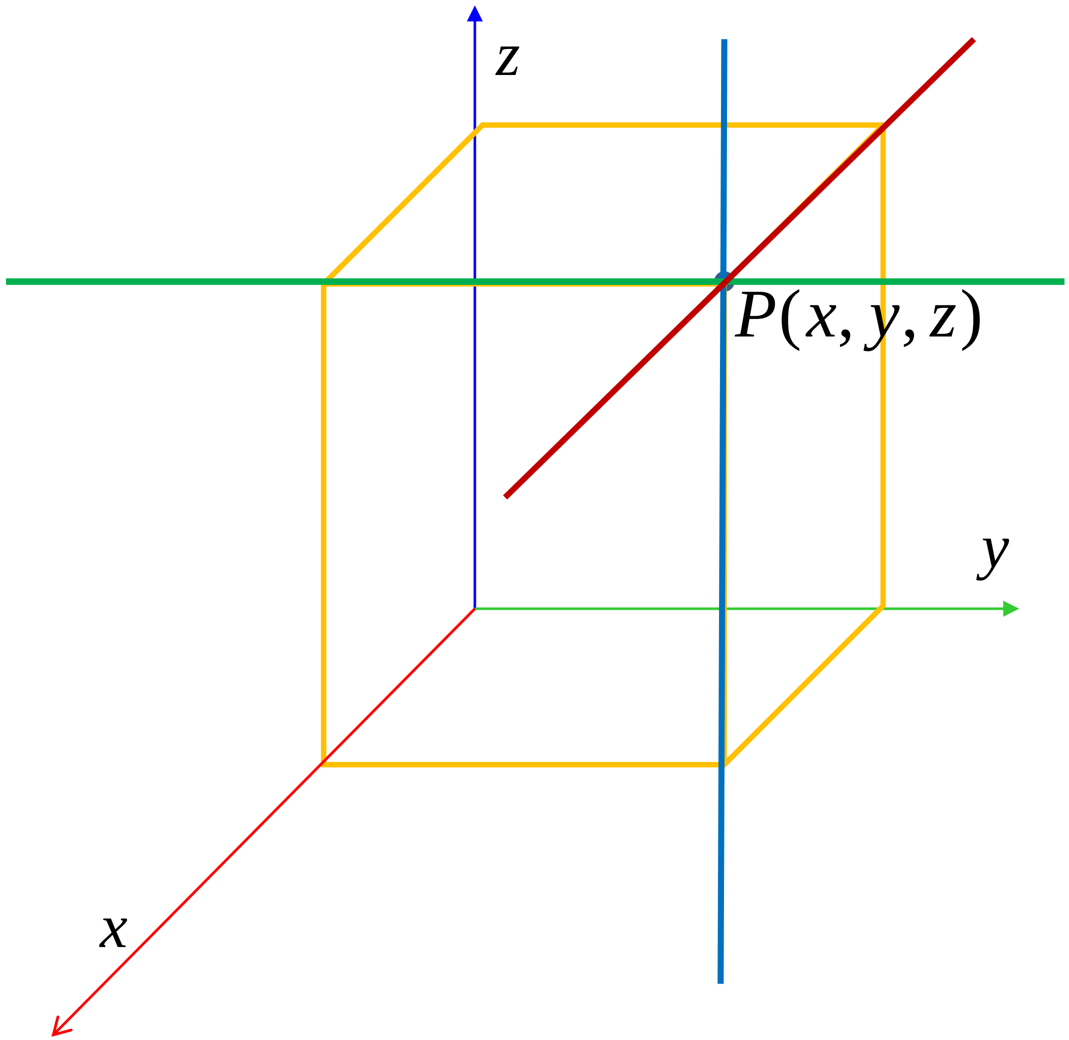
ravan

$z = const$ $\quad z = c_3$

ravan

KOORDINATNI SISTEMI U PROSTORU

Dekartov pravougli koordinatni sistem



$$P(x, y, z)$$

$$x = c_1$$
$$y = c_2$$

prava

$$x = c_1$$
$$z = c_3$$

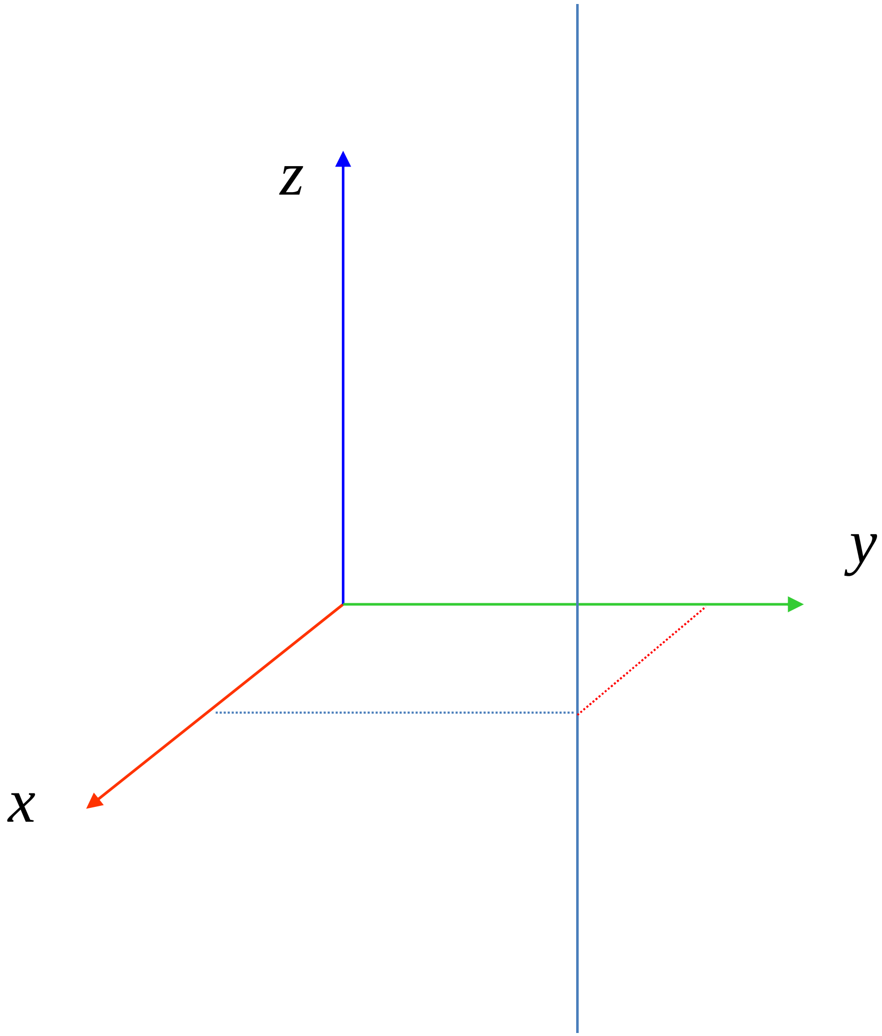
prava

$$y = c_2$$
$$z = c_3$$

prava

KOORDINATNI SISTEMI U PROSTORU

Dekartov pravougli koordinatni sistem



$$P(x, y, z)$$

$$x = c_1 \qquad y = c_2$$

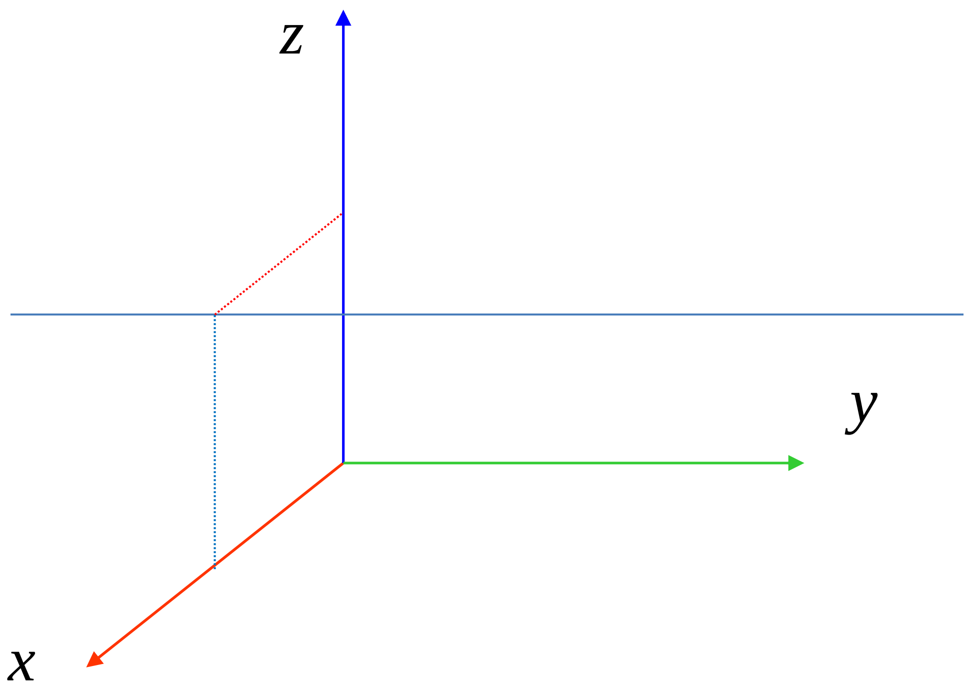
Prava paralelna sa *z*-osom

$$P(c_1, c_2, 0)$$

U Oxy ravni

KOORDINATNI SISTEMI U PROSTORU

Dekartov pravougli koordinatni sistem



$$P(x, y, z)$$

$$x = c_1 \qquad z = c_3$$

Prava paralelna sa *z*-osom

$$P(c_1, 0, c_3)$$

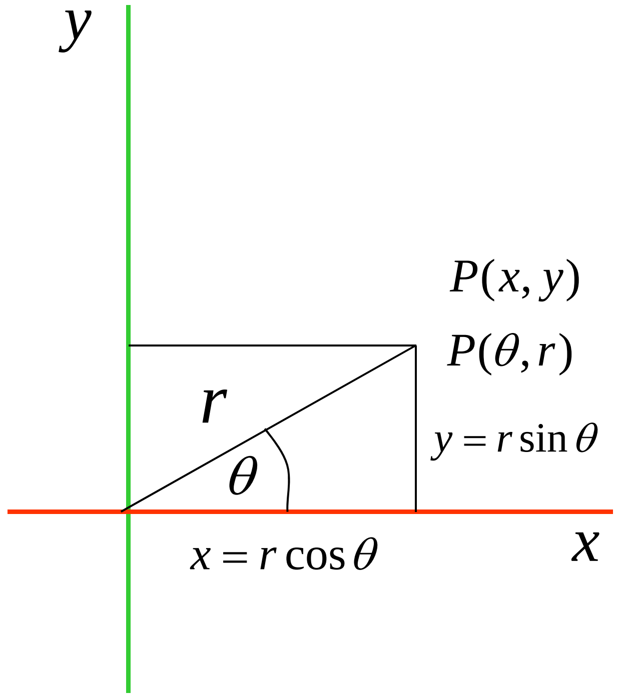
U Oxy ravni

KOORDINATNI SISTEMI U PROSTORU

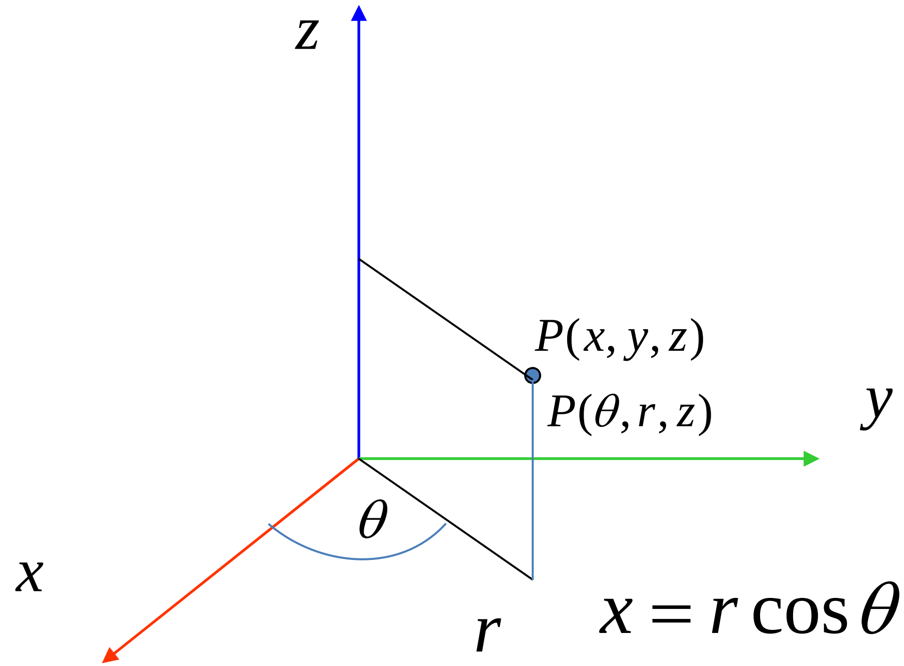
Cilindrični koordinatni sistem

Polarni koordinatni sistem u ravni

U ravni Oxz polarni koordinatni sistem



Cilindrični koordinatni sistem



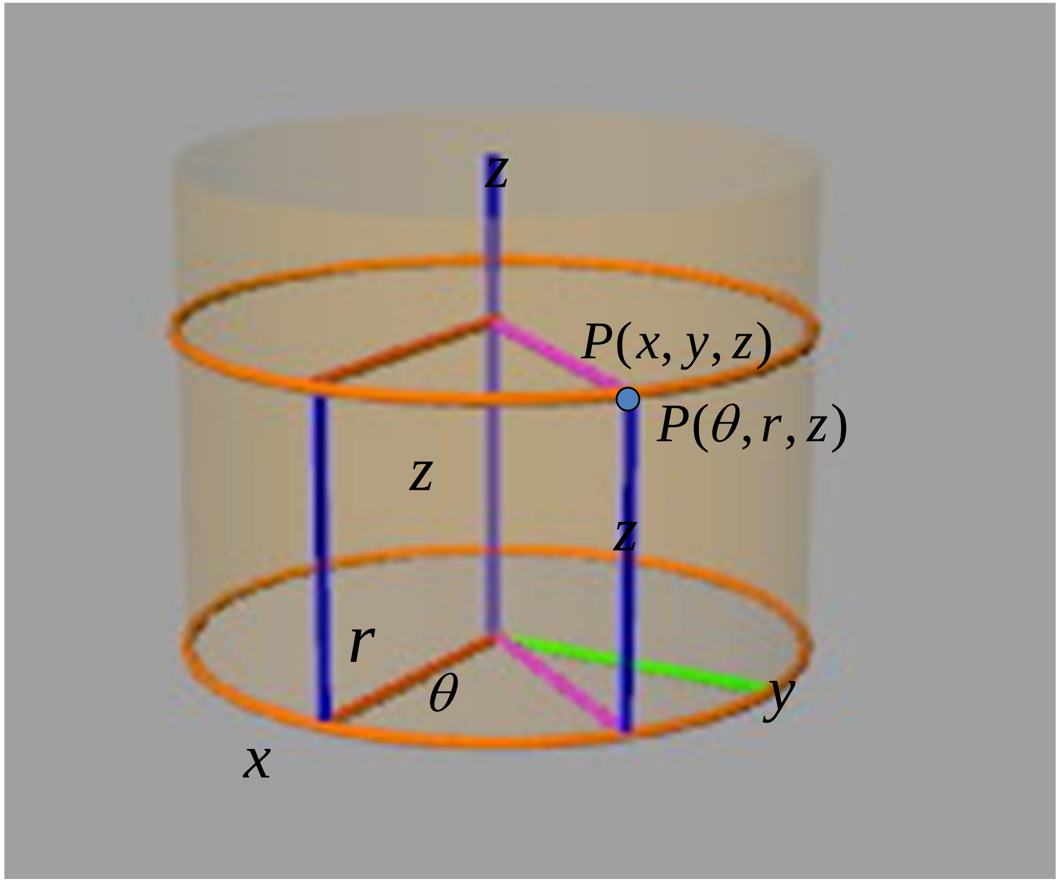
$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$z = z$$

KOORDINATNI SISTEMI U PROSTORU

Cilindrični koordinatni sistem



$$P(x, y, z)$$

$$P(\theta, r, z)$$

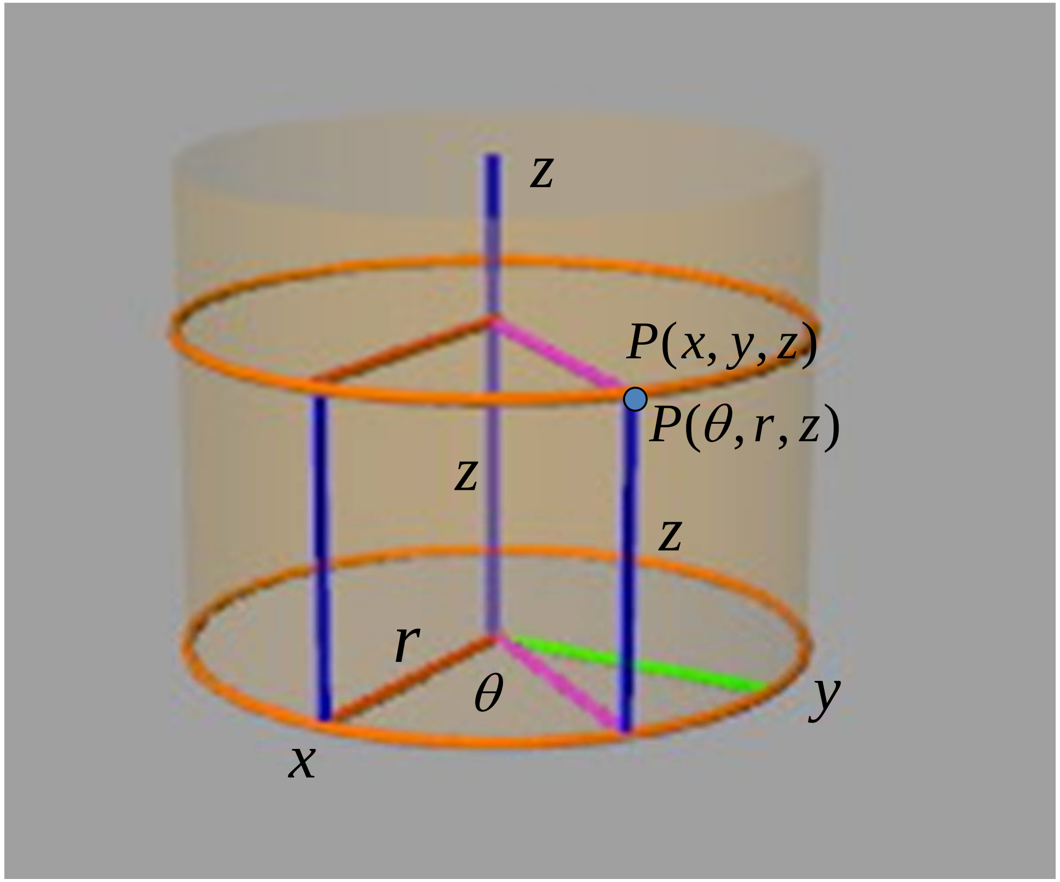
$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$z = z$$

KOORDINATNI SISTEMI U PROSTORU

Cilindrični koordinatni sistem



$$P(x, y, z)$$

$$P(\theta, r, z)$$

$$\theta = const$$

poluravan

$$r = const$$

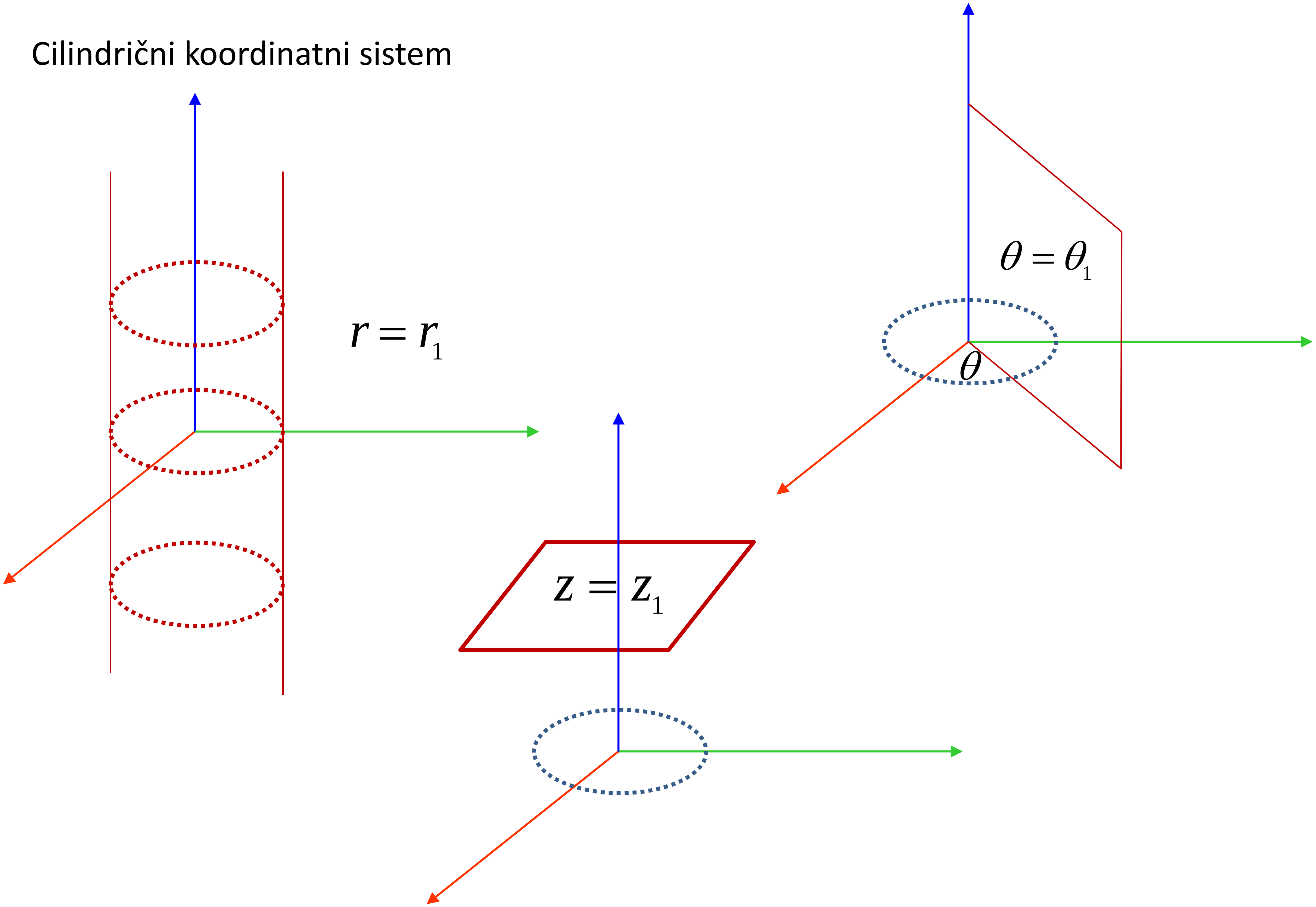
cilindar

$$z = const$$

ravan

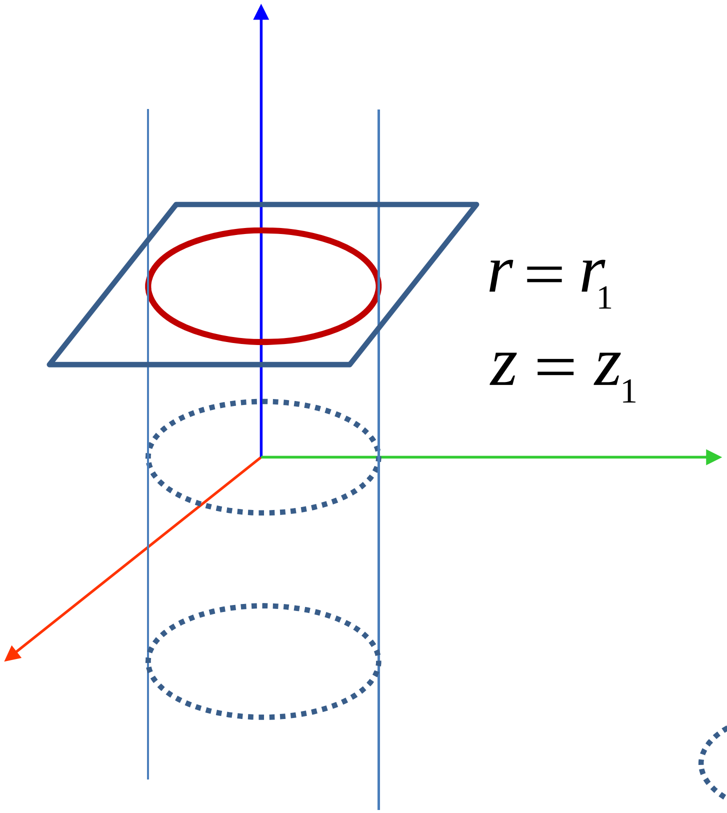
KOORDINATNI SISTEMI U PROSTORU

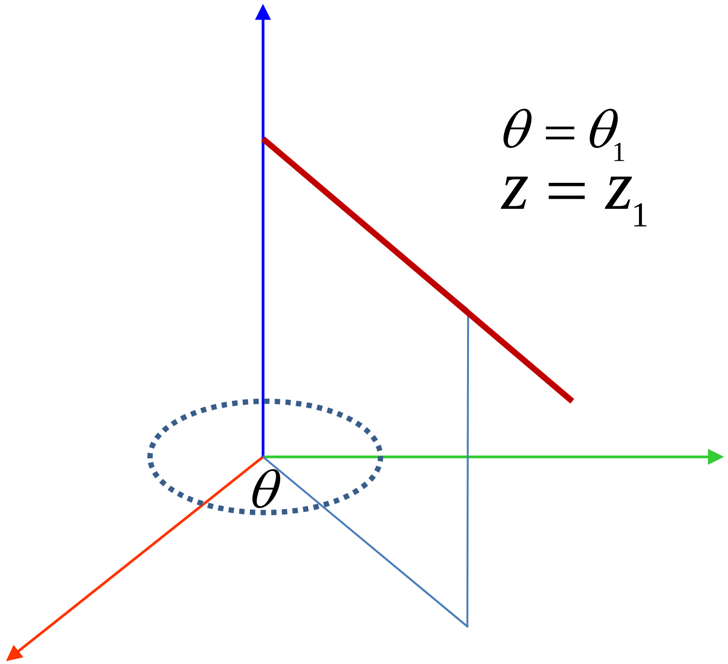
Cilindrični koordinatni sistem

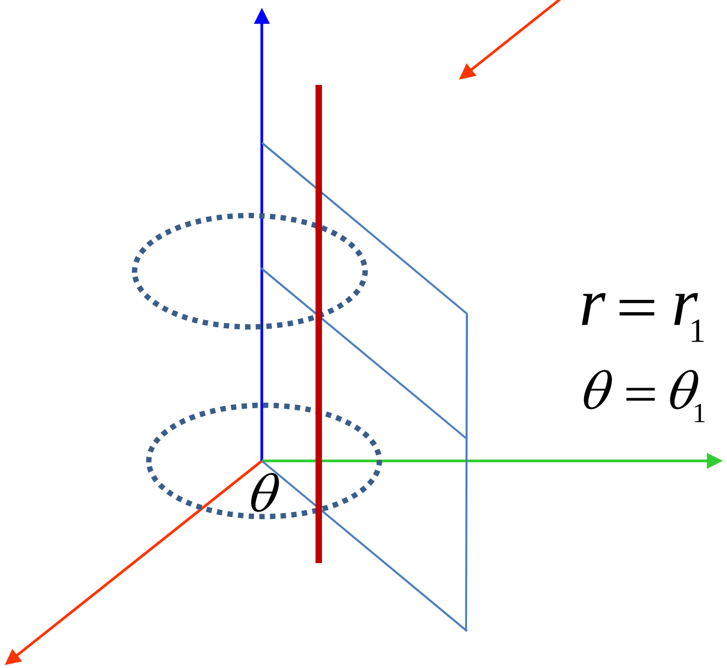


KOORDINATNI SISTEMI U PROSTORU

Cilindrični koordinatni sistem

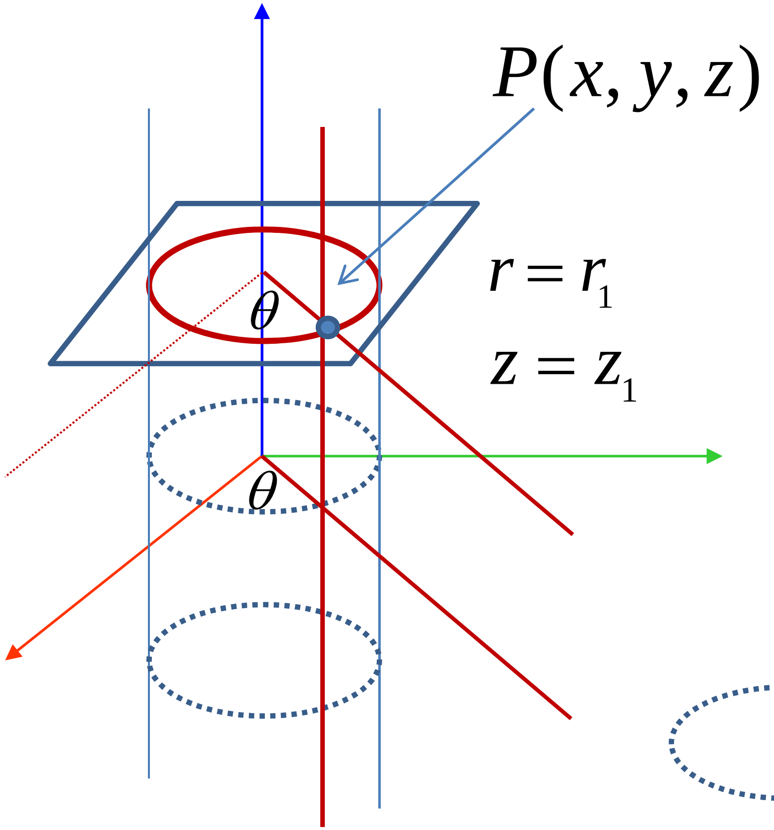


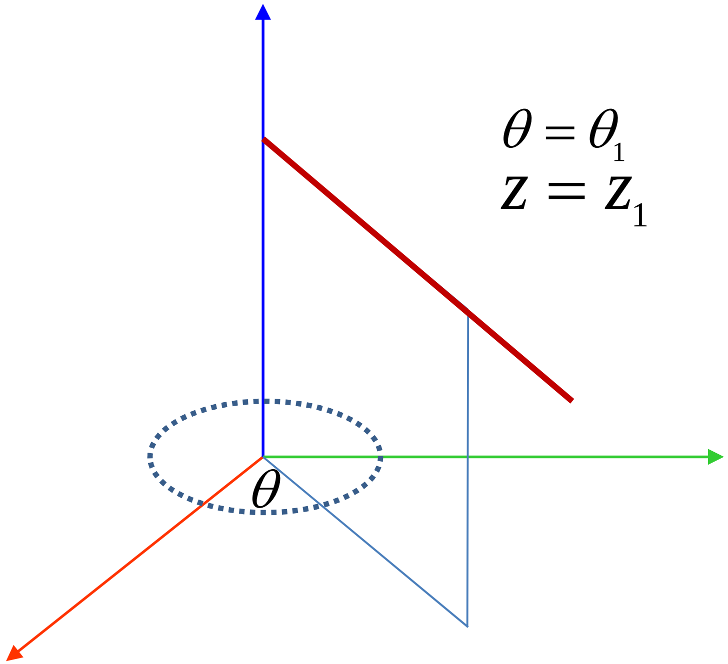


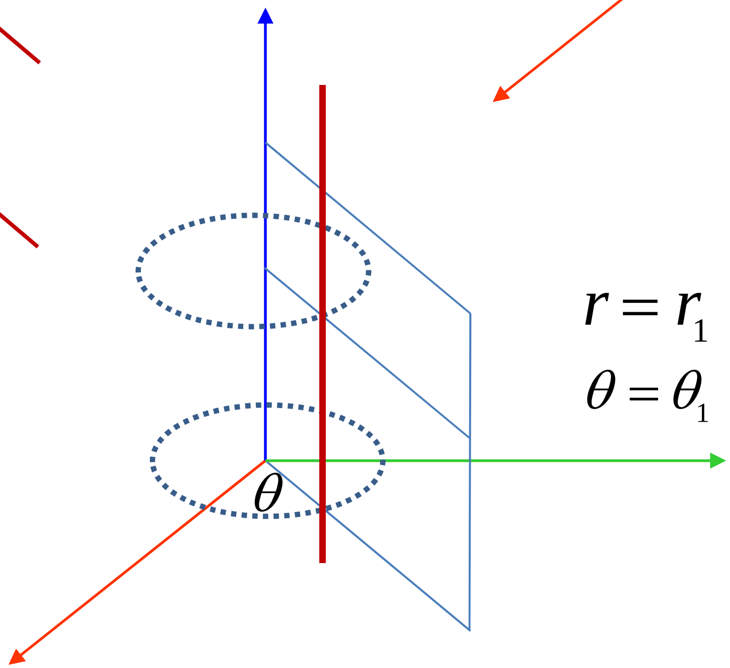


KOORDINATNI SISTEMI U PROSTORU

Cilindrični koordinatni sistem

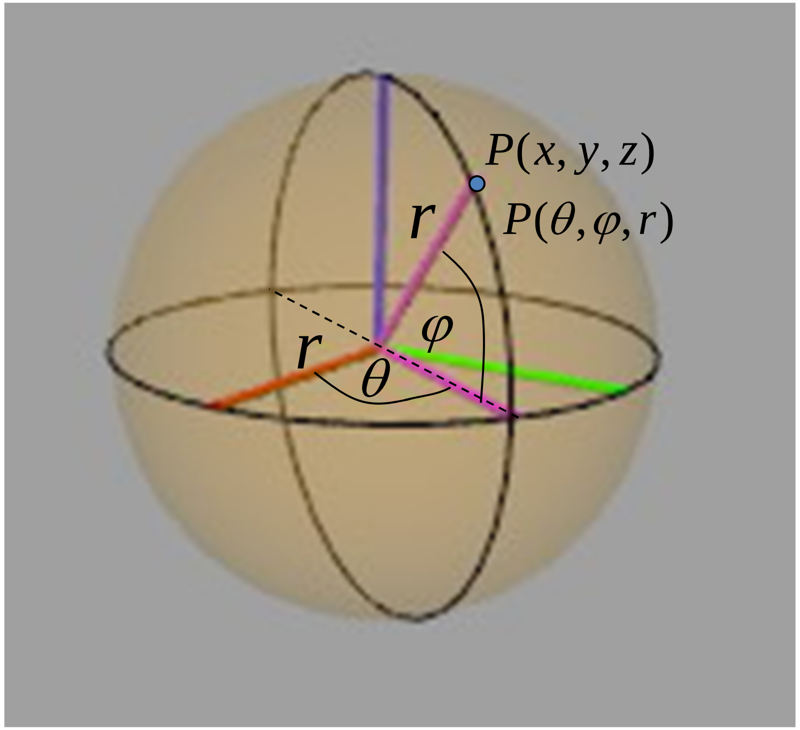






KOORDINATNI SISTEMI U PROSTORU

Sferni koordinatni sistem



$$P(x,y,z)$$

$$P\,(\theta\,,\varphi\,,r\,)$$

Odnos Dekartovih i sfernih koordinata

$$x = r\cos\varphi\cos\theta$$

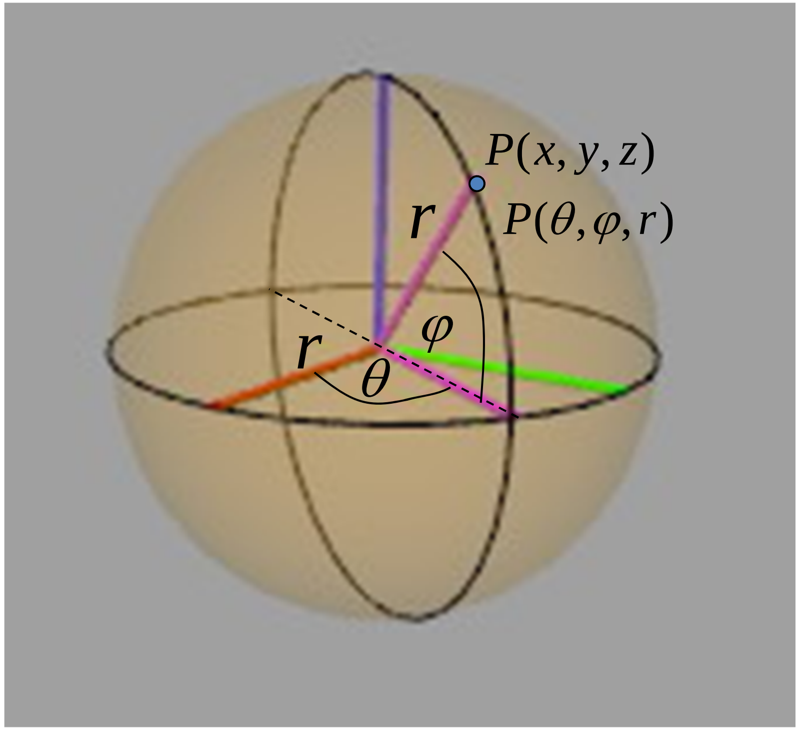
$$y = r\cos\varphi\sin\theta$$

$$z = r\sin\varphi$$

$$0 \le \theta \le 2\pi \qquad -\frac{\pi}{2} \le \varphi \le \frac{\pi}{2}$$

KOORDINATNI SISTEMI U PROSTORU

Sferni koordinatni sistem



$$0 \le \theta \le 2\pi \qquad -\frac{\pi}{2} \le \varphi \le \frac{\pi}{2}$$

$$P(x,y,z)$$

$$P(\theta,\varphi,r)$$

$$\theta = const$$

poluravan

$$\varphi = const$$

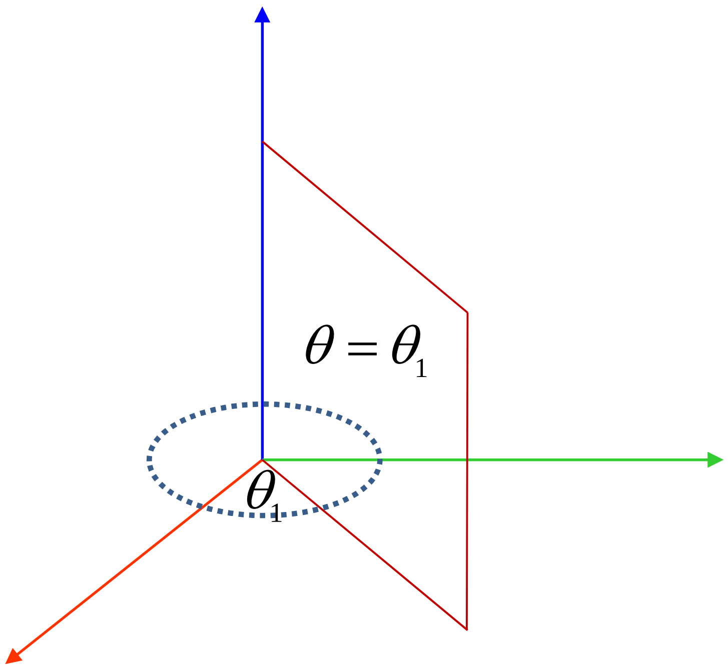
konus

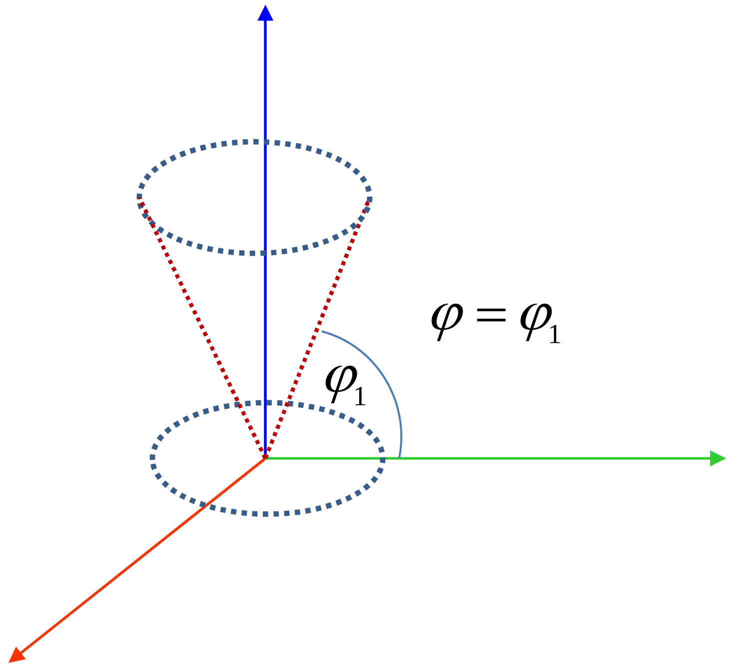
$$r = const$$

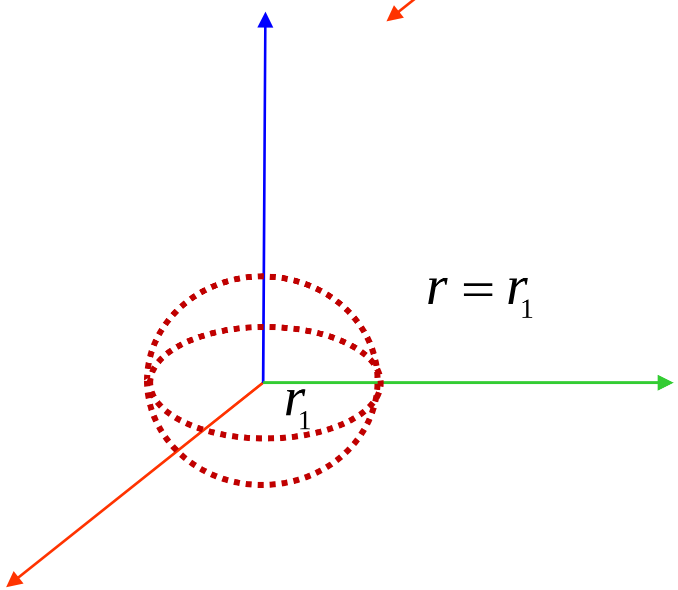
sfera

KOORDINATNI SISTEMI U PROSTORU

Sferni koordinatni sistem

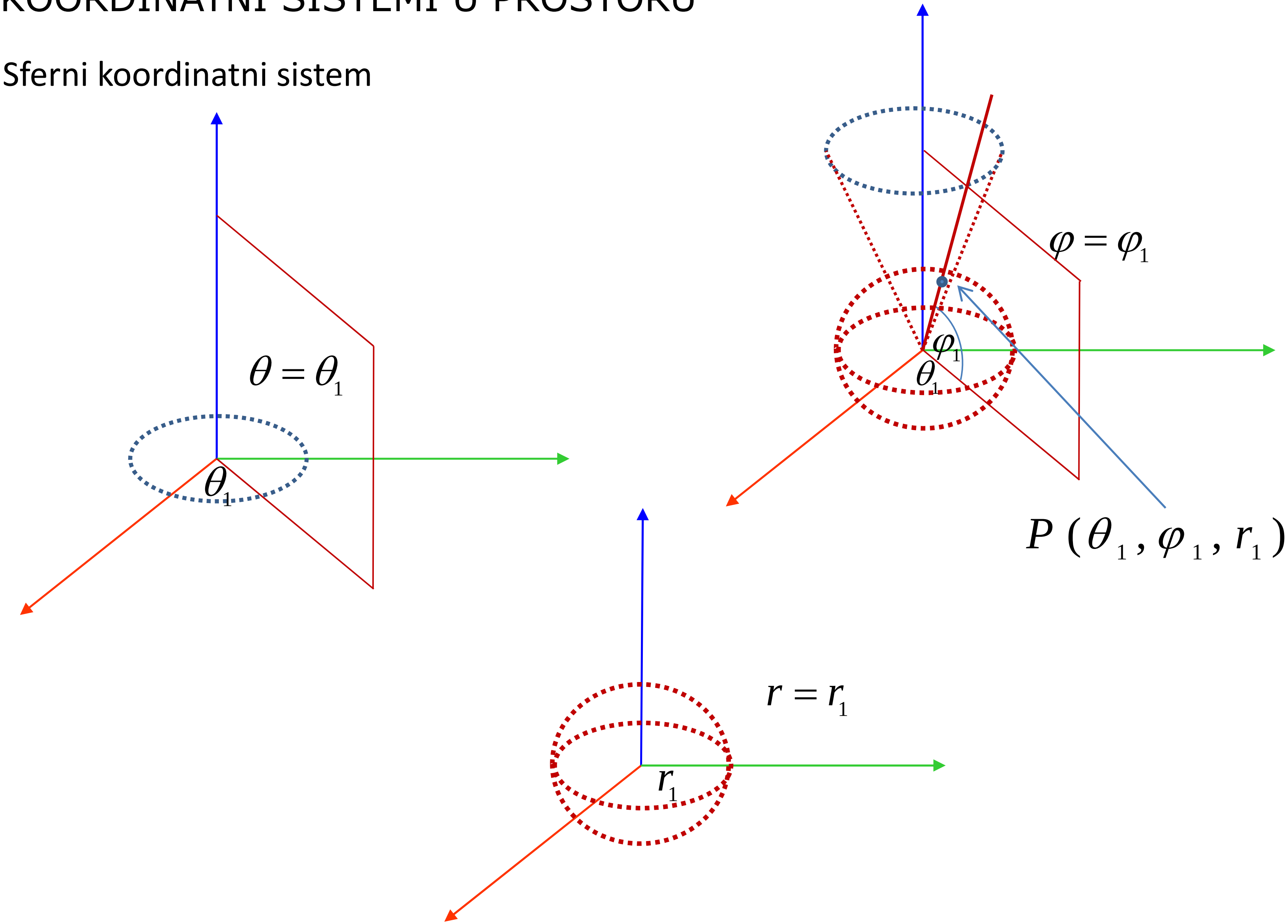






KOORDINATNI SISTEMI U PROSTORU

Sferni koordinatni sistem



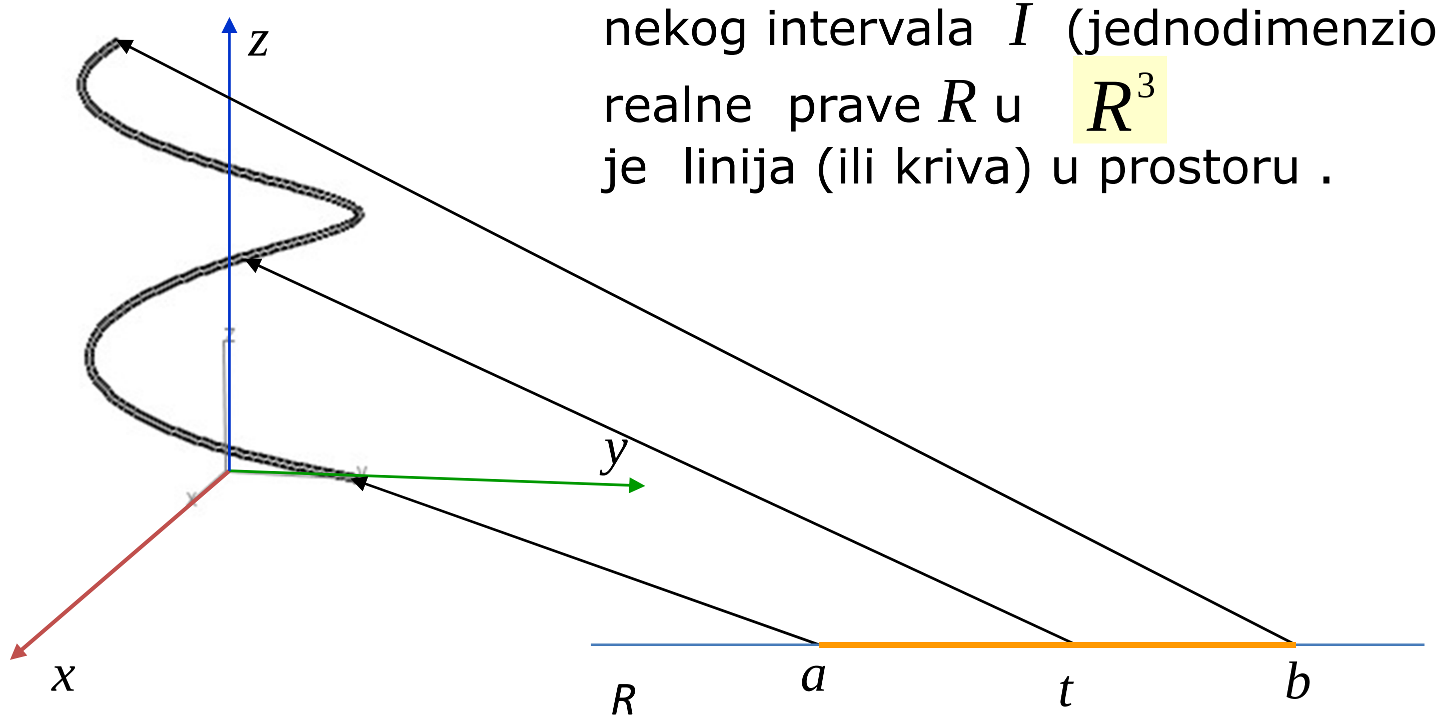
LINIJE (KRIVE) U PROSTORU

$$I = [a, b] \subset R$$

Preslikavanje

$$\alpha : I \rightarrow R^2, \quad t \in I$$

nekog intervala I (jednodimenzionalne) realne prave R u R^3 je linija (ili kriva) u prostoru .



R – parametarska prava

LINIJE (KRIVE) U PROSTORU

$$\alpha : I \to R^2$$

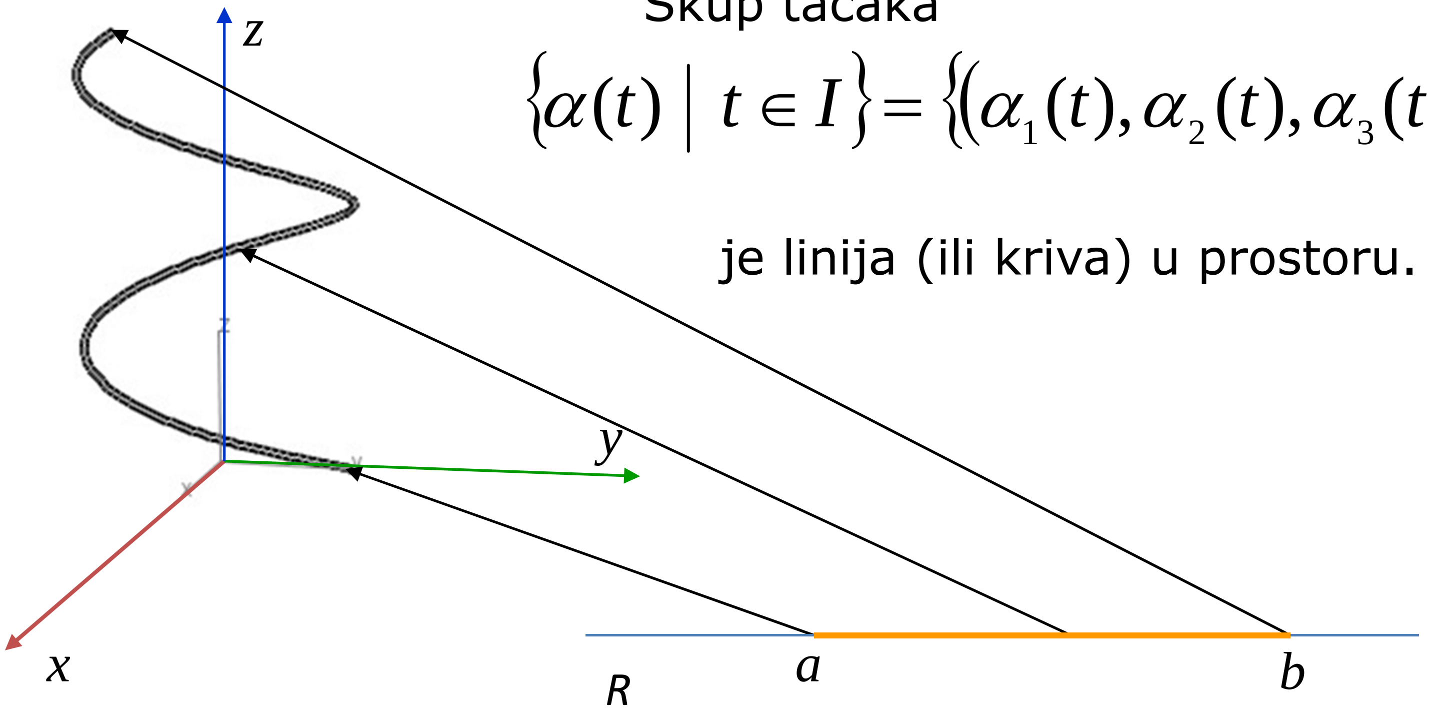
$$I = [a, b] \subset R$$

$$t \in R \qquad \alpha(t) = \left(\alpha_1(t), \alpha_2(t), \alpha_3(t)\right)$$

Skup tačaka

$$\{\alpha(t) \mid t \in I\} = \{\left(\alpha_1(t), \alpha_2(t), \alpha_3(t)\right) \mid t \in I\}$$

je linija (ili kriva) u prostoru.



R – parametarska prava

LINIJE (KRIVE) U PROSTORU

$$\alpha : I \to R^2$$

$$\alpha(t) = \left(\alpha_1(t), \alpha_2(t), \alpha_3(t)\right)$$

$$a \le t \le b$$

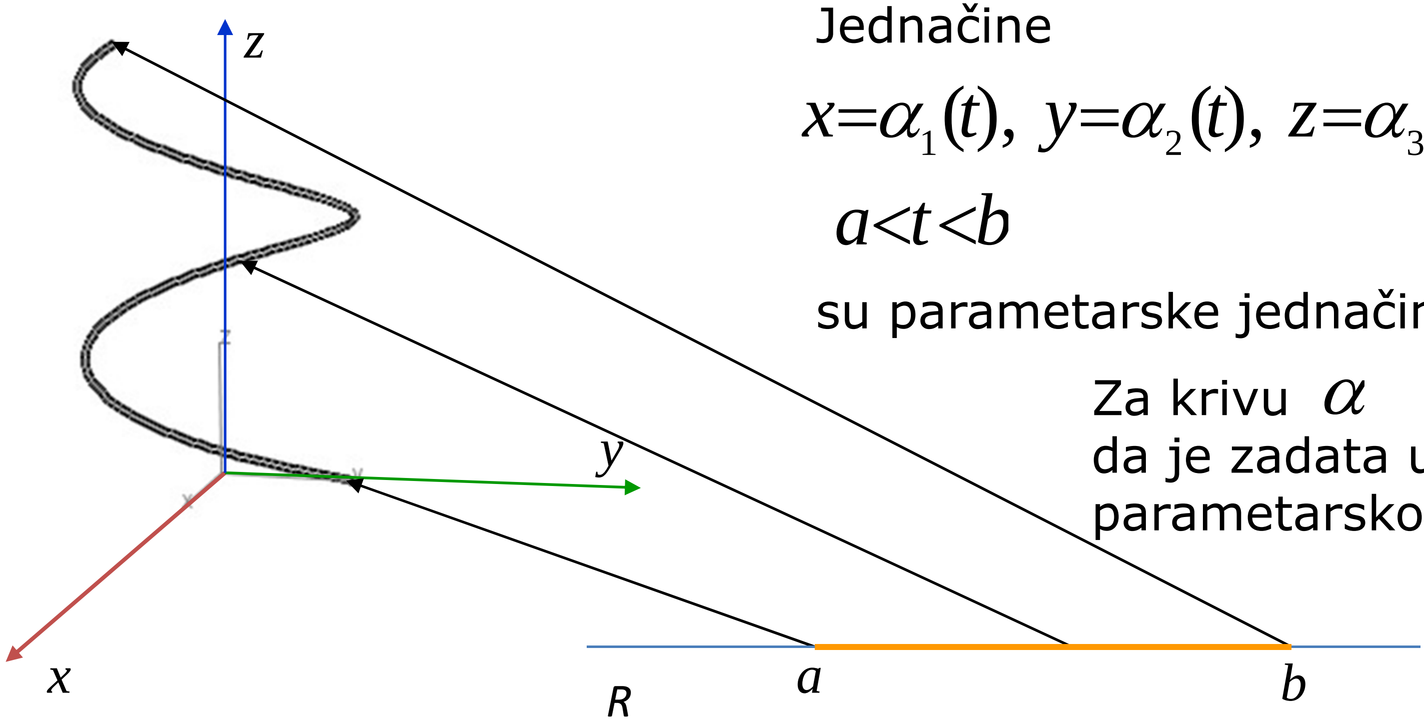
Jednačine

$$x=\alpha_1(t),\ y=\alpha_2(t),\ z=\alpha_3(t)$$

$$a<t<b$$

su parametarske jednačine te krive.

Za krivu α se kaže da je zadata u parametarskom obliku.



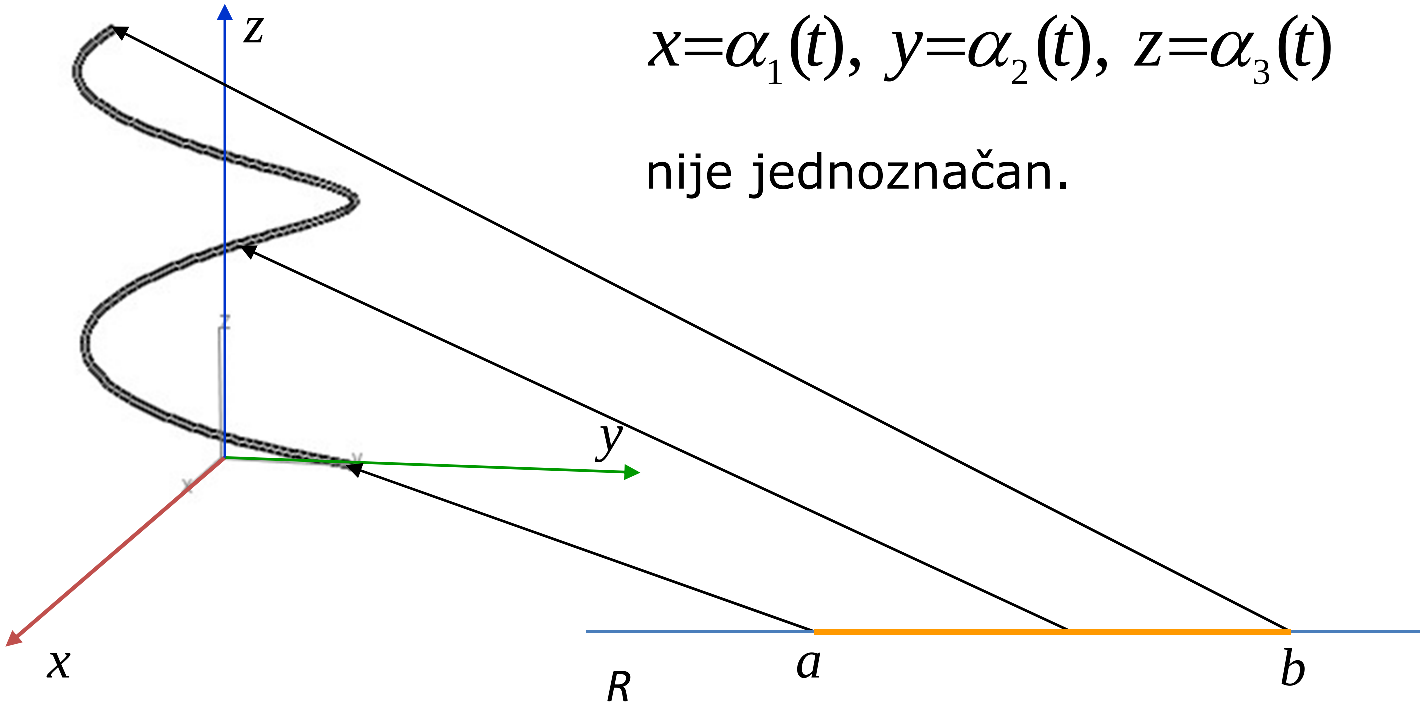
R – parametarska prava

LINIJE (KRIVE) U PROSTORU

Matematički prikaz krive linije C
u prostoru u parametarskom
obliku

$$x=\alpha_1(t),\ y=\alpha_2(t),\ z=\alpha_3(t) \qquad a<t<b$$

nije jednoznačan.



LINIJE (KRIVE) U PROSTORU

Ukoliko je

$$\varphi : J \to I$$

Diferencijabilna rastuća funkcija koja vrši preslikavanje nekog intervala $J = [c,d]$ neke parametarske prave τ na interval $I = [a,b]$ tada su

$$\varphi'(\tau) > 0$$

$$x=\alpha_1(\varphi(\tau)),\ y=\alpha_2(\varphi(\tau)),\ z=\alpha_3(\varphi(\tau)) \qquad c<\tau<d$$

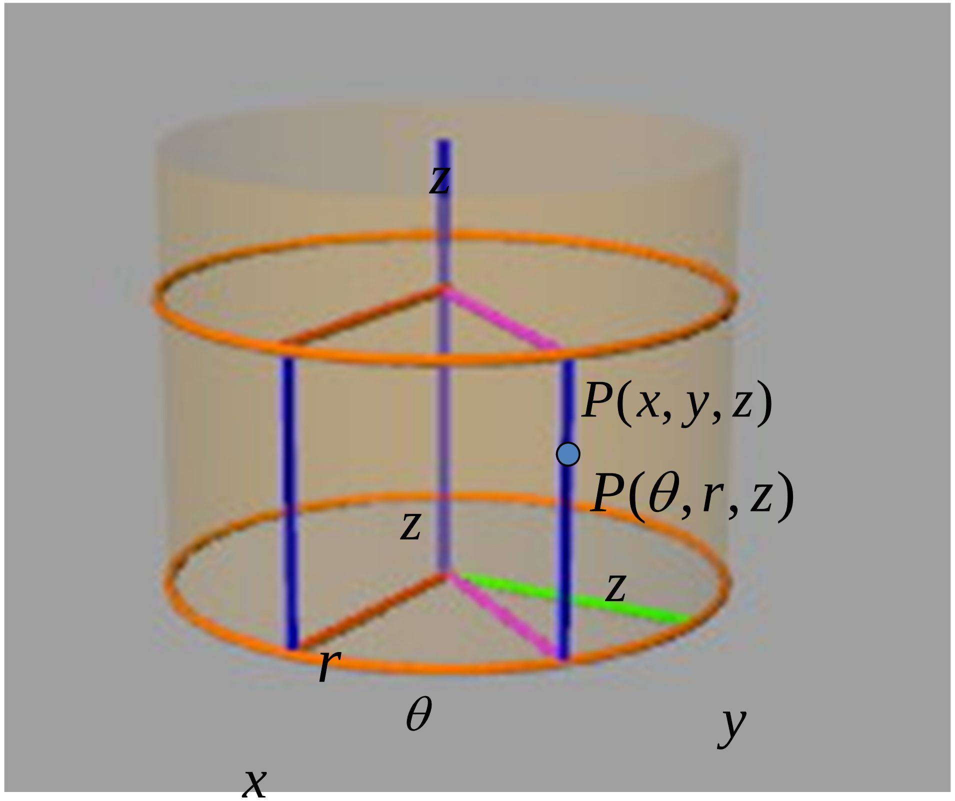
takodje parametarske jednačine iste krive C .

Kaže se da je izvršena reparametrizacija i nove jednačine su:

$$x=\beta_1(\tau),\ y=\beta_2(\tau),\ z=\beta_3(\tau) \qquad c<\tau<d$$

LINIJE (KRIVE) U PROSTORU

Cilindrični koordinatni sistem

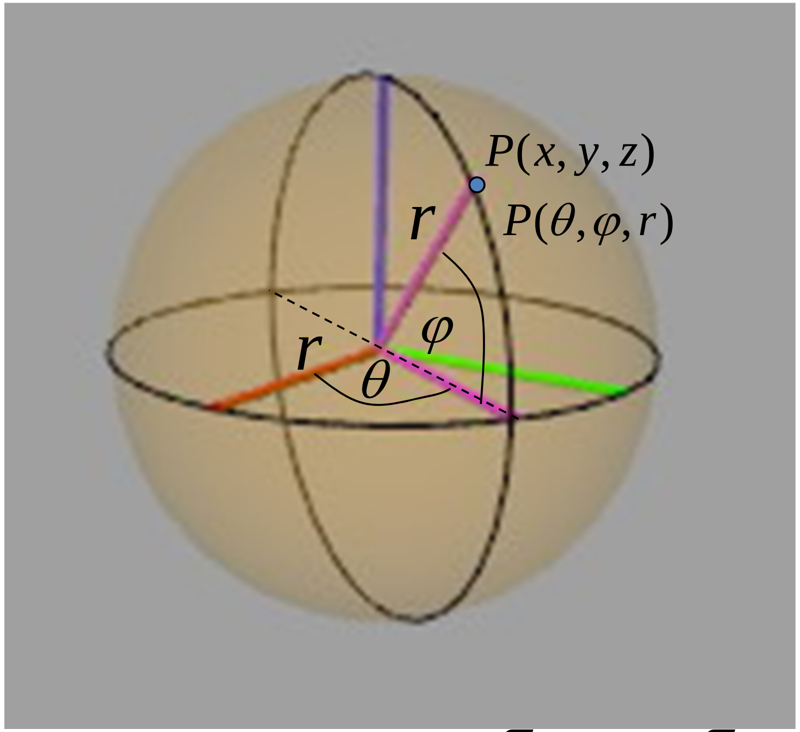


Parametarski zadata kriva linija u prostoru u cilindričnom koordinatnom sistemu:

$$\theta = \varphi_1(t)$$
$$r = \varphi_2(t) \qquad t_1 \le t \le t_2$$
$$z = \varphi_3(t)$$

LINIJE (KRIVE) U PROSTORU

Sferni koordinatni sistem



$$0 \le \theta \le 2\pi \qquad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$$

Parametarski zadata kriva linija u prostoru u sfernom koordinatnom sistemu:

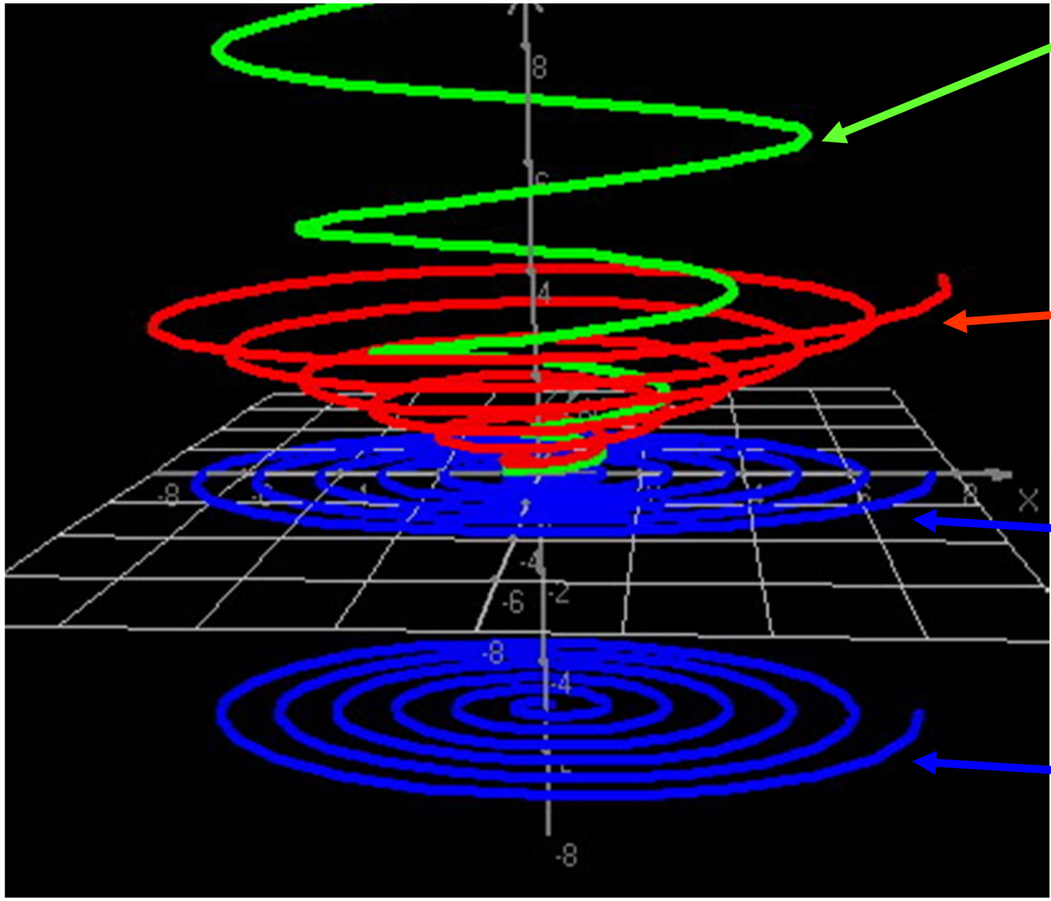
$$\theta = \psi_1(t)$$

$$\varphi = \psi_2(t) \qquad t_1 \le t \le t_2$$

$$r = \psi_3(t)$$

LINIJE (KRIVE) U PROSTORU

Primeri



4
$$x = 0.2t\cos t \quad y = 0.1t\sin t \quad z = 0.01t^2 \qquad 0 \le t \le 12\pi$$

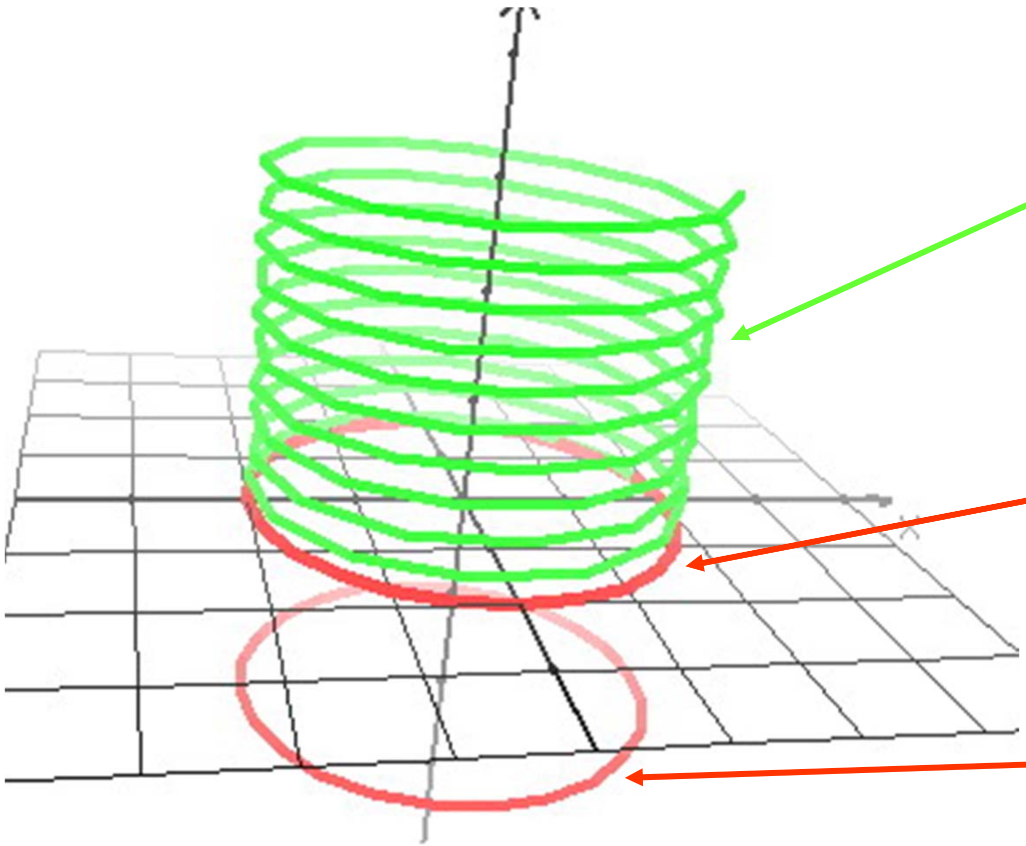
3
$$x = 0.2t\cos t \quad y = 0.1t\sin t \quad z = 0.1t \qquad 0 \le t \le 12\pi$$

1
$$x = 0.2t\cos t \quad y = 0.1t\sin t \quad z = 0 \qquad 0 \le t \le 12\pi$$

2
$$x = 0.2t\cos t \quad y = 0.1t\sin t \quad z = -5 \qquad 0 \le t \le 12\pi$$

LINIJE (KRIVE) U PROSTORU

Primeri



3
$$x = 4\cos t, \quad y = 4\sin t, \quad z = 0.1t \qquad 0 \le t \le 20\pi$$

1
$$x = 4\cos t, \quad y = 4\sin t, \quad z = 0 \qquad 0 \le t \le 20\pi$$

2
$$x = 4\cos t, \quad y = 4\sin t, \quad z = -4 \qquad 0 \le t \le 20\pi$$

LINIJE (KRIVE) U PROSTORU

$$X(t) = \alpha_1(t)$$
$$Y(t) = \alpha_2(t)$$
$$Z(t) = \alpha_3(t)$$

$$\alpha(t) = \left(\alpha_1(t), \alpha_2(t), \alpha_3(t)\right), \qquad t \in I$$ Kriva u prostoru

$$\alpha_{xy}(t) = \left(\alpha_1(t), \alpha_2(t), 0\right), \qquad t \in I$$ Projekcija krive na Oxy -ravan

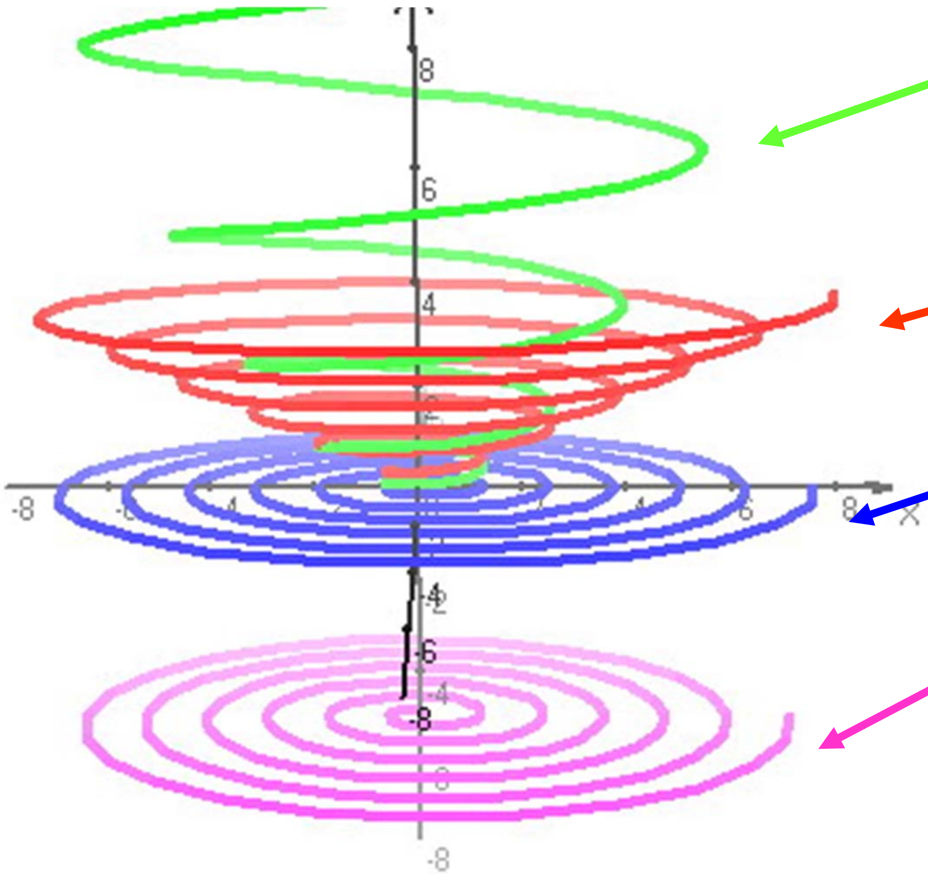
$$\alpha_{xz}(t) = \left(\alpha_1(t), 0, \alpha_3(t)\right), \qquad t \in I$$ Projekcija krive na Oxz -ravan

$$\alpha_{yz}(t) = \left(0, \alpha_2(t), \alpha_3(t)\right), \qquad t \in I$$ Projekcija krive na Oyz -ravan

LINIJE (KRIVE) U PROSTORU

Primeri

$$x = 0.2t\cos t \qquad y = 0.1t\sin t \qquad 0 \le t \le 12\pi$$



4 $z = 0.01t^2$

3 $z = 0.1t$

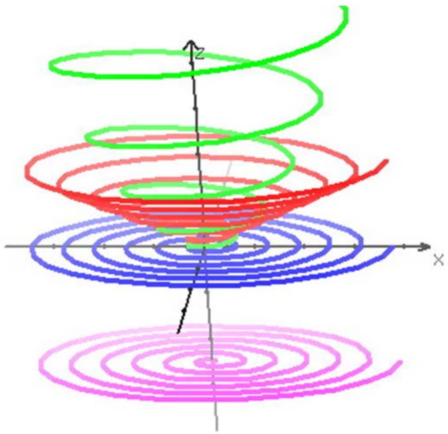
1 $z = 0$

Projekcija krivih 2,3 i 4 na Oxy - ravan

2 $z = -5$

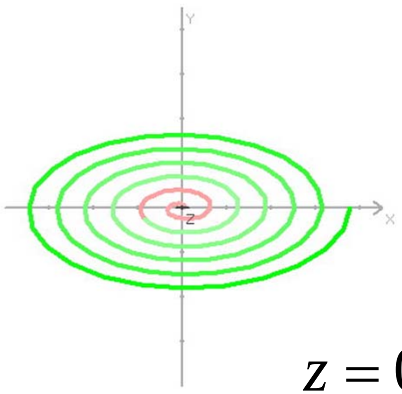
LINIJE (KRIVE) U PROSTORU

Primeri

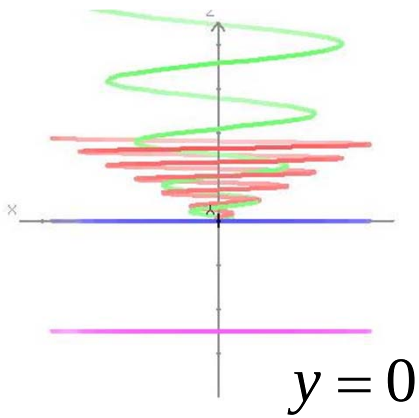


Projekcije na koordinatne ravni

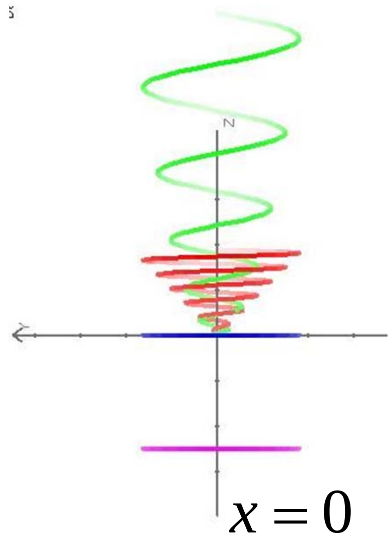
Pogledi: odozgo, spreda, sa bočne strane



$z = 0$



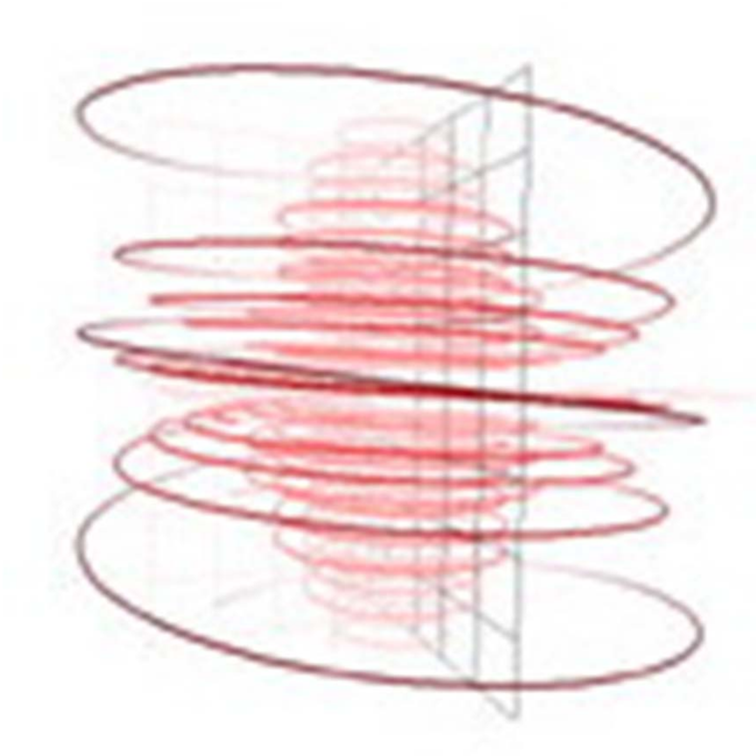
$y = 0$



$x = 0$

LINIJE (KRIVE) U PROSTORU

Primeri



$$x = (1-10)\sin t$$

$$y = (1-10)\cos t$$

$$z = t/(10-1)$$

$$-10 \le t \le 10 \qquad \text{Step } 0.1$$

$$x = a\sin t$$

$$y = a\cos t$$

$$z = t/(10-a)$$

$$a = 1,2,\ldots,10$$

$$-10 \le t \le 10$$

Primer krivih u prostoru

Projekcija na Oxy - ravan su koncentrični krugovi:

$$x = (1-10)\sin t$$

$$y = (1-10)\cos t$$

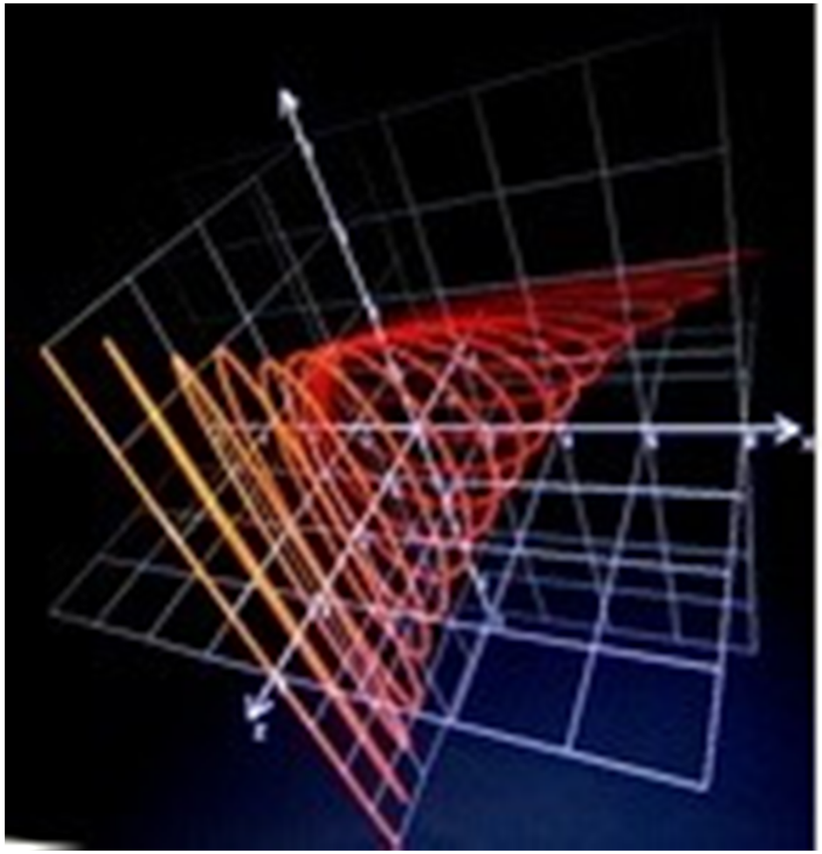
$$z = 0$$

Darko Kadvanj

LINIJE (KRIVE) U PROSTORU

$$x = a\cos t$$
$$y = b\sin t$$
$$z = c$$

Primeri



Ivana Kula

$$x = (8-0)\cos t$$
$$y = (0-8)\sin t$$
$$z = (-8,\ldots,8)$$

$0 \le t \le 2\pi$ Step 0.3

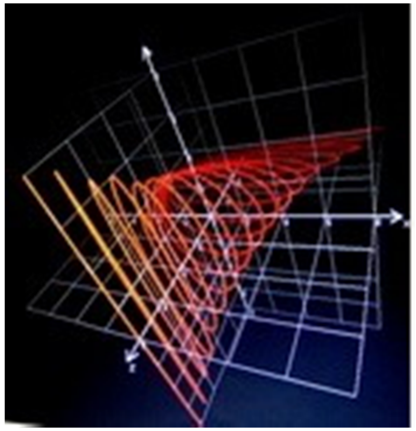
17 elipsi u ravnima paralelnim sa Oxy ravni

Projekcija na Oxy - ravan su elipse:

$$x = (8-0)\cos t$$
$$y = (0-8)\sin t$$
$$z = 0$$

LINIJE (KRIVE) U PROSTORU

Primeri



$$x = (8-0)\cos t$$
$$y = (0-8)\sin t$$
$$z = (-8,\dots,8)$$

$0 \le t \le 2\pi$ Step 0.3

17 elipsi u ravnima paralelnim sa Oxy ravni

$$x = a\cos t \qquad a = 8,7,\dots,-1,0,1,\dots,7,8$$
$$y = b\sin t \qquad b = 0,1,\dots,7,8,7,\dots,1,0$$
$$z = c \qquad c = -8,-7,\dots,0,1,\dots,7,8$$

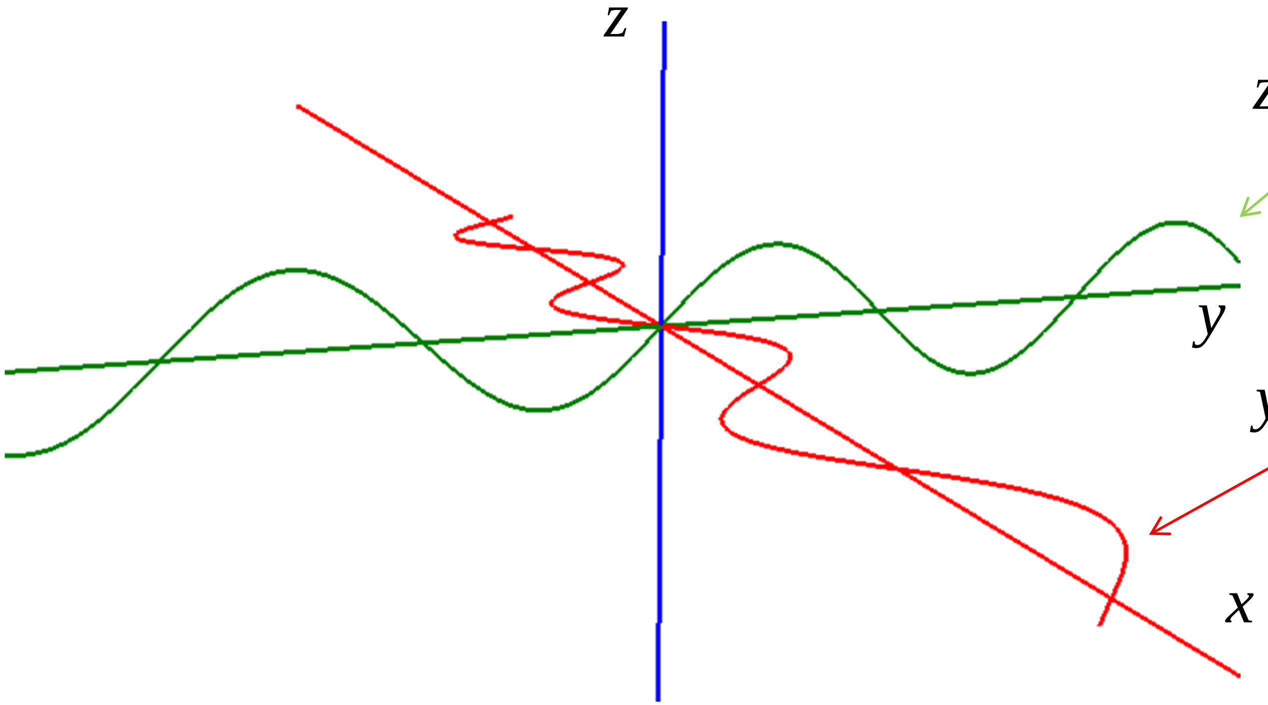
$$x = a\cos t$$
$$y = (8-a)\sin t \qquad a = 8,7,\dots,-1,0,1,\dots,7,8$$
$$z = c \qquad c = -8,-7,\dots,0,1,\dots,7,8$$

Ivana Kula

LINIJE (KRIVE) U PROSTORU

Ravne krive

Kriva u prostoru koja leži u nekoj ravni naziva se ravnom krivom.

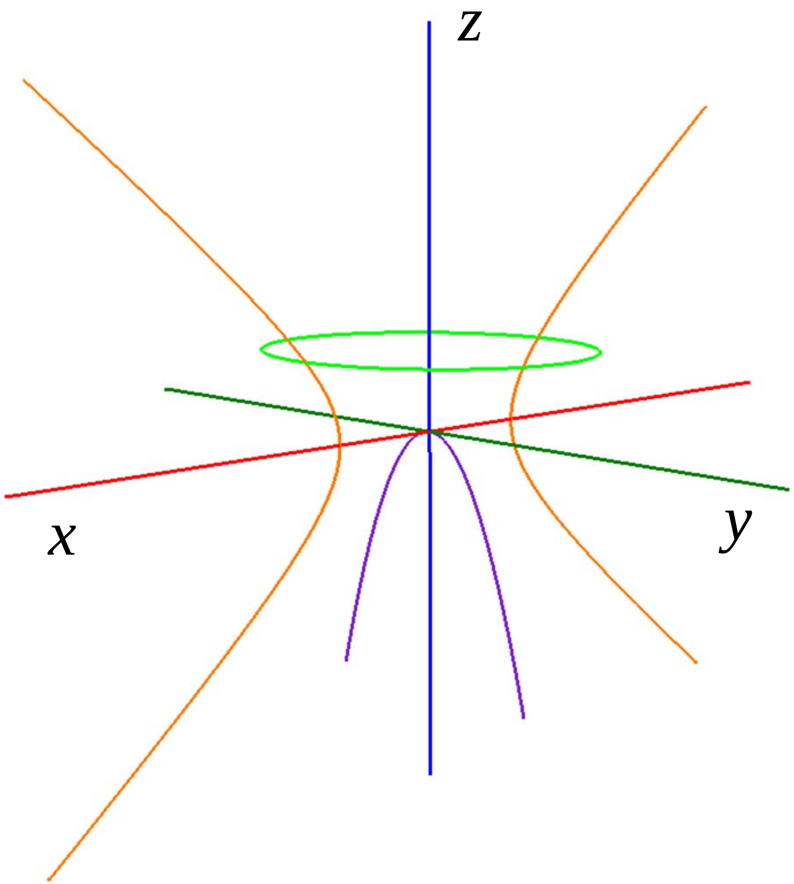


$$z = \sin y, \quad x = 0$$

$$y = \sin x, \quad z = 0$$

LINIJE (KRIVE) U PROSTORU

Ravne krive



elipsa

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, \quad z=c=const$$

---

parabola

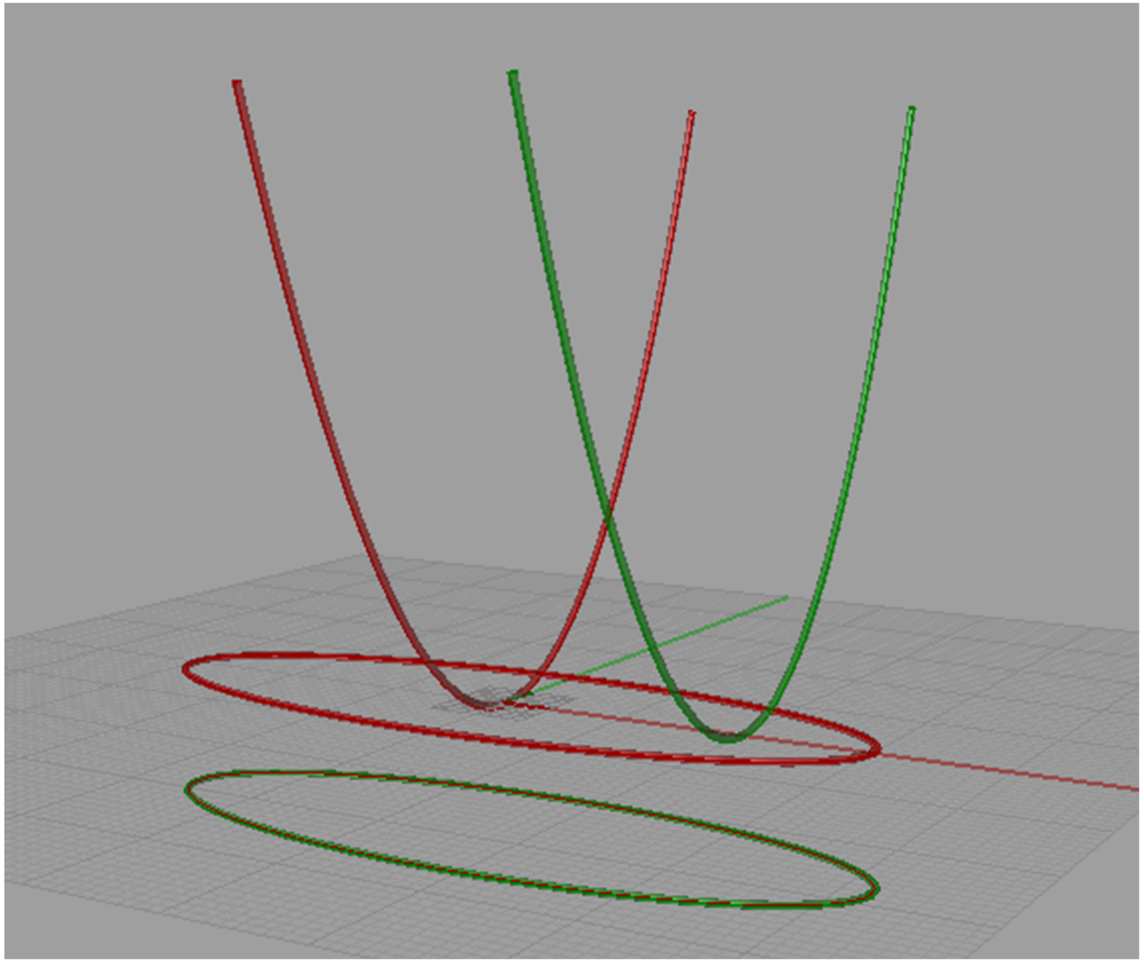
$$y^2=2pz, \quad x=0$$

---

hiperbola

$$\frac{x^2}{a^2}-\frac{z^2}{b^2}=1, \quad y=0$$

LINIJE (KRIVE) U PROSTORU



parabola

$$y^2 = 2pz, \quad x = 0$$

parabola

$$y^2 = 2pz, \quad x = c \neq 0$$

elipsa

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad z = c = 0$$

elipsa

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad z = c \neq 0$$

LINIJE (KRIVE) U PROSTORU

Tangenta, normala, binormala

$$I=[a,b]\subset R$$

Kriva u prostoru $\alpha : I \to R^3$

$$\alpha(t)=(\alpha_1(t),\alpha_2(t),\alpha_3(t)), \qquad t\in I$$

$$\alpha'(t)=(\alpha_1'(t),\alpha_2'(t),\alpha_3'(t)), \qquad t\in I$$

$$\alpha''(t)=(\alpha_1''(t),\alpha_2''(t),\alpha_3''(t)), \qquad t\in I$$

$$\alpha'''(t)=(\alpha_1'''(t),\alpha_2'''(t),\alpha_3'''(t)), \qquad t\in I$$

LINIJE (KRIVE) U PROSTORU

Tangenta, normala, binormala

$$I=[a,b]\subset R \qquad \alpha : I \to R^3$$

$$\alpha(t)=\left(\alpha_1(t),\alpha_2(t),\alpha_3(t)\right) \qquad |\alpha(t)|=\sqrt{\alpha_1^{\,2}(t)+\alpha_2^{\,2}(t)+\alpha_3^{\,2}(t)}$$

$$\alpha'(t)=\left(\alpha_1'(t),\alpha_2'(t),\alpha_3'(t)\right) \qquad |\alpha'(t)|=\sqrt{\alpha_1{'}^2(t)+\alpha_2{'}^2(t)+\alpha_3{'}^2(t)}$$

$$\alpha''(t)=\left(\alpha_1''(t),\alpha_2''(t),\alpha_3''(t)\right) \qquad |\alpha''(t)|=\sqrt{\alpha_1{''}^2(t)+\alpha_2{''}^2(t)+\alpha_3{''}^2(t)}$$

$$\alpha'''(t)=\left(\alpha_1'''(t),\alpha_2'''(t),\alpha_3'''(t)\right) \qquad |\alpha'''(t)|=\sqrt{\alpha_1{'''}^2(t)+\alpha_2{'''}^2(t)+\alpha_3{'''}^2(t)}$$

LINIJE (KRIVE) U PROSTORU

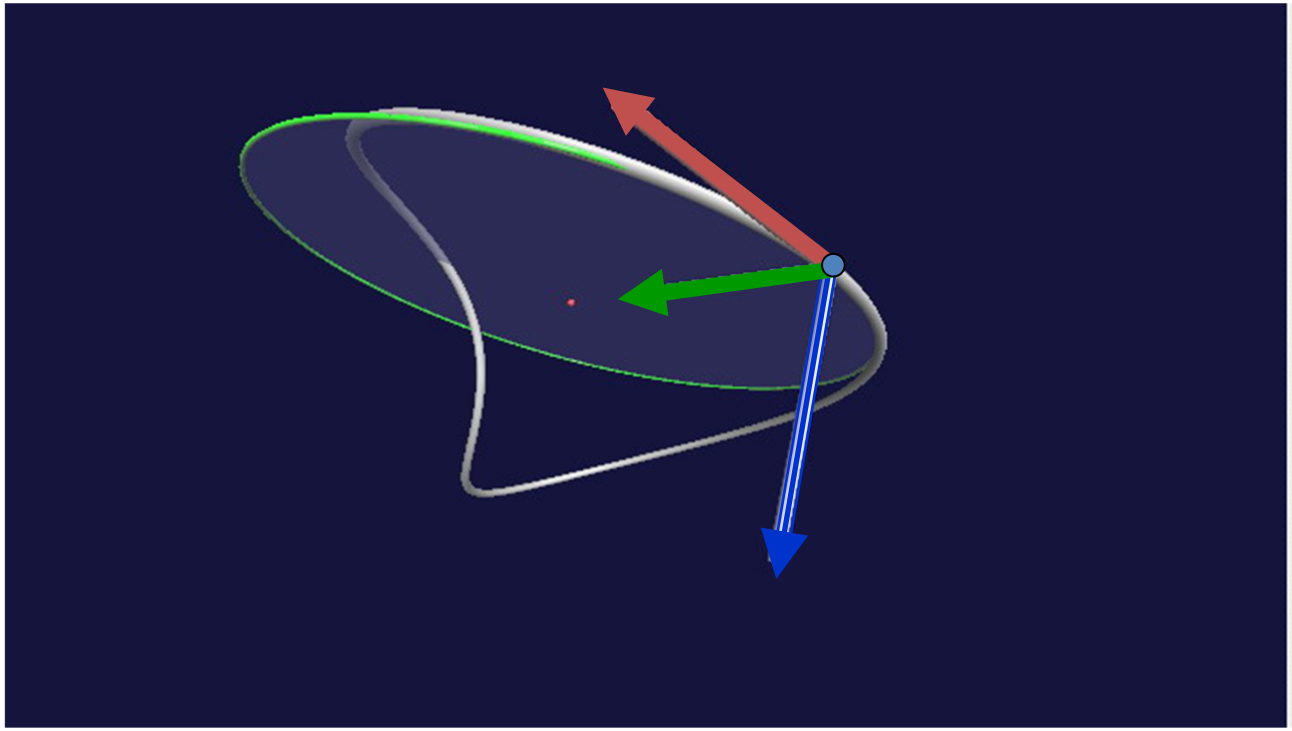
Tangenta, normala, binormala

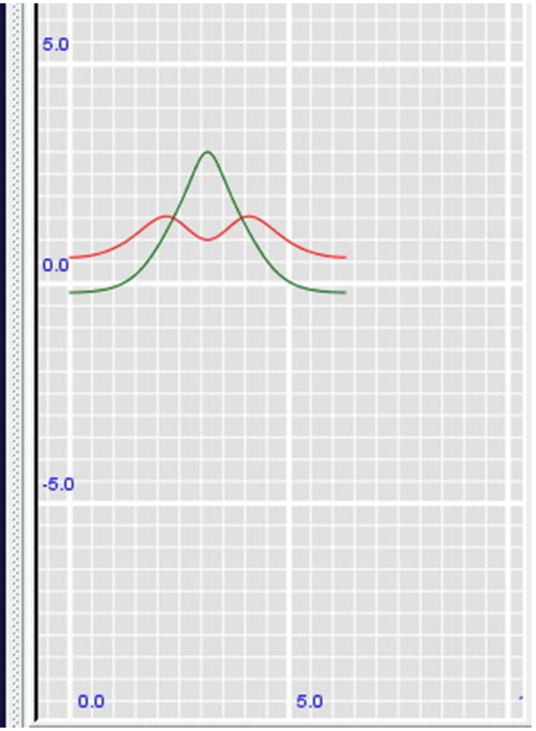
Tangenta

Normala

Binormala

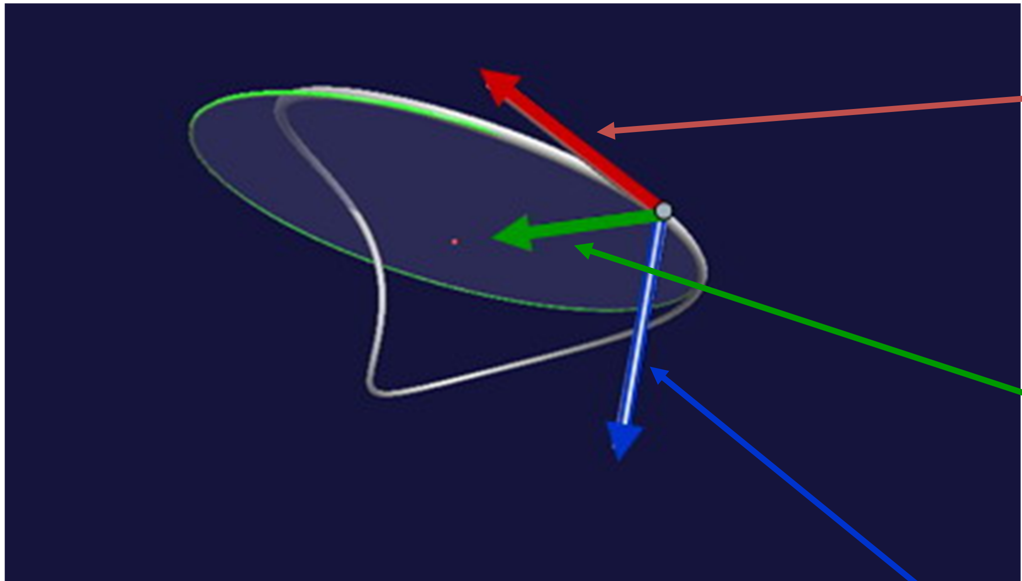
$$\hat{T}(t)=\frac{\alpha'(t)}{|\alpha'(t)|} \qquad \hat{N}(t)=\frac{T'(t)}{|T'(t)|} \qquad \hat{B}(t)=T(t)\times N(t)$$





LINIJE (KRIVE) U PROSTORU

Tangenta, normala, binormala



Jedinični vektor tangente:

$$\hat{T}(t) = \frac{\alpha'(t)}{|\alpha'(t)|}$$

Jedinični vektor normale:

$$\hat{N}(t) = \frac{\hat{T}'(t)}{|\hat{T}'(t)|}$$

Jedinični vektor binormale:

$$\hat{B}(t) = \hat{T}(t) \times \hat{N}(t)$$

$\hat{T}(t), \hat{N}(t), \hat{B}(t)$ — Frene-ov trijedar

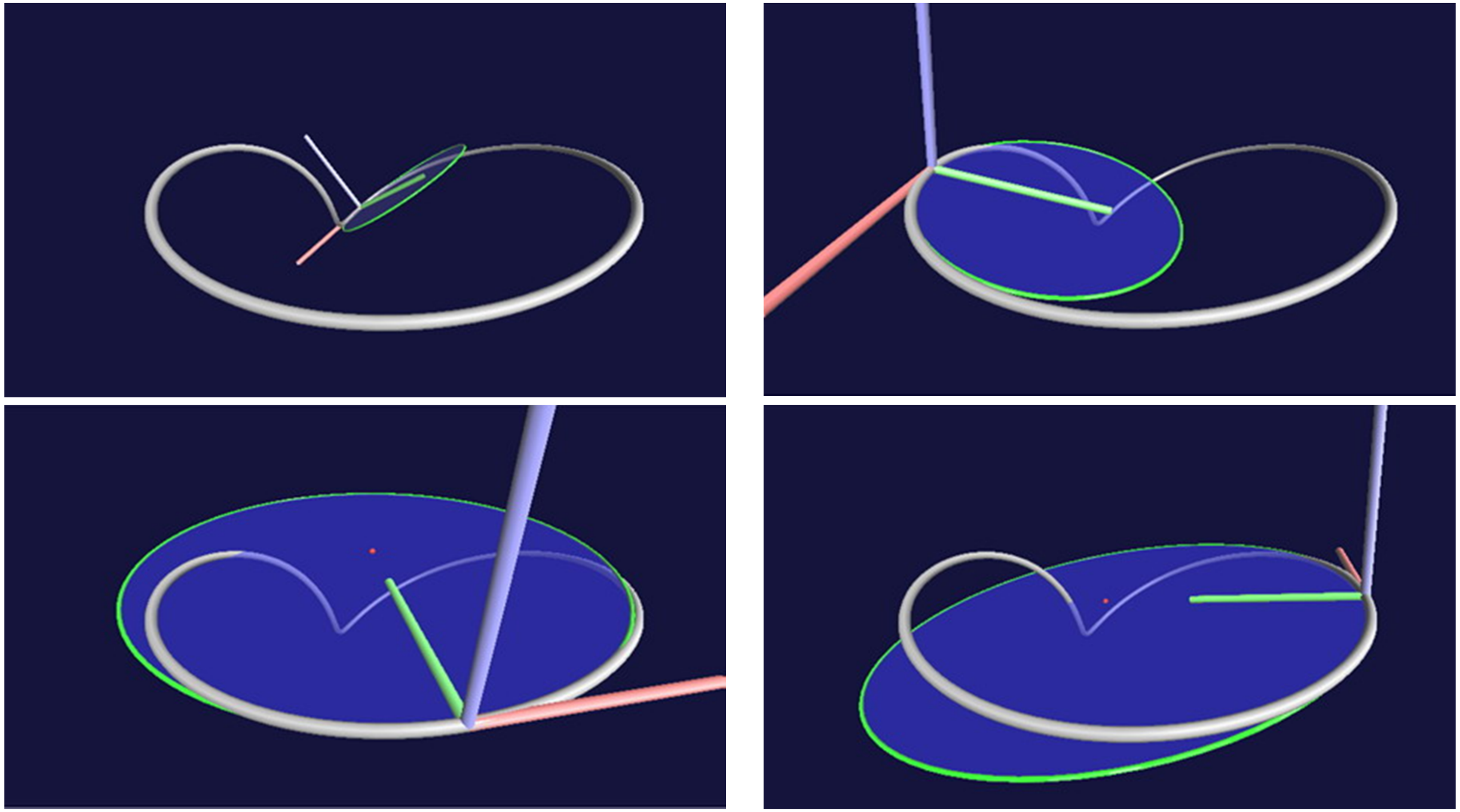
Pokretni prirodni trijedar

LINIJE (KRIVE) U PROSTORU

Primer

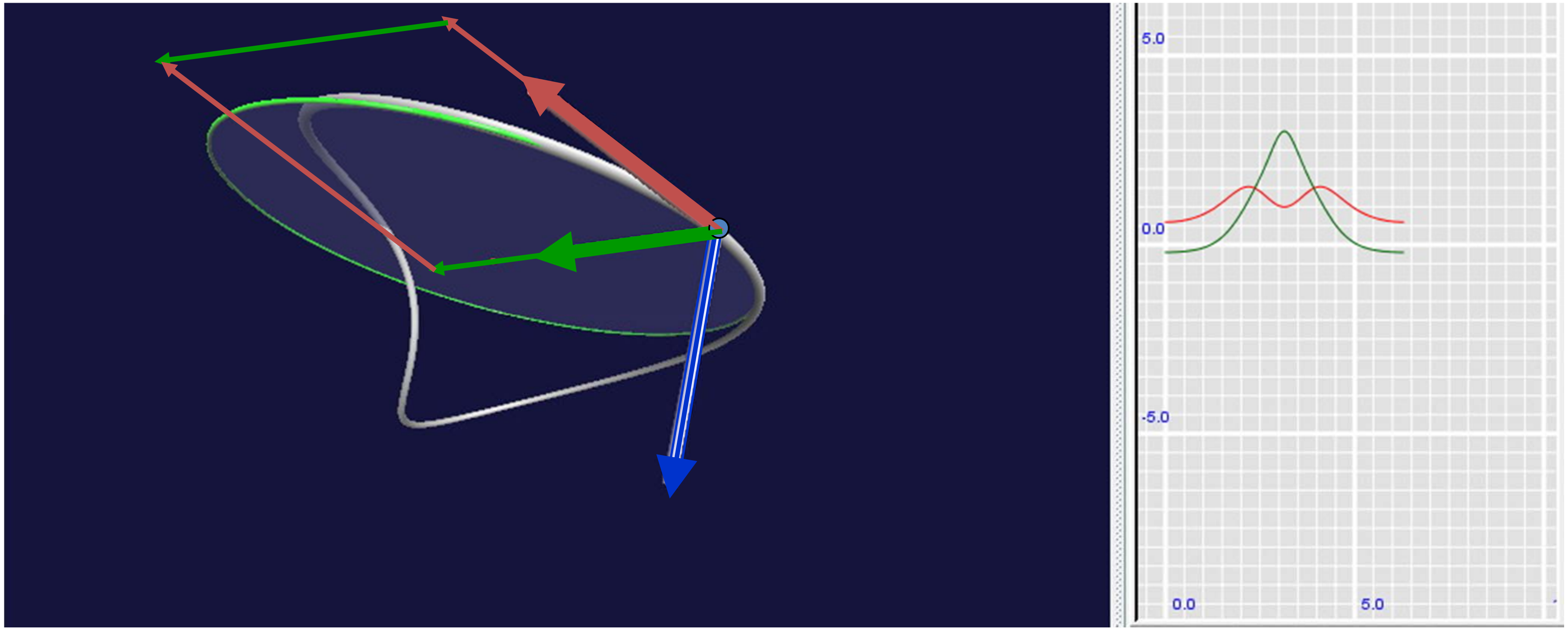
$$x = \cos t \cdot (1 + \cos t)$$
$$y = \sin t$$
$$z = 1 + \cos t$$

$$0 \le t \le 2\pi$$



LINIJE (KRIVE) U PROSTORU

$T(t),\ \ N(t)$ - oskulatorna ravan

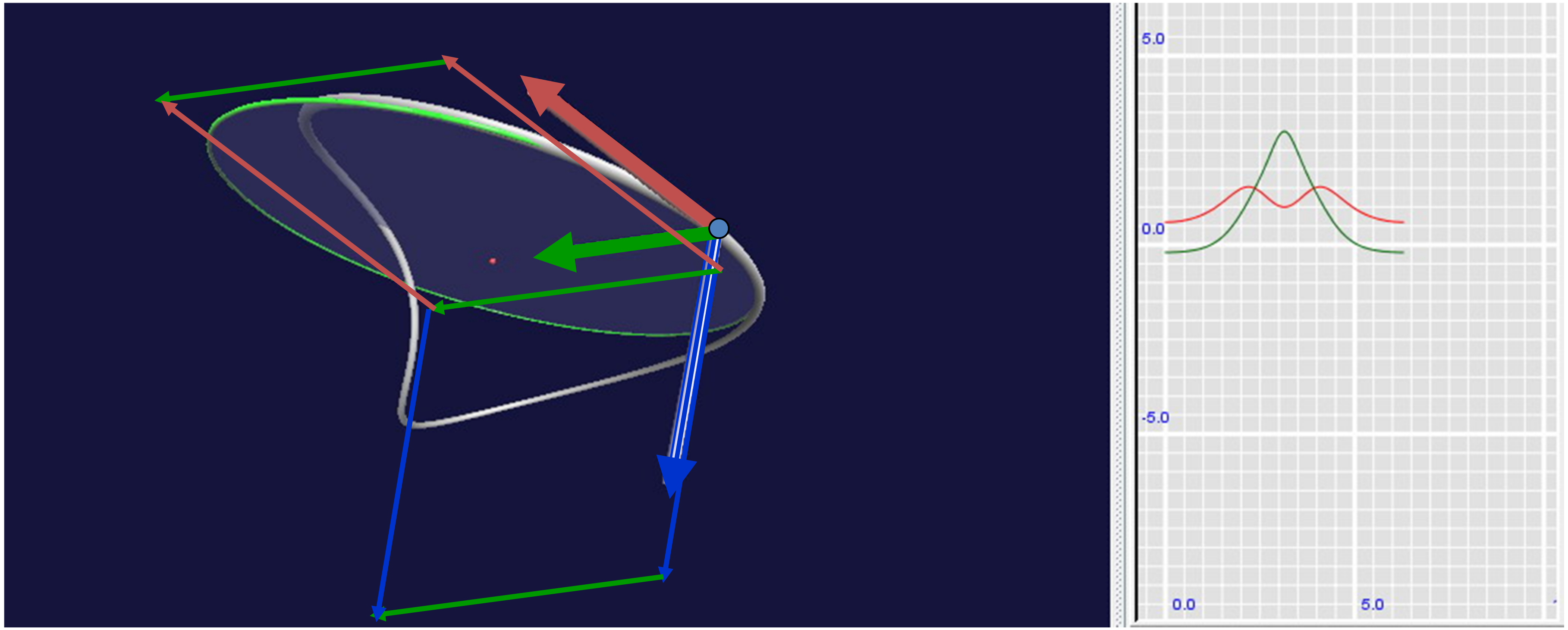


LINIJE (KRIVE) U PROSTORU

$T(t), \; N(t)$ - oskulatorna ravan

$N(t), \; B(t)$ - normalna ravan

$N(t), \; B(t)$ - normalna ravan



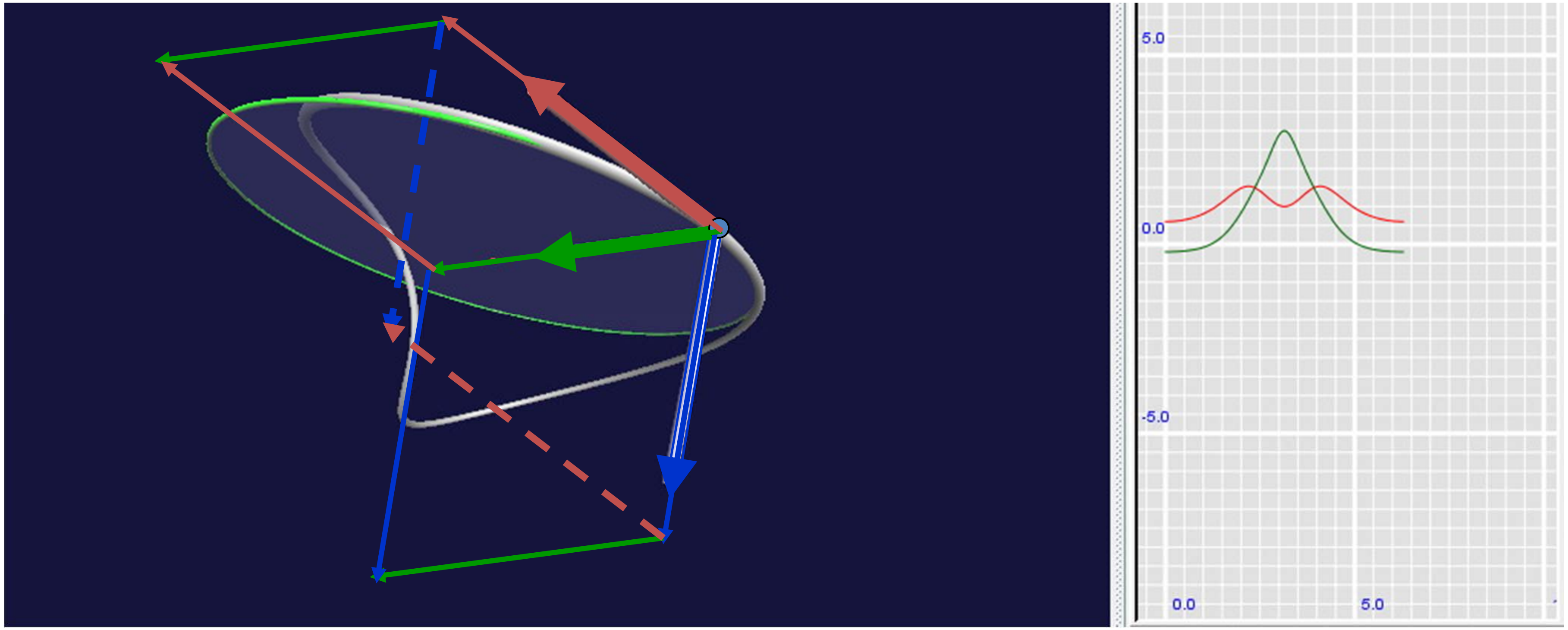
LINIJE (KRIVE) U PROSTORU

$T(t),\ N(t)$ - oskulatorna ravan

$N(t),\ B(t)$ - normalna ravan

$T(t),\ B(t)$ - rektifikaciona ravan

$T(t),\ B(t)$ - rektifikaciona ravan



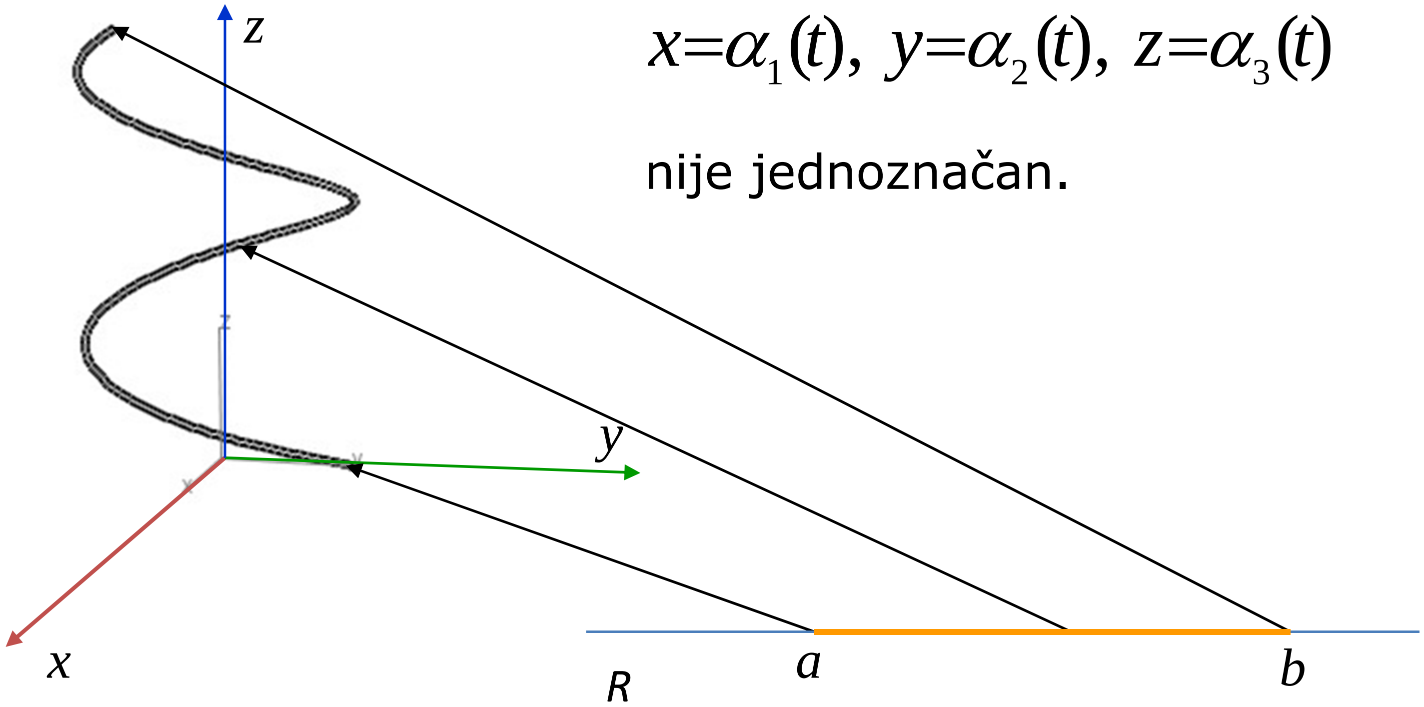
LINIJE (KRIVE) U PROSTORU

Reparametrizacija

Matematički prikaz krive linije C
u prostoru u parametarskom
obliku

$$x=\alpha_1(t),\ y=\alpha_2(t),\ z=\alpha_3(t) \qquad a<t<b$$

nije jednoznačan.



LINIJE (KRIVE) U PROSTORU

Reparametrizacija

Ukoliko je

$\varphi : J \rightarrow I$ $\varphi'(\tau) > 0$

Diferencijabilna rastuća funkcija koja vrši preslikavanje nekog intervala $J = [c, d]$ neke parametarske prave τ na interval $I = [a, b]$ tada su

$$x=\alpha_1(\varphi(\tau)),\ y=\alpha_2(\varphi(\tau)),\ z=\alpha_3(\varphi(\tau)) \qquad c<\tau<d$$

takodje parametarske jednačine iste krive C .

Kaže se da je izvršena reparametrizacija i nove jednačine su:

$$x=\beta_1(\tau),\ y=\beta_2(\tau),\ z=\beta_3(\tau) \qquad c<\tau<d$$

LINIJE (KRIVE) U PROSTORU

Reparametrizacija

$$\alpha(t) = \left(\alpha_1(t), \alpha_2(t), \alpha_3(t)\right), \qquad t \in I$$

$$I = [a, b] \qquad J = [c, d]$$

$$\varphi : J \to I \qquad t = \varphi(\tau), \quad \tau \in J$$

$$\alpha(t) = \alpha(\varphi(\tau)) = \beta(\tau), \quad \tau \in J$$

$$\beta'(\tau) = \alpha'(\varphi(\tau)) \cdot \varphi'(\tau), \quad \tau \in J$$

LINIJE (KRIVE) U PROSTORU

Reparametrizacija

$$\alpha(t) = \big(\alpha_1(t), \alpha_2(t), \alpha_3(t)\big), \qquad t \in I \qquad\qquad t = \varphi(\tau), \quad \tau \in J$$

$$\beta(\tau) = \big(\beta_1(\tau), \beta_2(\tau), \beta_3(\tau)\big), \qquad \tau \in J$$

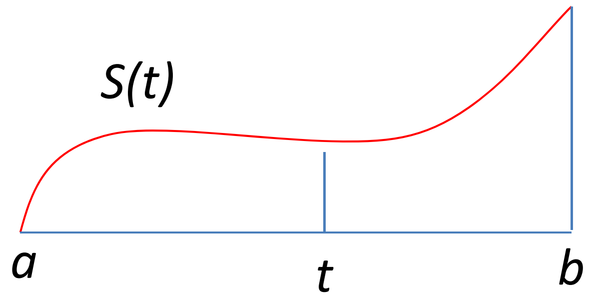
$$X(t) = \alpha_1(t) \qquad\qquad X(t) = \beta_1(\tau)$$

$$Y(t) = \alpha_2(t) \qquad a \le t \le b \qquad Y(t) = \beta_2(\tau) \qquad c \le \tau \le d$$

$$Z(t) = \alpha_3(t) \qquad\qquad Z(t) = \beta_3(\tau)$$

$$t = \varphi(\tau), \quad \tau \in J$$

LINIJE (KRIVE) U PROSTORU



Prirodna reparametrizacija

$$\alpha(t)=\left(\alpha_1(t),\alpha_2(t),\alpha_3(t)\right),\qquad t\in I \qquad I=[a,b]$$

$$s'(t)=\sqrt{\alpha_1'^2(t)+\alpha_2'^2(t)+\alpha_3'^2(t)}$$

$$s(t)=\int_a^t s'(t)\,dt=\int_a^t\sqrt{\alpha_1'^2(t)+\alpha_2'^2(t)+\alpha_3'^2(t)}\,dt$$

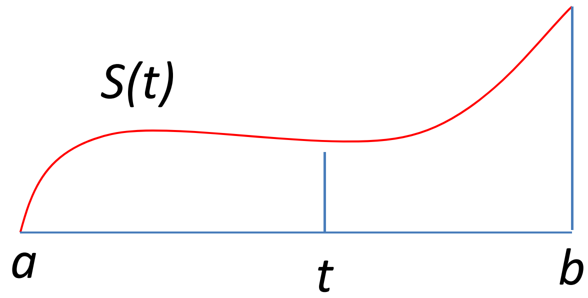
$$s(a)=0 \qquad I=[a,b] \qquad J=[0,s(b)]$$

$$\varphi: I\to J \qquad \varphi^{-1}: J\to I \qquad s=\varphi(t) \qquad t=\varphi^{-1}(s)$$

$$t\in I, \qquad s\in J$$

LINIJE (KRIVE) U PROSTORU

Prirodna reparametrizacija



$$s(a)=0 \quad I=[a,b] \quad J=[0,s(b)]$$

$$\varphi : I \to J \qquad \varphi^{-1} : J \to I \qquad s=\varphi(t) \quad t=\varphi^{-1}(s)$$

$$t \in I, \quad s \in J$$

$$\alpha(t)=\alpha(\varphi^{-1}(s))=\gamma(s)$$

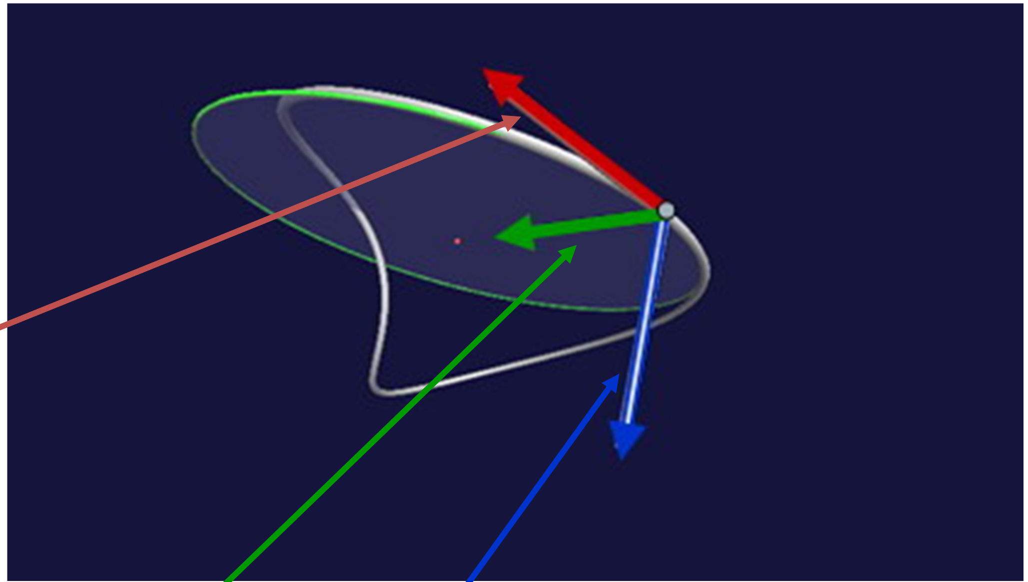
$$\alpha(t)=(\alpha_1(t),\alpha_2(t),\alpha_3(t)), \qquad t \in I$$

$$\gamma(s)=(\gamma_1(s),\gamma_2(s),\gamma_3(s)), \qquad s \in J$$

LINIJE (KRIVE) U PROSTORU

Prirodna reparametrizacija

$$\left|\gamma'(s)\right| = 1$$



$$\hat{T}(s) = \gamma'(s)$$

$$\hat{N}(s) = \frac{\hat{T}'(s)}{\left|\hat{T}'(s)\right|} = \frac{\gamma''(s)}{\left|\gamma''(s)\right|}$$

$$\hat{B}(s) = T(s) \times N(s)$$

LINIJE (KRIVE) U PROSTORU

Prirodna reparametrizacija

Tangenta

Normala

Binormala

Jedinični vektori tangente, normale i binormale su invarijantni u odnosu na parametrizaciju:

$$\hat{T}(t)\Big(=\hat{T}(\tau)=\hat{T}(s)\Big)=\frac{\alpha'(t)}{|\alpha'(t)|}=\frac{\beta'(\tau)}{|\beta'(\tau)|}=\gamma'(s)$$

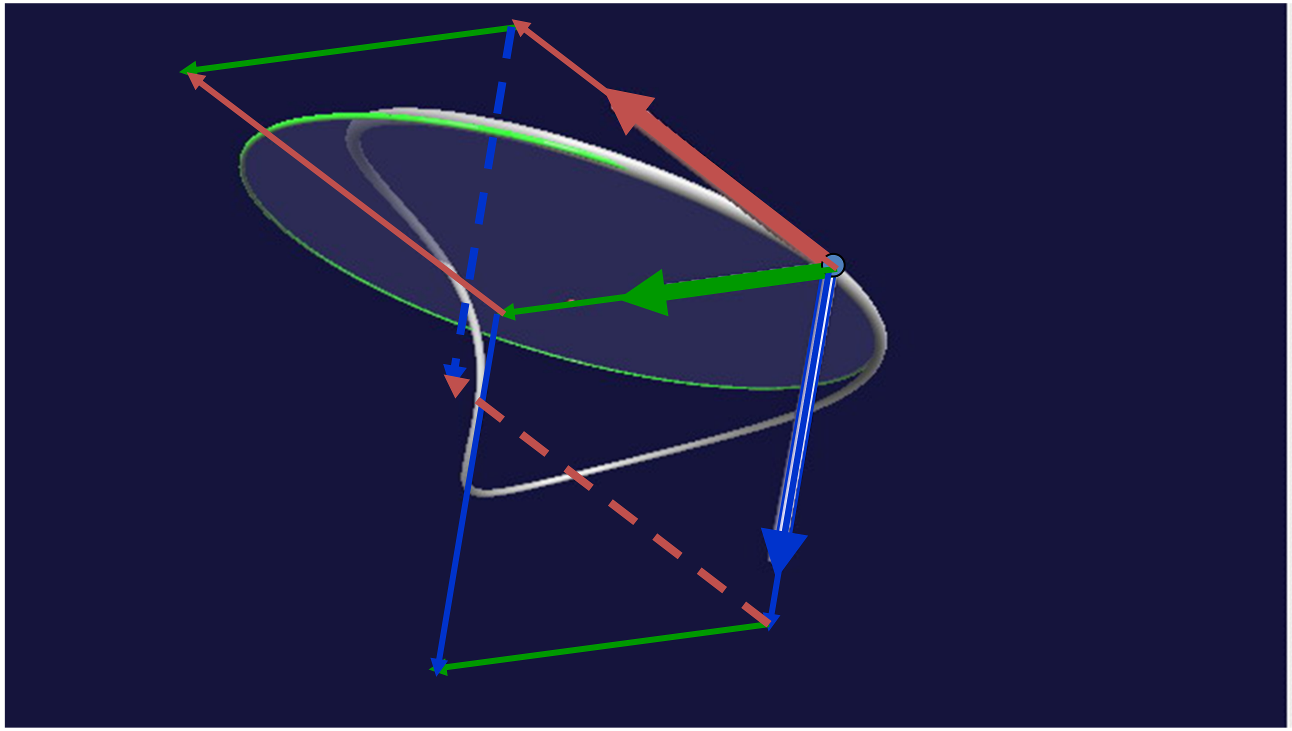
$$\hat{N}(t)\Big(=\hat{N}(\tau)=\hat{N}(s)\Big)=\frac{T'(t)}{|T'(t)|}=\frac{T'(\tau)}{|T'(\tau)|}=\frac{\gamma''(s)}{|\gamma''(s)|}$$

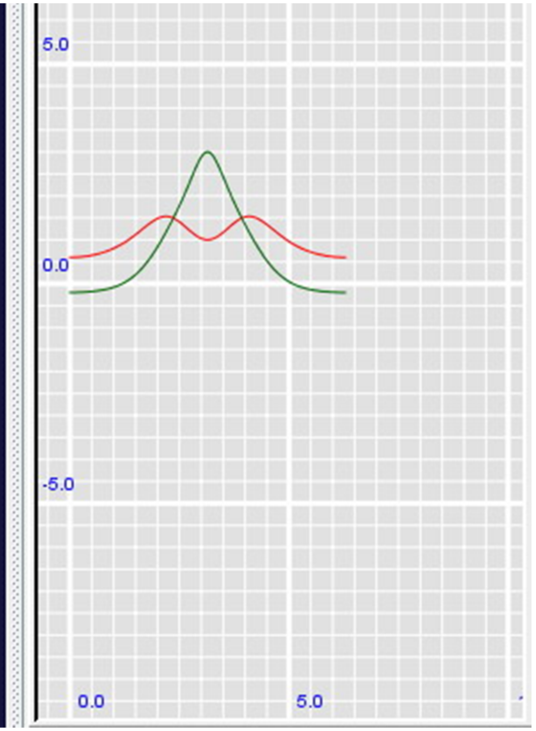
$$\hat{B}(t)\Big(=\hat{B}(\tau)=\hat{B}(s)\Big)=\hat{T}(t)\times\hat{N}(t)=\frac{\gamma'(s)\times\gamma''(s)}{|\gamma''(s|}$$

LINIJE (KRIVE) U PROSTORU

Krivina

$$K(s) = \left| \frac{d\hat{T}(s)}{ds} \right| = \left| \hat{T}'(s) \right| = \left| \gamma''(s) \right|$$





LINIJE (KRIVE) U PROSTORU

Krivina

$$\alpha(t) = \big(\alpha_1(t), \alpha_2(t), \alpha_3(t)\big), \qquad t \in I$$

$$X(t) = \alpha_1(t)$$

$$Y(t) = \alpha_2(t) \qquad a \le t \le b$$

$$Z(t) = \alpha_3(t)$$

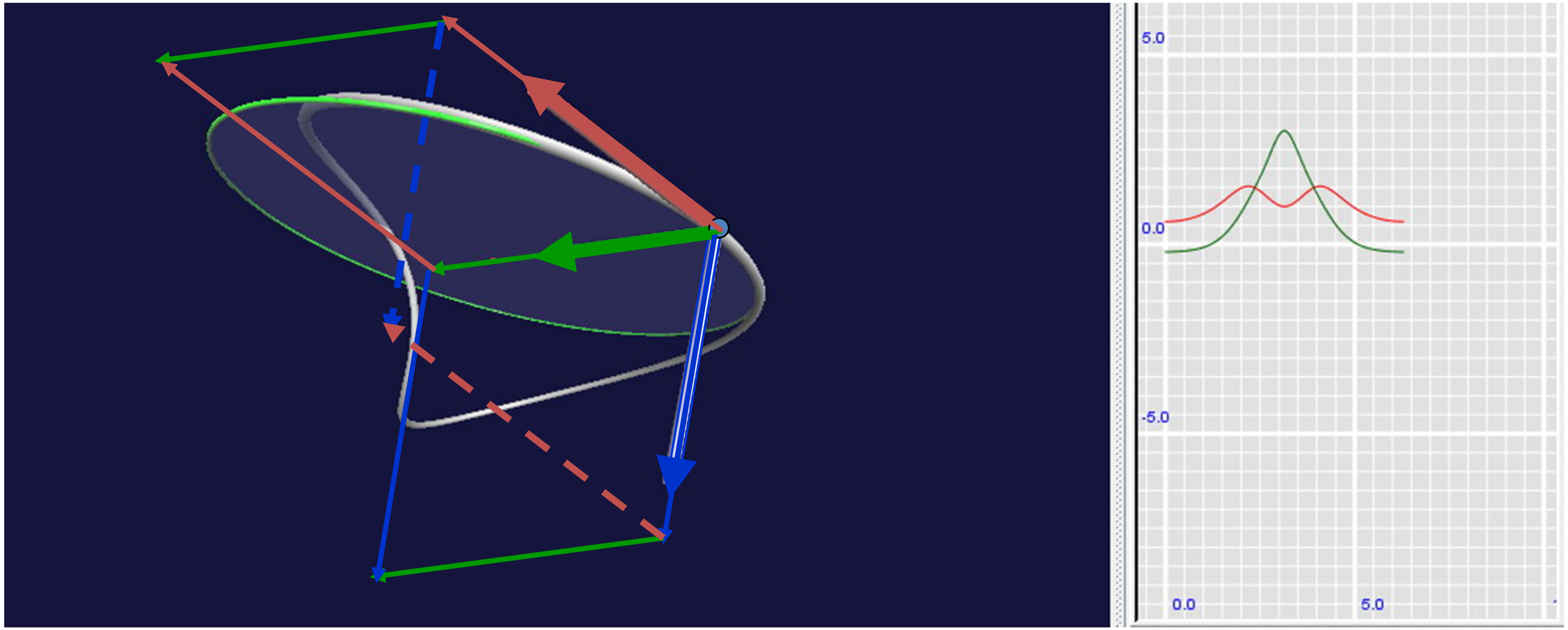
$$K(t) = \frac{\left|\alpha'(t) \times \alpha''(t)\right|}{\left|\alpha'(t)\right|^3}$$

$$K(t) = \frac{\sqrt{(\alpha_3''\alpha_2' - \alpha_2''\alpha_3')^2 + (\alpha_2''\alpha_1' - \alpha_1''\alpha_2')^2 + (\alpha_1''\alpha_3' - \alpha_3''\alpha_1')^2}}{\left(\alpha_1'^2 + \alpha_2'^2 + \alpha_3'^2\right)^{\frac{3}{2}}}$$

LINIJE (KRIVE) U PROSTORU

Krivina

Poluprečnik krivine: $$R = \frac{1}{K}$$ Krug krivine

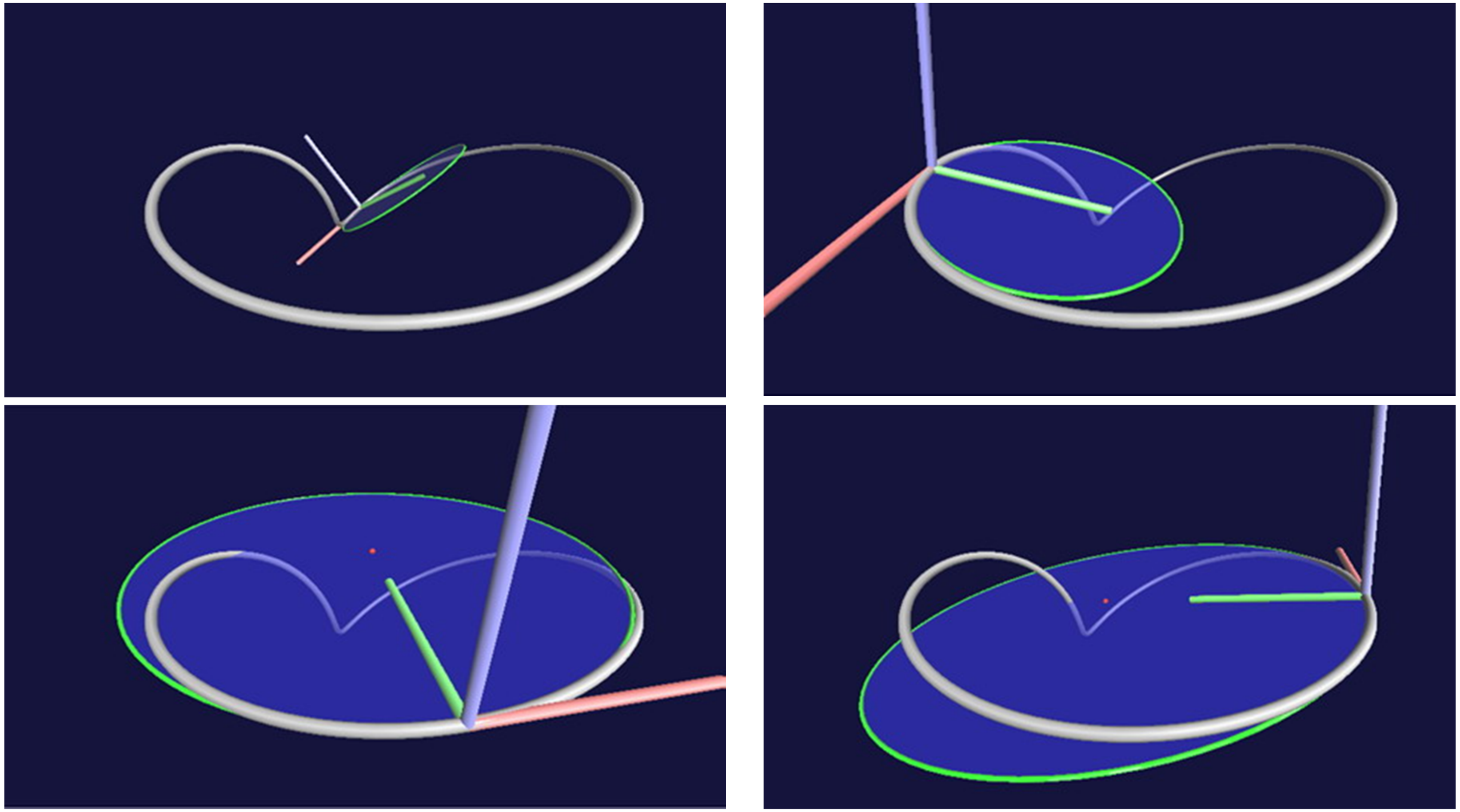


LINIJE (KRIVE) U PROSTORU

Krivina - primer

$$x = \cos t \cdot (1 + \cos t)$$
$$y = \sin t$$
$$z = 1 + \cos t$$

$$0 \le t \le 2\pi$$



LINIJE (KRIVE) U PROSTORU

Geometrijsko mesto centara krugova krivina neke krive je evoluta te krive. Sama ta kriva, za svoju evolutu je evolventa.