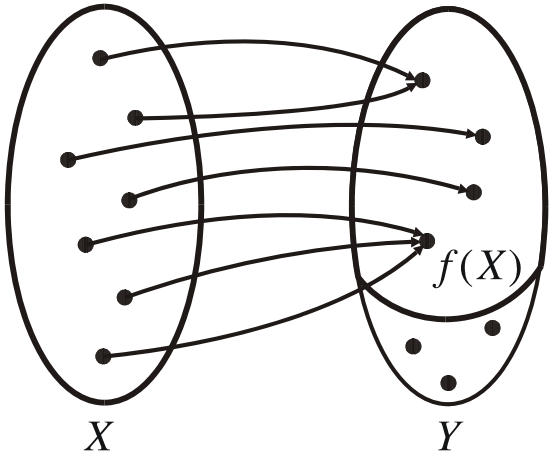
PRESLIKAVANJE (FUNKCIJA)

Neka su dati skupovi *X* i *Y*. Ako je svakom elementu $x \in X$ po nekom zakonu f dodeljen (pridružen) tačno jedan određen element $y \in Y$, kaže se da je f ***funkcija*** ili ***preslikavanje*** skupa *X* **u** skup *Y* i piše se

$$f: X \to Y$$



PRESLIKAVANJE (FUNKCIJA)

Funkcija $f: X \to Y$ se može zadati i opisati pomoću jednakosti

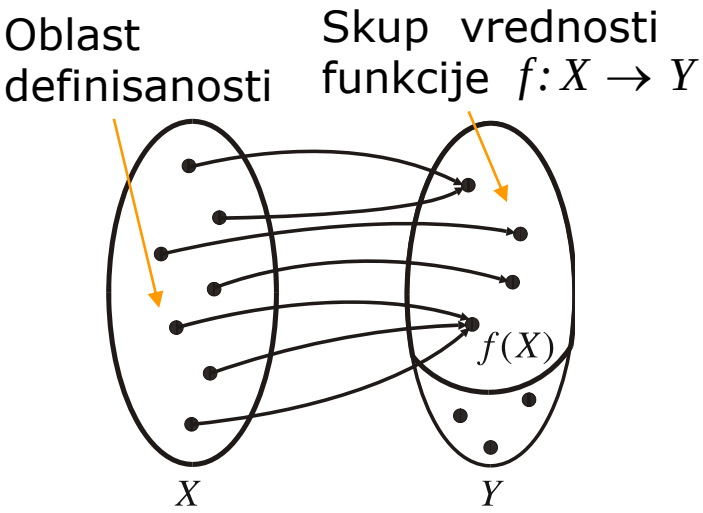
$$y = f(x),\ x \in X$$

kojom se svakoj pojedinačnoj vrednosti $x \in X$ dodeljuje

tačno jedna određena vrednost $y \in Y$.

Skup vrednosti funkcije $f : X \to Y$

$$f(X) = \{y \mid y = f(x), x \in X\} \subseteq Y$$



PRESLIKAVANJE (FUNKCIJA)

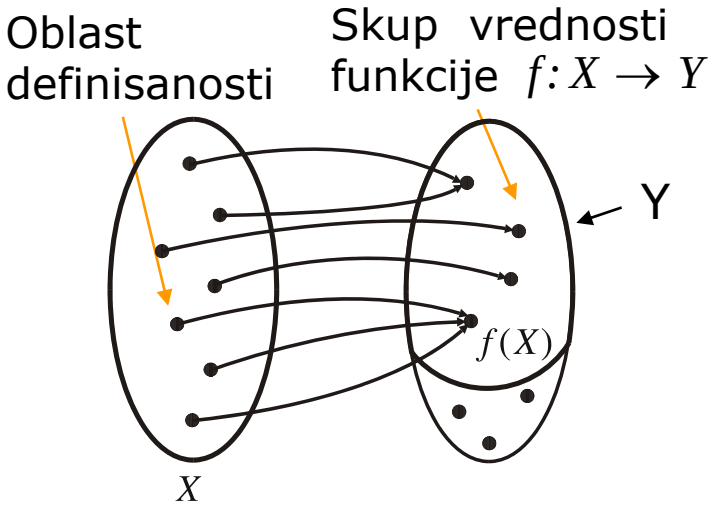
Skup vrednosti funkcije $f : X \to Y$

$$f(X) = \left\{ y \,\middle|\, y = f(x), x \in X \right\} \subseteq Y$$

Ako je

$$f(X) = Y$$

kaže se da funkcija f vrši preslikavanje skupa X na skup Y

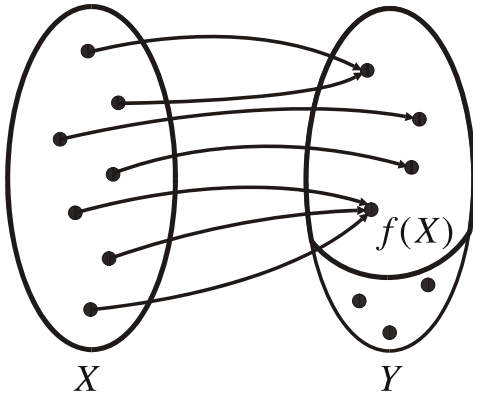


PRESLIKAVANJE (FUNKCIJA)

Jednoznačnost preslikavanja

$$x_1 = x_2 \Rightarrow f(x_1) = f(x_2)$$

$$f(x_1) \neq f(x_2) \Rightarrow x_1 \neq x_2$$



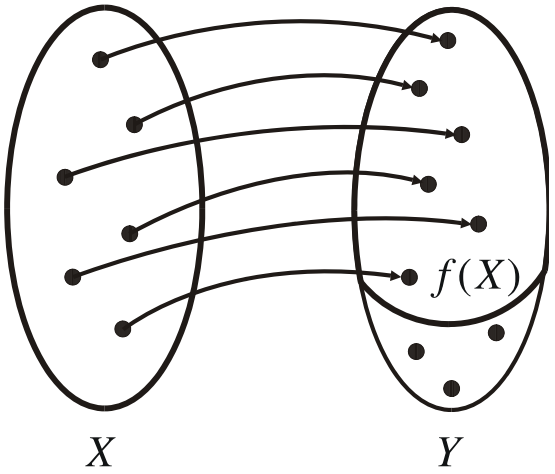
PRESLIKAVANJE (FUNKCIJA)

Ako važi obrnuta implikacija

$$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$$

odnosno

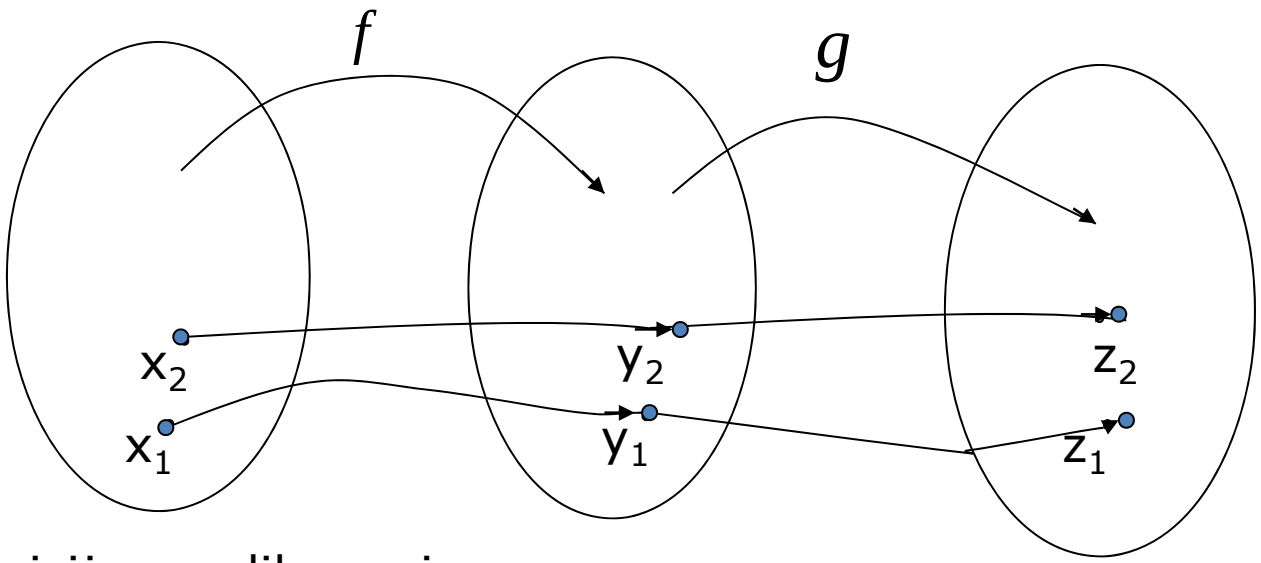
$$x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$$



preslikavanje je obostrano jednoznačno ili 1-1 preslikavanje i predstavlja uzajamnu biunivoku korespodenciju između skupova ***X*** i ***f(X).***

PRESLIKAVANJE (FUNKCIJA)

$f: X \to Y$ $\quad$ $g: Y \to Z$



Kompozicija preslikavanja

$$f \circ g\,(x) = g(\,f\,(x))$$

$$f \circ g : X \to Z$$

PRESLIKAVANJE (FUNKCIJA)

$I : X \to X \qquad I = I(x) = x$ Identično preslikavanje

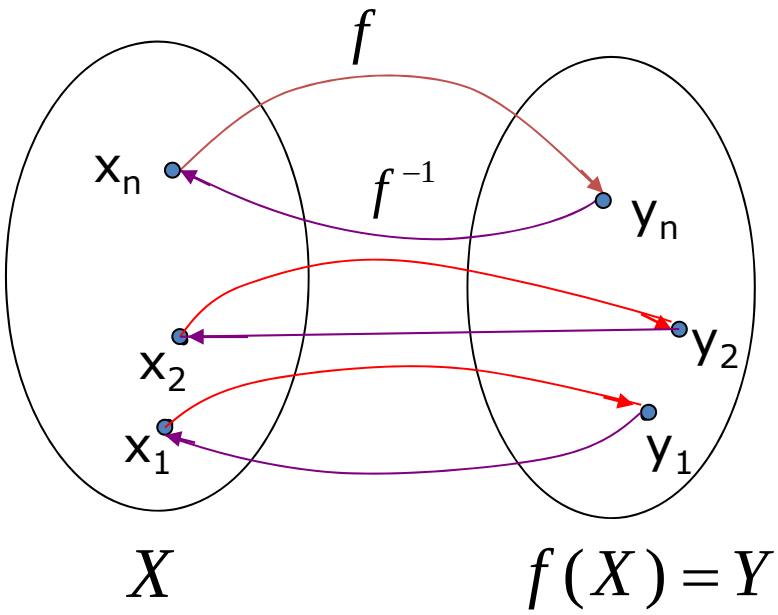
$f : X \to Y \qquad f(X) = Y$

Ako postoji funkcija $f^{-1} : Y \to X$ takva da je

$$f \circ f^{-1} = f^{-1} \circ f = I$$

funkcija f^{-1} je inverzna za funkciju f .

PRESLIKAVANJE (FUNKCIJA)



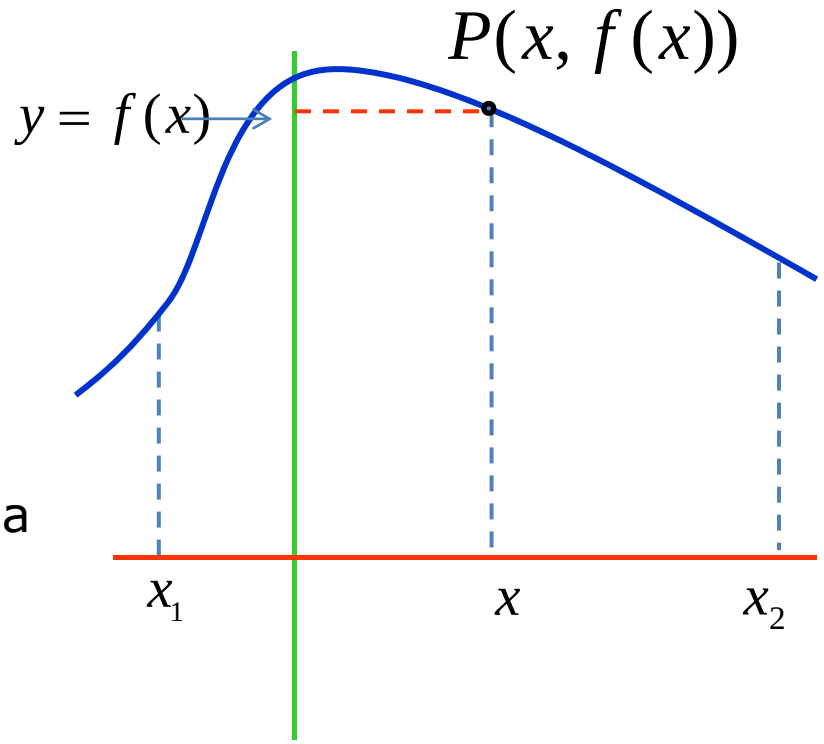
f - obostrano jednoznačno prelikavanje i

$$f(X) = Y$$

Inverzno preslikavanje

$$f \circ f^{-1}(x) = f^{-1}(f(x)) = x$$

LINIJE (KRIVE) U RAVNI



$f : R \to R$

$y = f(x) \qquad x \in R$

$y = f(x)$ eksplicitno zadata funkcija

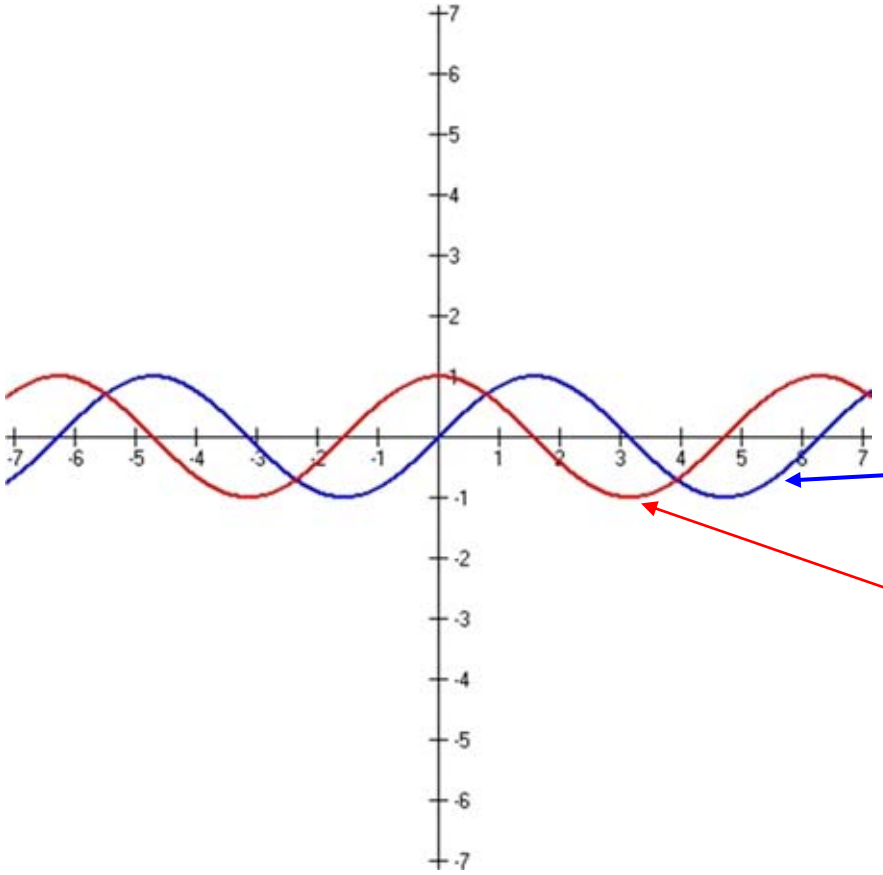
Skup tačaka

$\{(x, f(x)), \quad x \in R\}$

je grafik funkcije $y = f(x)$ i predstavlja liniju (ili krivu) u ravni.
Jednačina $y = f(x)$ je jednačina krive.

LINIJE (KRIVE) U RAVNI

Primeri



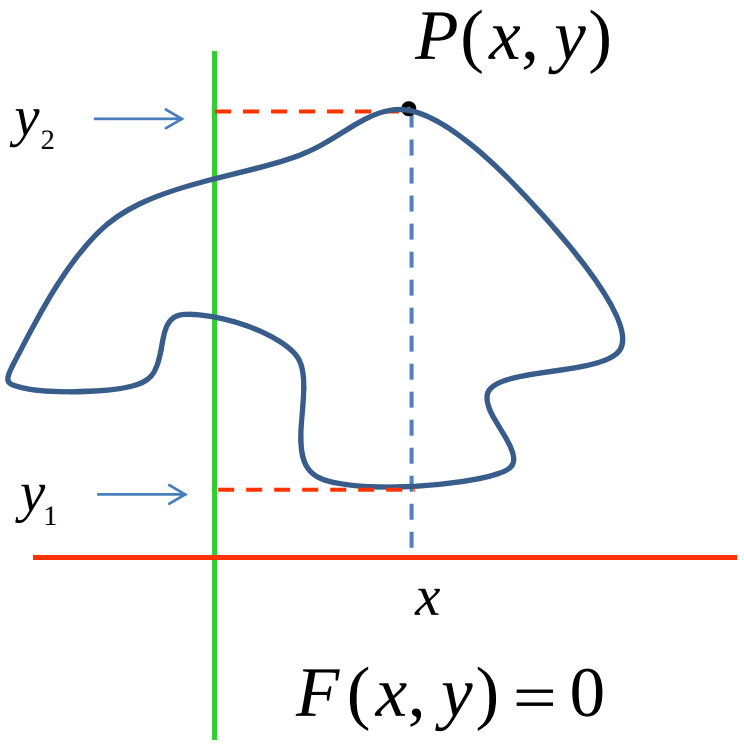
$y = \sin x$

$y = \cos x$

LINIJE (KRIVE) U RAVNI

Implicitno zadata funkcija

$$F(x,y)=0 \quad x,y \in R$$



Skup tačaka

$$\{(x,y),\ \ F(x,y)=0\}$$

je grafik funkcije f i predstavlja liniju (ili krivu) u ravni.

Jednačina $F(x,y)=0$ je jednačina krive.

LINIJE (KRIVE) U RAVNI

Krive drugog reda

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0 \qquad A^2 + B^2 + C^2 \neq 0$$

Svodjenjem na kanonski oblik, odnosno transformacijom koordinatnog sistema (translacijom i rotacijom) dobija se jedna od sledećih krivih:

Krug: $x^2 + y^2 = r^2$

Elipsa: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Hiperbola: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$

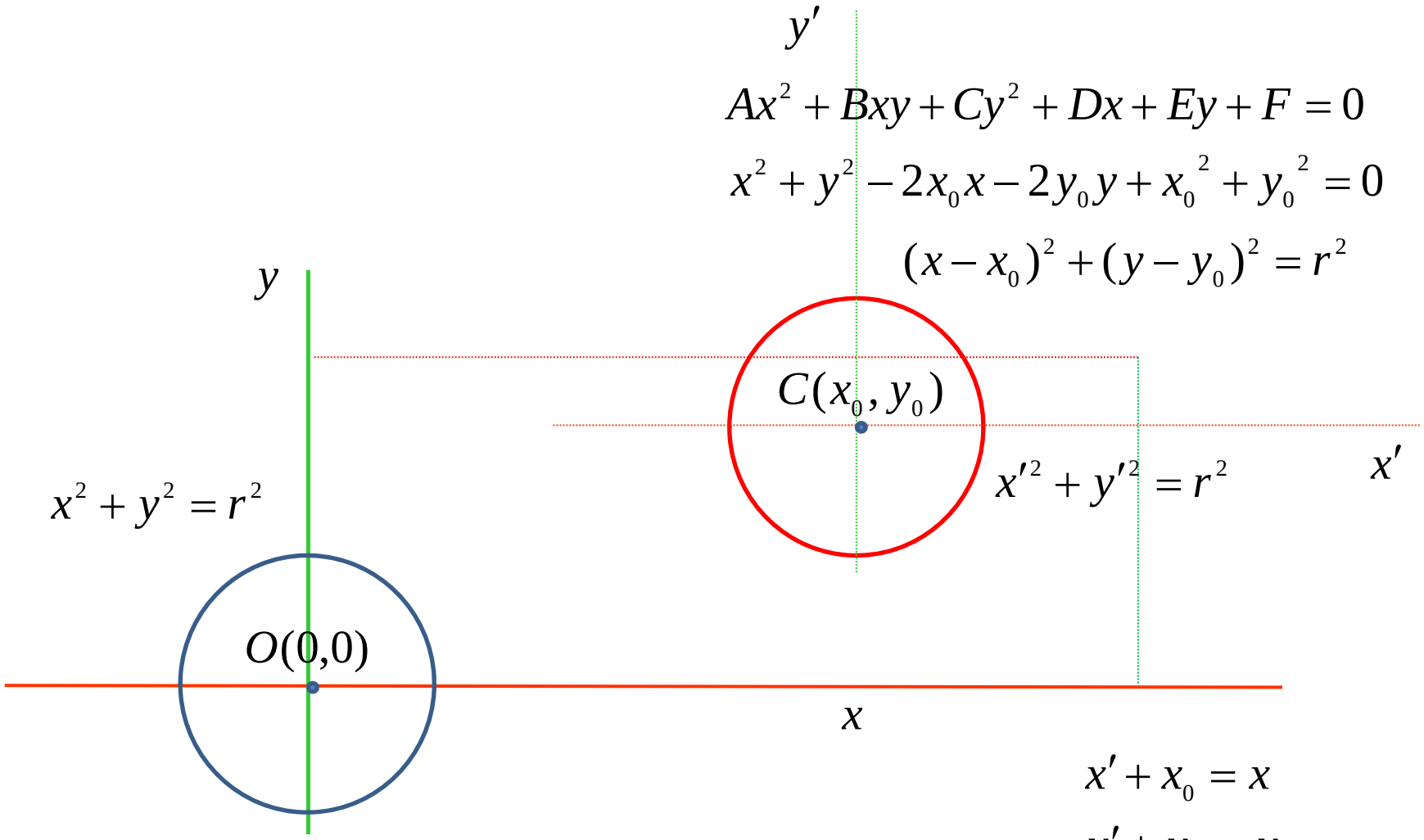
Parabola: $x^2 = 2py$ $y^2 = 2px$

Dve prave koje se seku ili su paralelne

Jedna tačka ili prazan skup.

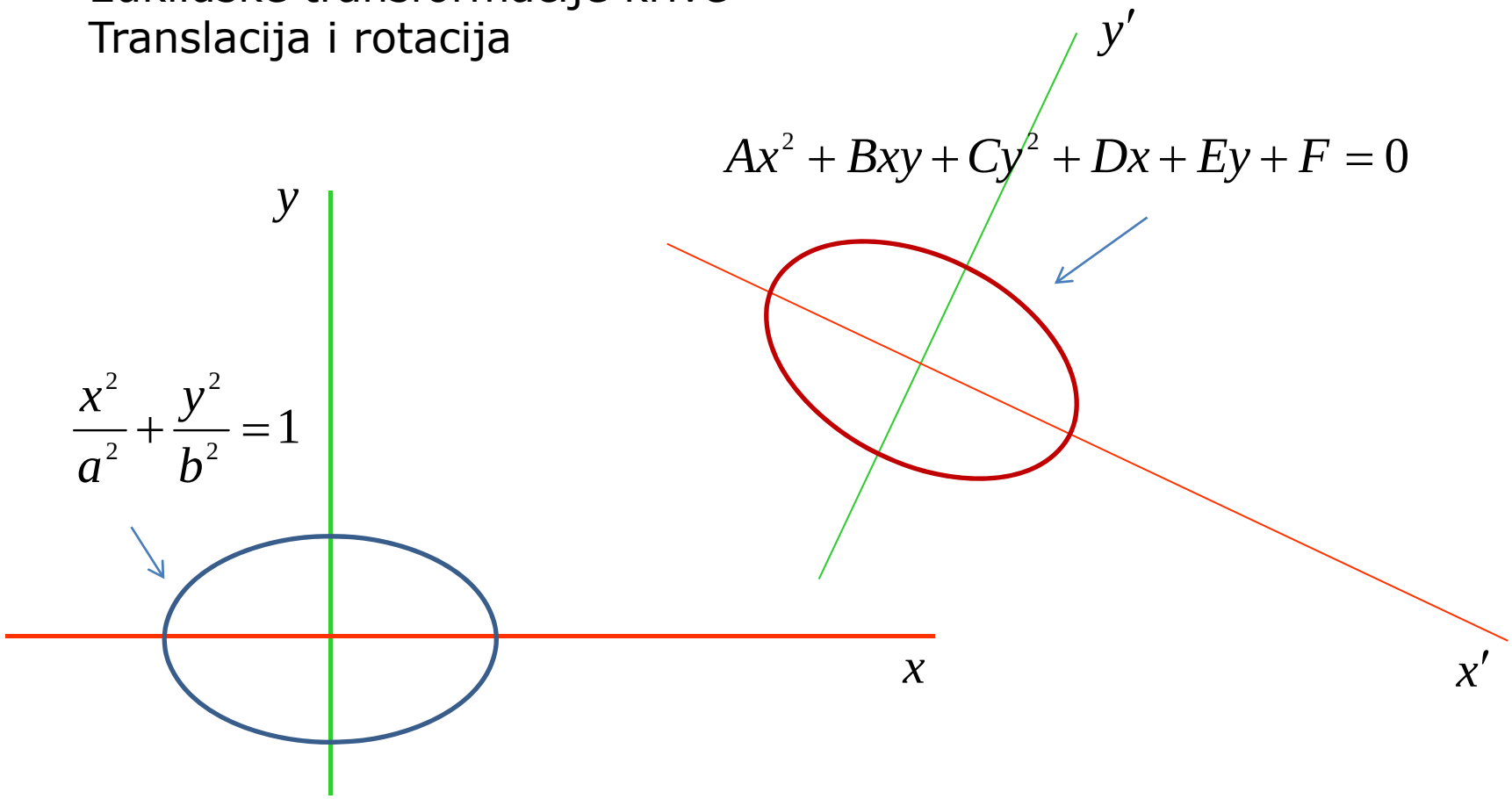
LINIJE (KRIVE) U RAVNI

KRUG



ELIPSA

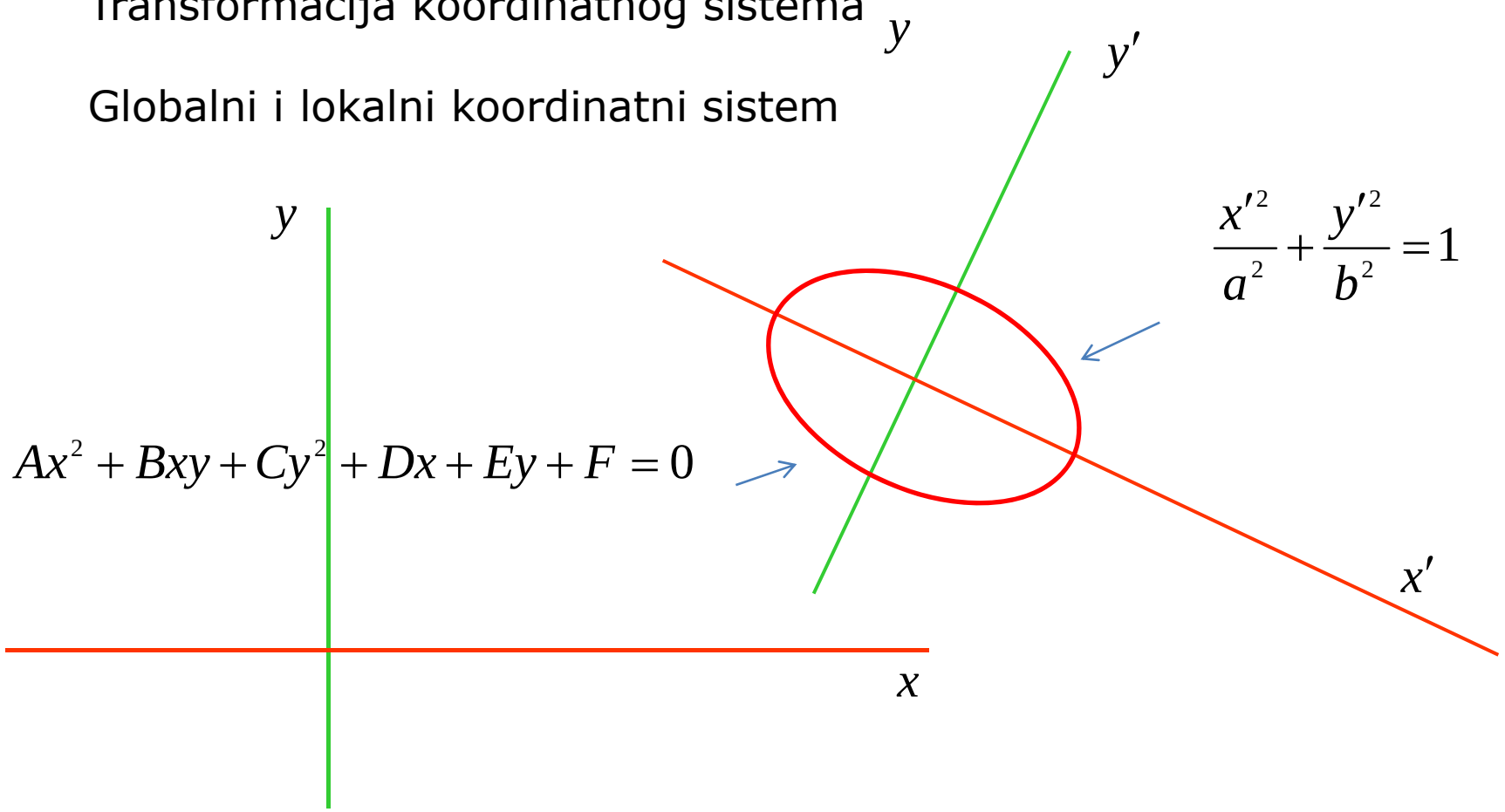
Euklidske transformacije krive
Translacija i rotacija



LINIJE (KRIVE) U RAVNI

Transformacija koordinatnog sistema

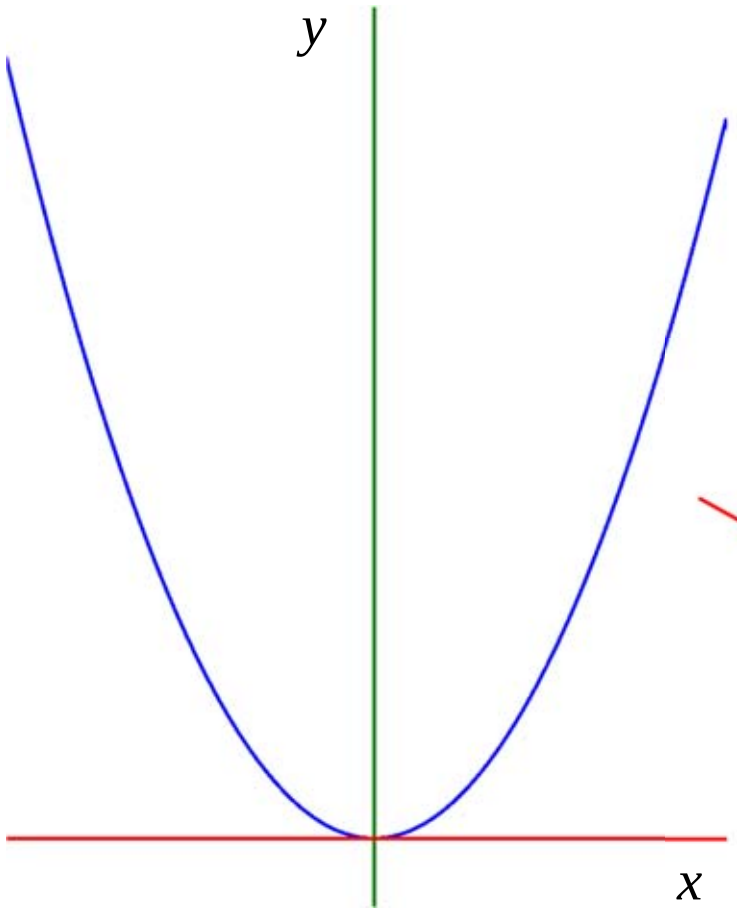
Globalni i lokalni koordinatni sistem



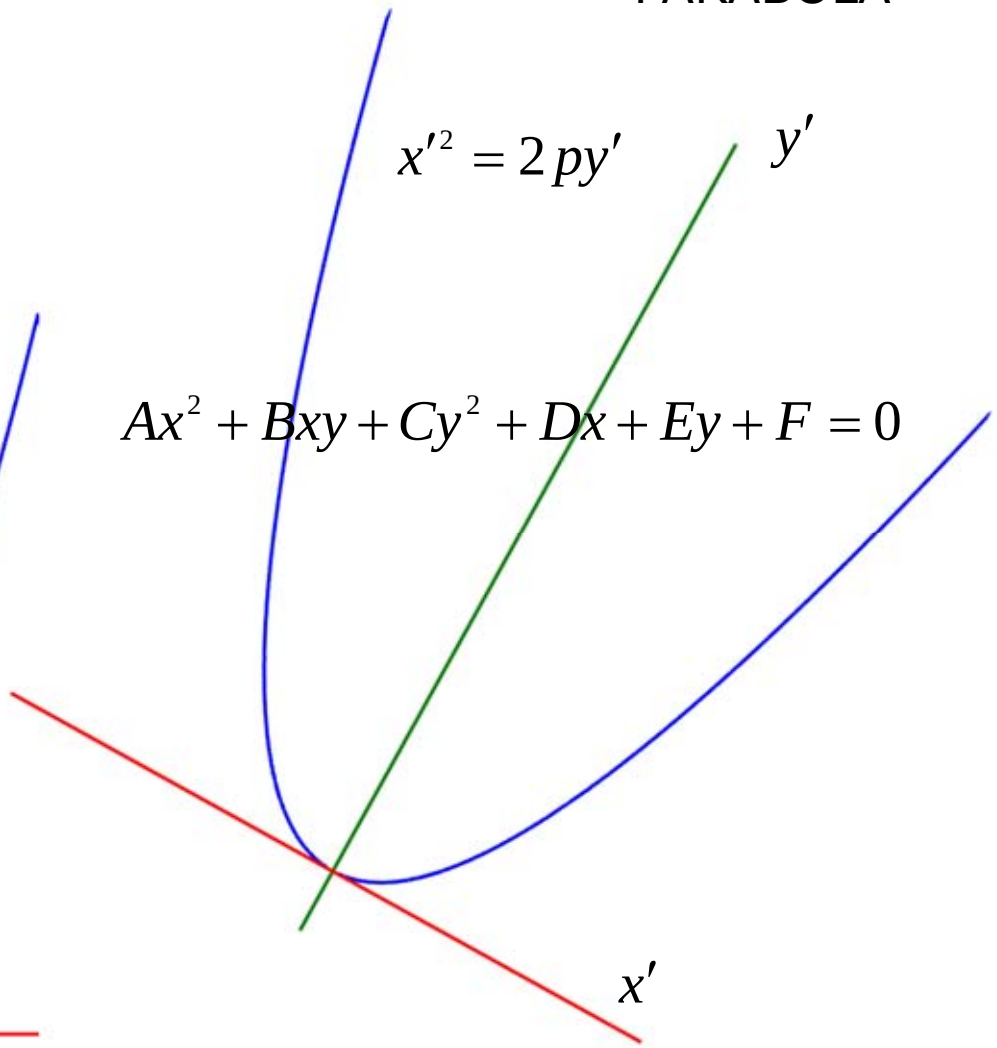
LINIJE (KRIVE) U RAVNI

PARABOLA

$x^2 = 2py$



$x'^2 = 2py'$

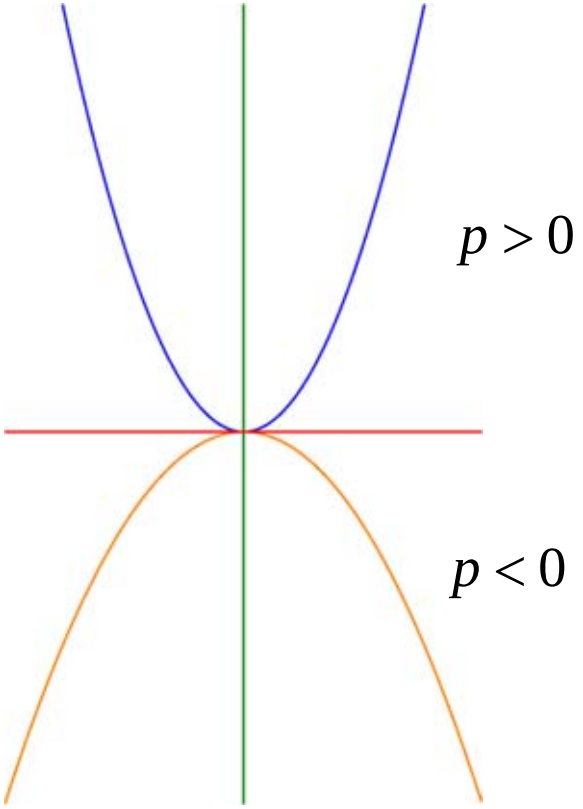


$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

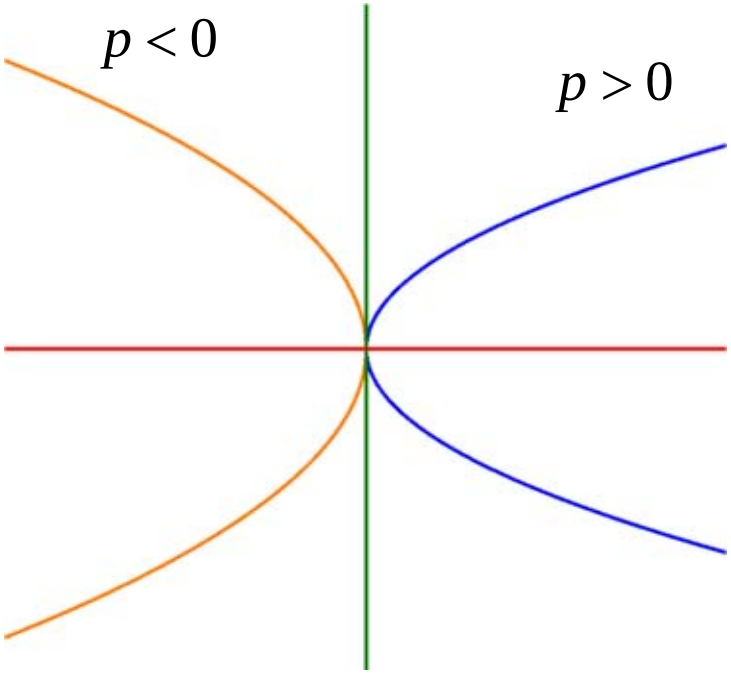
LINIJE (KRIVE) U RAVNI

PARABOLA

$$x^2 = 2py$$



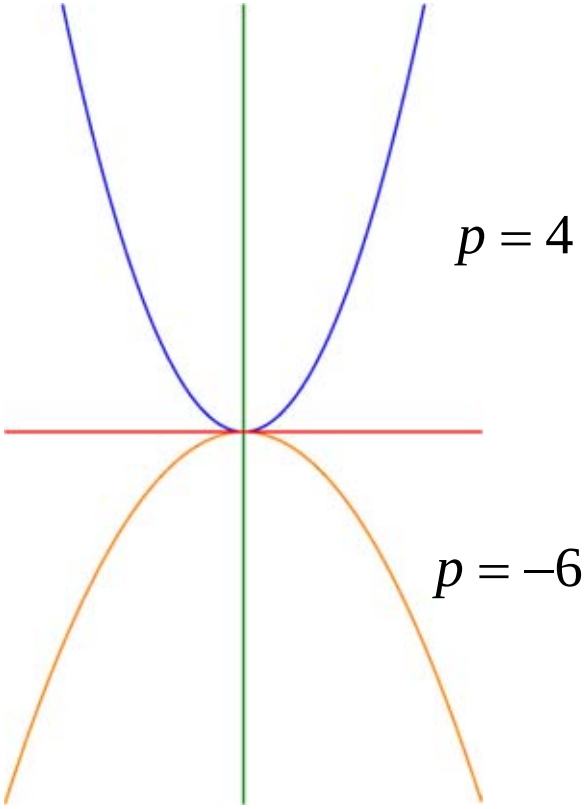
$$y^2 = 2px$$



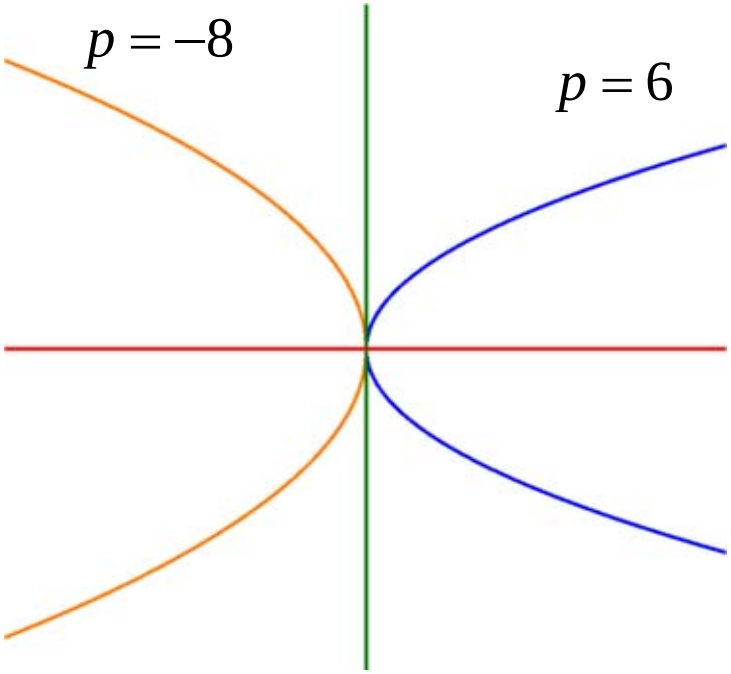
LINIJE (KRIVE) U RAVNI

PARABOLA

$$x^2 = 2py$$



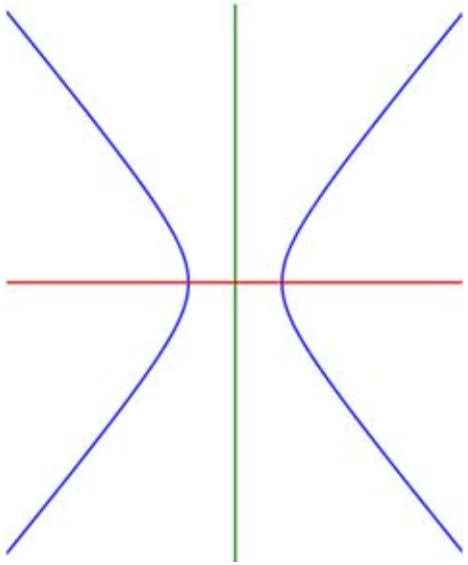
$$y^2 = 2px$$



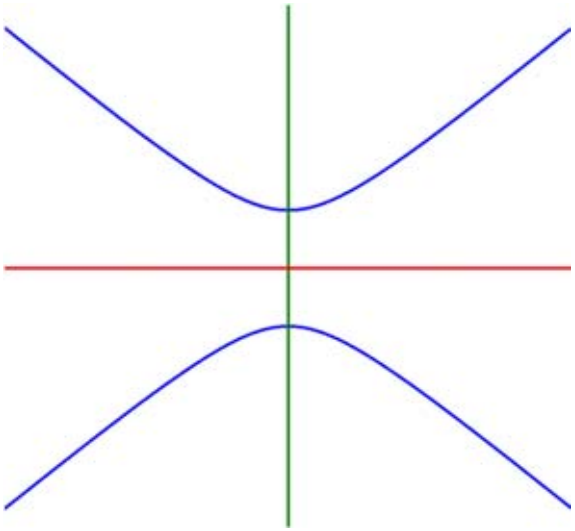
LINIJE (KRIVE) U RAVNI

HIPERBOLA

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$



$$\frac{y^2}{b^2}-\frac{x^2}{a^2}=1$$



LINIJE (KRIVE) U RAVNI

$I = [a,b] \subset R$ $\quad I = (a,b) \subset R$

$I = (-\infty,+\infty) = R$

$I = (a,+\infty) \subset R$

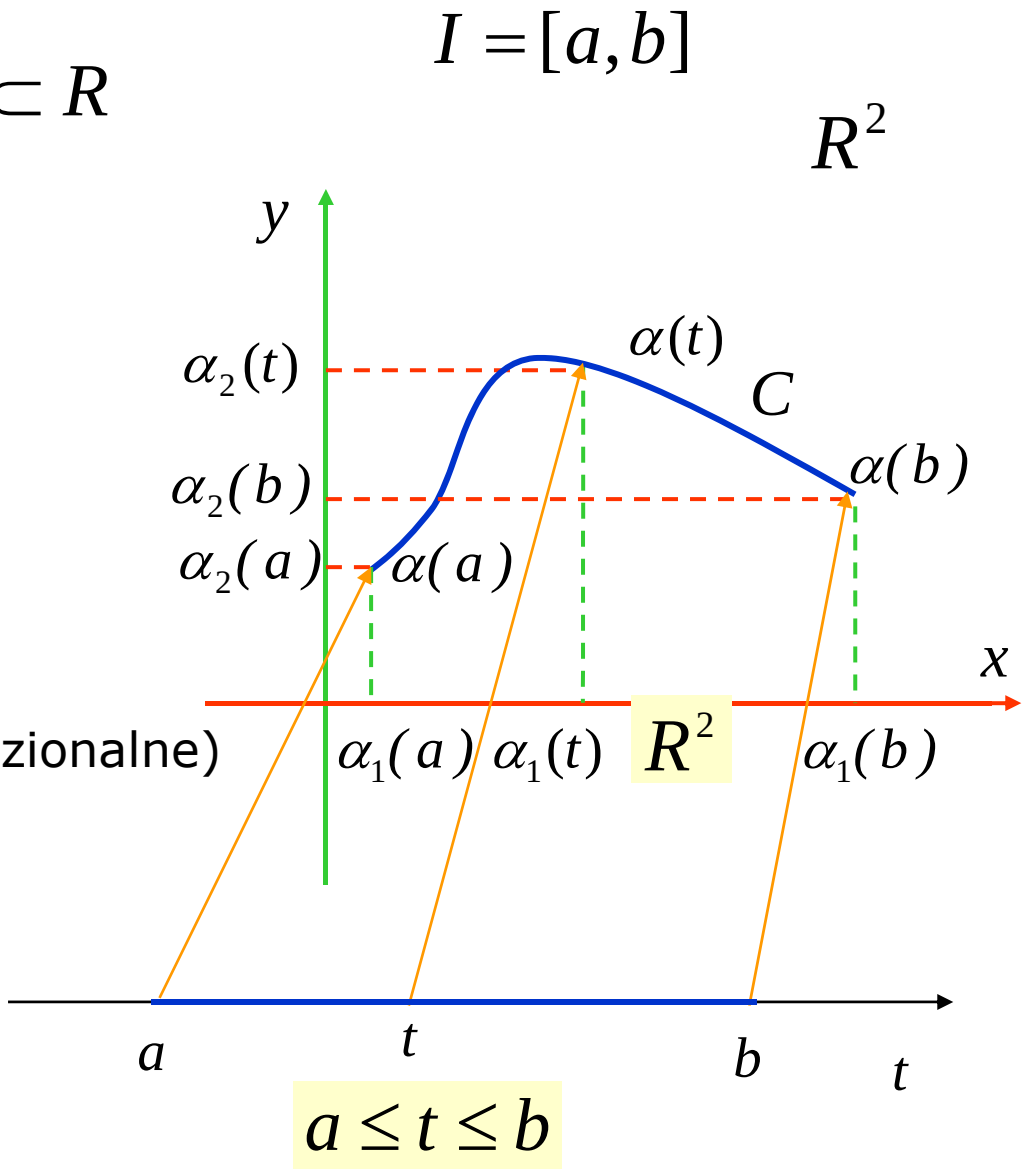
$I = (-\infty, a) \subset R$

Preslikavanje

$$\alpha : I \to R^2, \qquad t \in I$$

nekog intervala I (jednodimenzionalne) realne prave R u R^2 je linija (ili kriva) u ravni .

R – parametarska prava



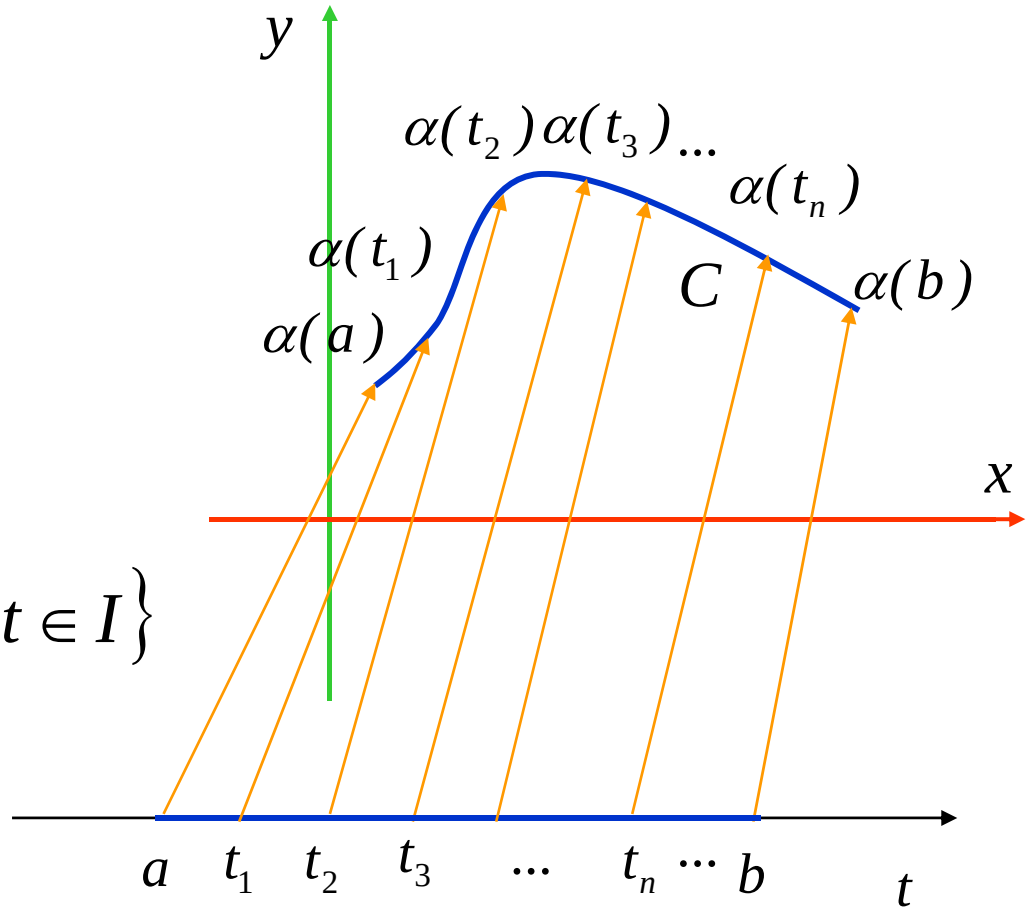
LINIJE (KRIVE) U RAVNI

$$I = [a,b] \subset R$$

$$\alpha : I \to R^2$$

$$\alpha(t) = (\alpha_1(t), \alpha_2(t))$$

$$t \in I$$



Skup tačaka

$$\{\alpha(t) \mid t \in I\} = \{(\alpha_1(t), \alpha_2(t)) \mid t \in I\}$$

je linija (ili kriva) u ravni.

LINIJE (KRIVE) U RAVNI

$$I = [a,b] \subset R$$

$$\alpha : I \to R^2$$

$$\alpha(t) = (\alpha_1(t), \alpha_2(t))$$

$$a \le t \le b$$

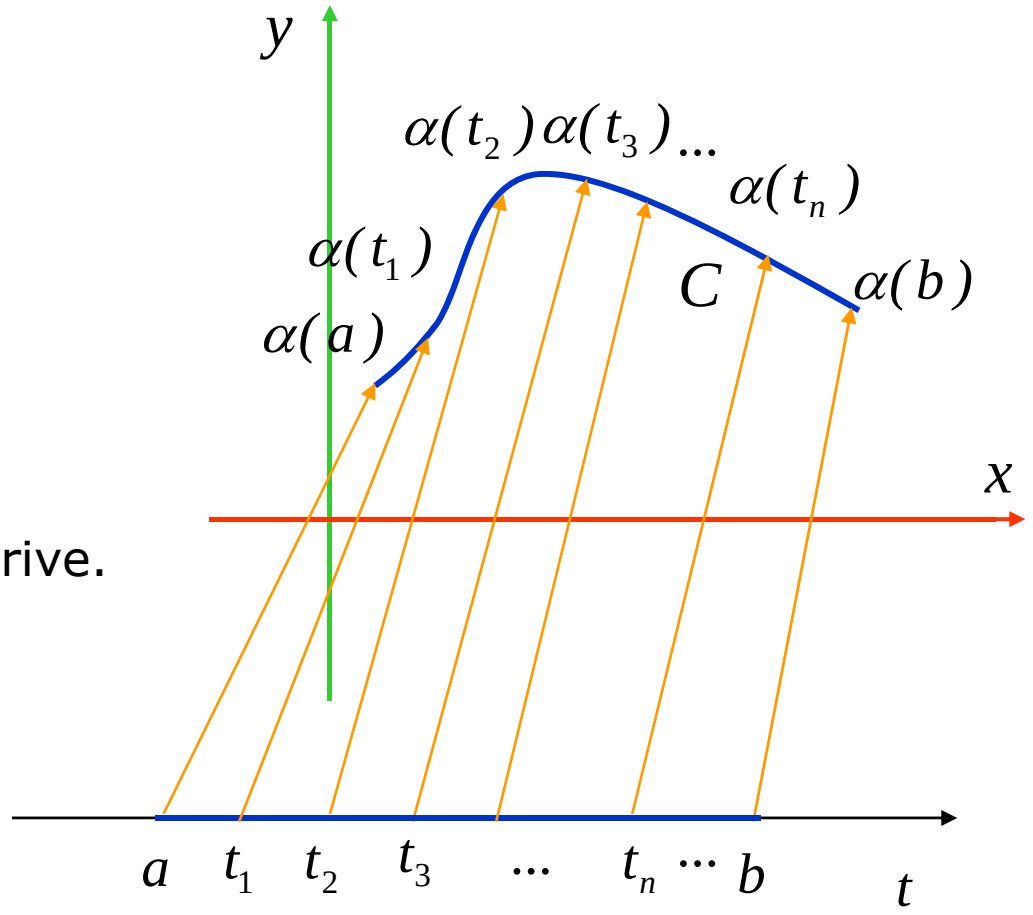
Jednačine

$$x = \alpha_1(t)$$

$$y = \alpha_2(t) \qquad a < t < b$$

su parametarske jednačine te krive.

Za krivu α se kaže da je zadata u parametarskom obliku.

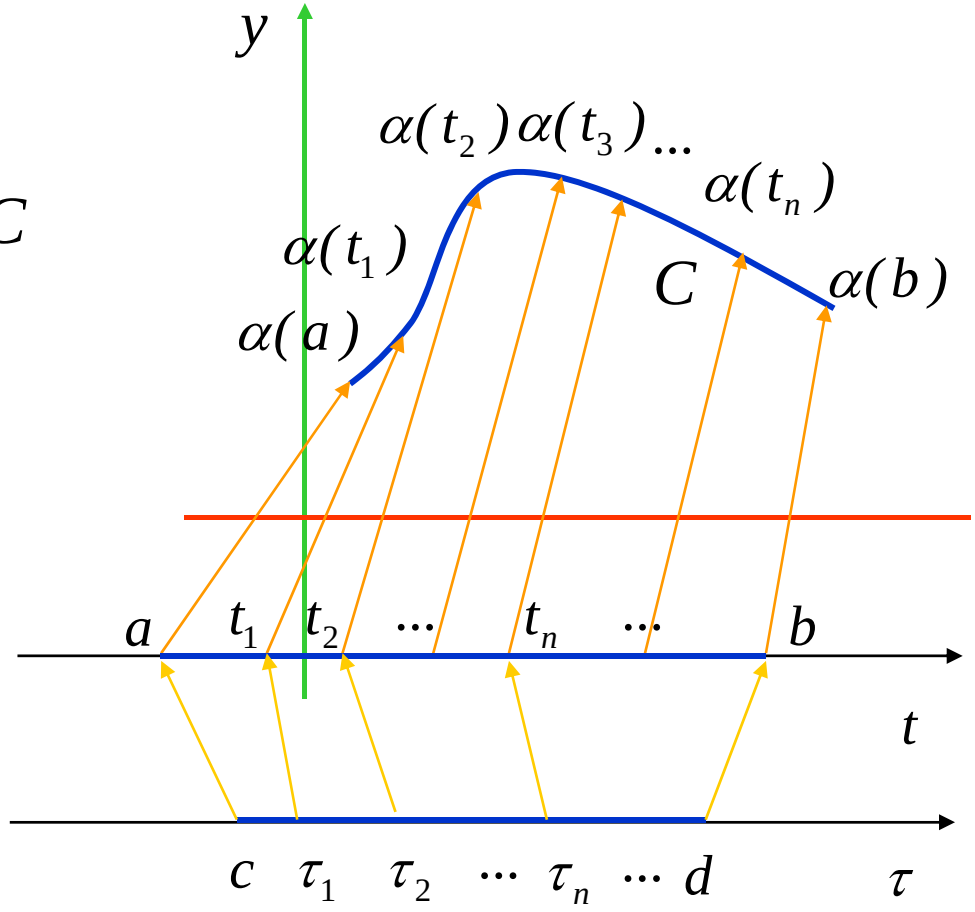


LINIJE (KRIVE) U RAVNI

Matematički prikaz krive linije C u ravni u parametarskom obliku

$$x=\alpha_1(t) \qquad a<t<b$$
$$y=\alpha_2(t)$$

nije jednoznačan.



LINIJE (KRIVE) U RAVNI

$$x=\alpha_1(t)$$
$$y=\alpha_2(t) \qquad a<t<b$$

Ukoliko je

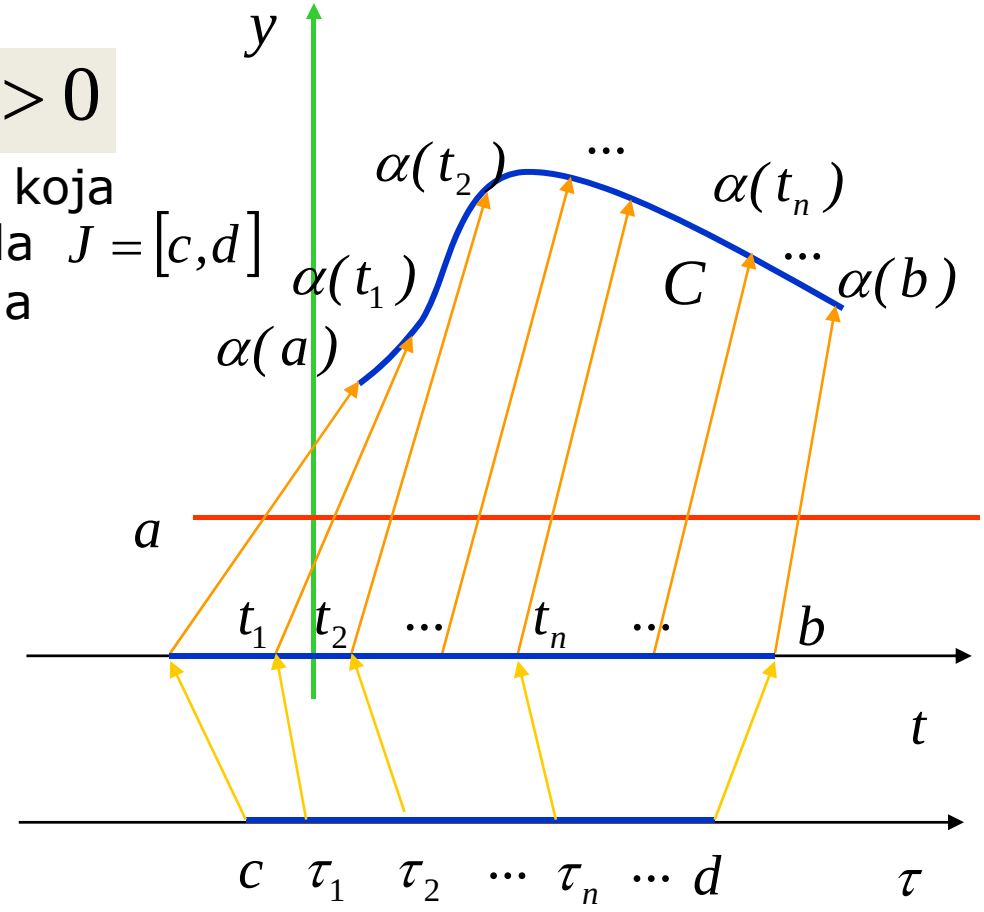
$$\varphi : J \to I \qquad \varphi'(\tau) > 0$$

Diferencijabilna rastuća funkcija koja vrši preslikavanje nekog intervala $J=[c,d]$ neke parametarske prave τ na interval $I=[a,b]$ tada su

$$x=\alpha_1(\varphi(\tau))$$
$$y=\alpha_2(\varphi(\tau)) \qquad c<\tau<d$$

takodje parametarske jednačine iste krive C .

Kaže se da je izvršena reparametrizacija.



LINIJE (KRIVE) U RAVNI

$$t = \varphi(\tau)$$

$$\alpha(t) = \beta(\tau)$$

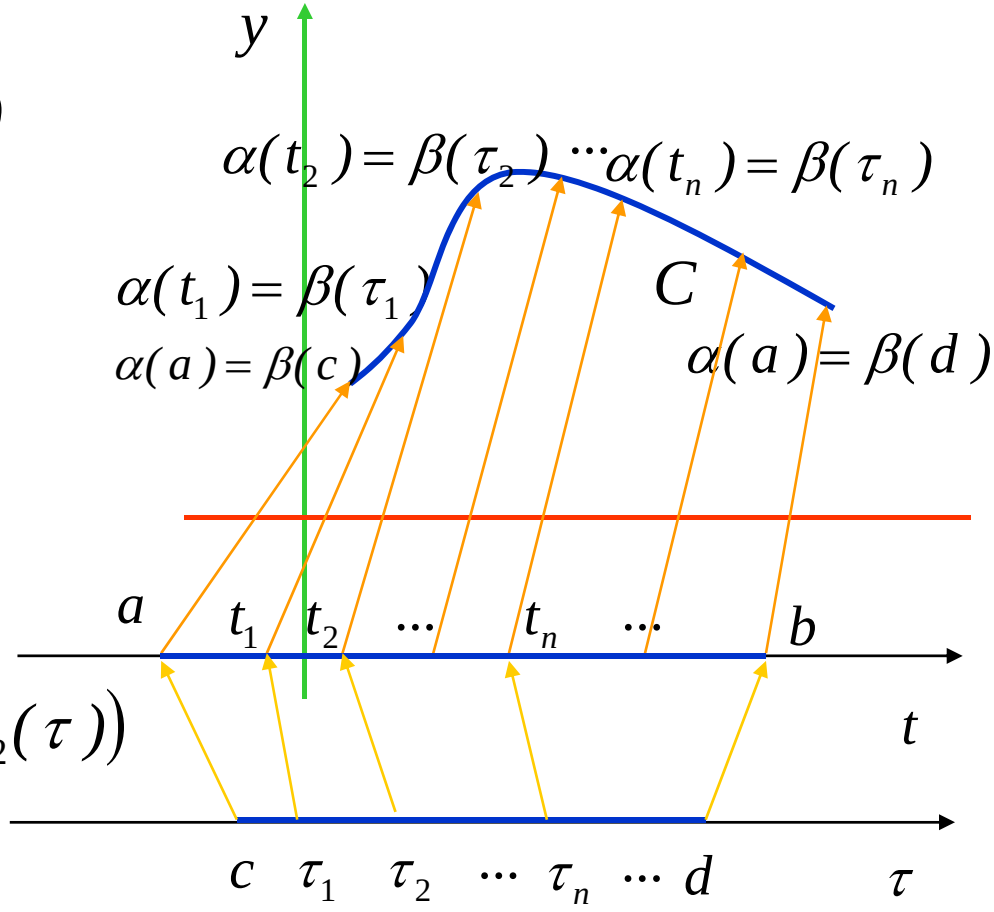
$$x = \alpha_1(t)$$
$$y = \alpha_2(t) \qquad a < t < b$$

$$t = \varphi(\tau)$$

$$x = \alpha_1(\varphi(\tau)) = \beta_1(\tau)$$
$$y = \alpha_2(\varphi(\tau)) = \beta_2(\tau) \qquad c < \tau < d$$

$$C : \alpha(t) = (\alpha_1(t), \alpha_2(t))$$

$$C : C(\varphi(\tau)) = \beta(\tau) = (\beta_1(\tau), \beta_2(\tau))$$



LINIJE (KRIVE) U RAVNI

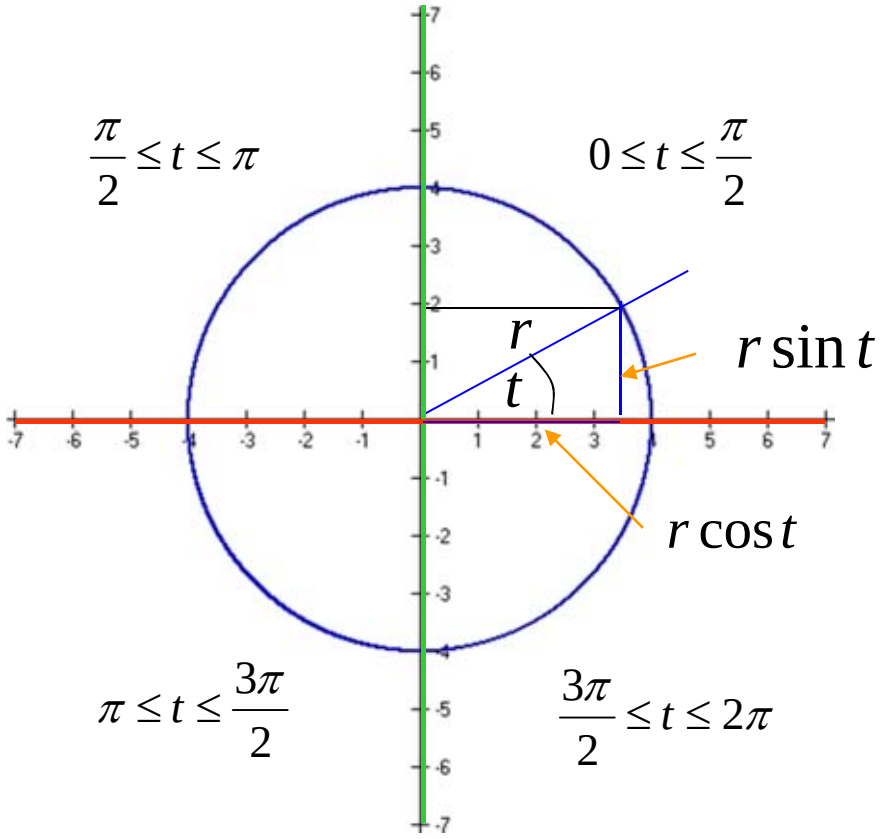
Parametarske jednačine kruga

$$C(0,0)$$

$$x = r\cos t$$

$$y = r\sin t$$

$$0 \le t \le 2\pi$$

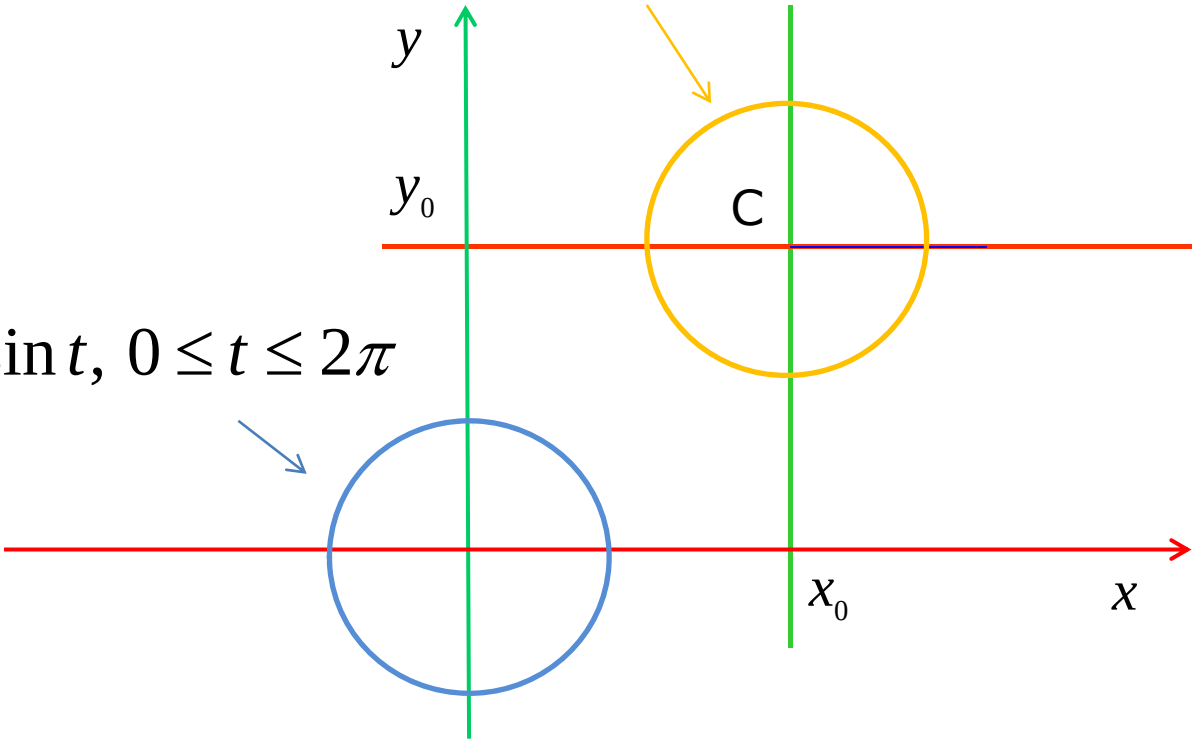


LINIJE (KRIVE) U RAVNI

$C(x_0, y_0)$

Parametarske jednačine kruga

$$x = x_0 + r\cos t,\ \ y = y_0 + r\sin t,\ 0 \le t \le 2\pi$$



$$x = r\cos t,\ \ y = r\sin t,\ 0 \le t \le 2\pi$$

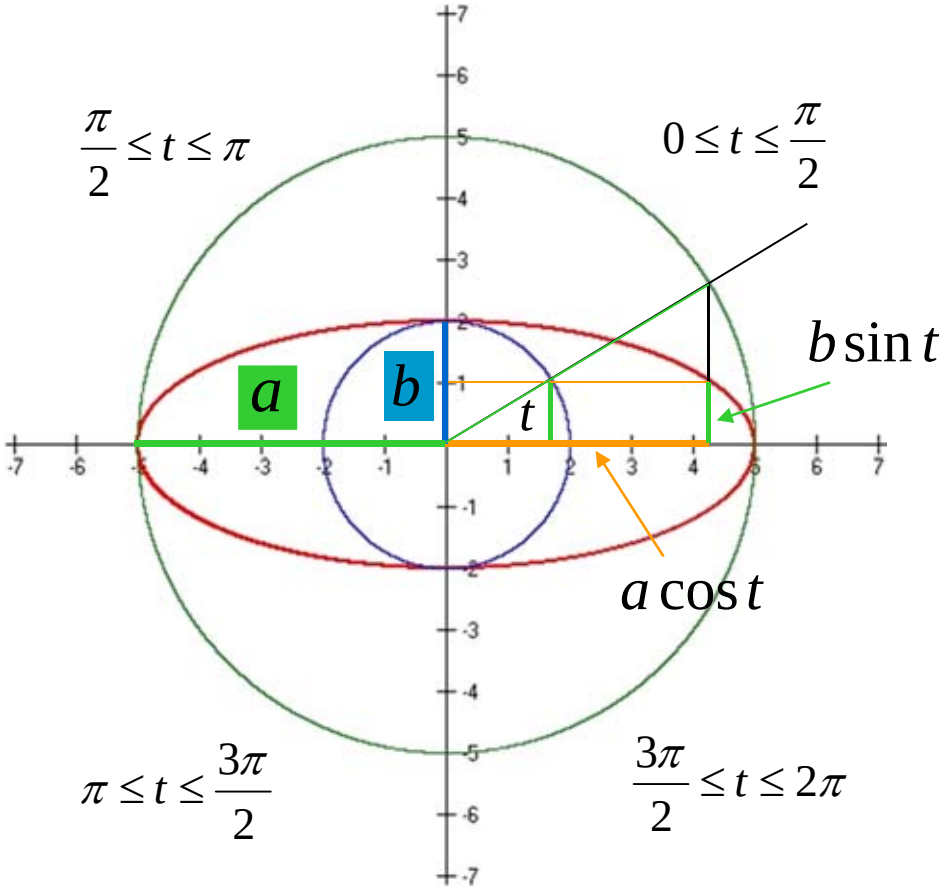
LINIJE (KRIVE) U RAVNI

Parametarske jednačine elipse

$$x = a\cos t$$

$$y = b\sin t$$

$$0 \le t \le 2\pi$$

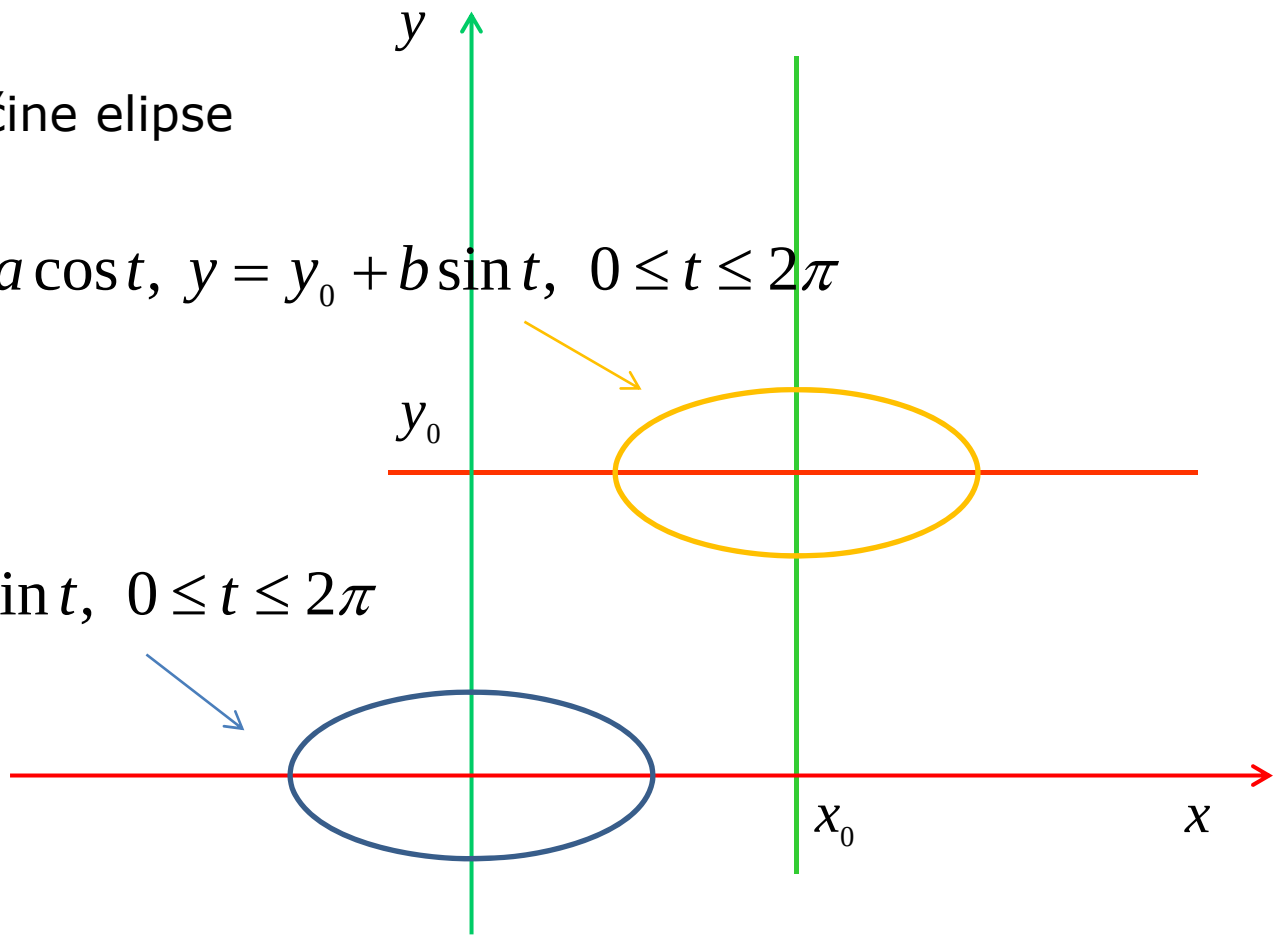


LINIJE (KRIVE) U RAVNI

Parametarske jednačine elipse

$$x = x_0 + a\cos t,\ y = y_0 + b\sin t,\ 0 \le t \le 2\pi$$

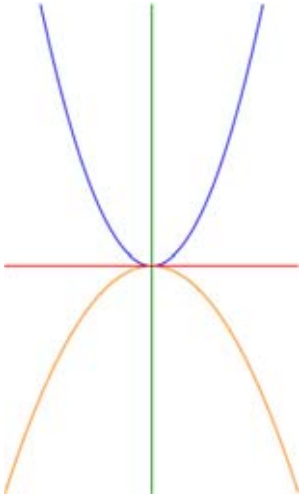
$$x = a\cos t,\ y = b\sin t,\ 0 \le t \le 2\pi$$



LINIJE (KRIVE) U RAVNI

Parametarske jednačine parabole

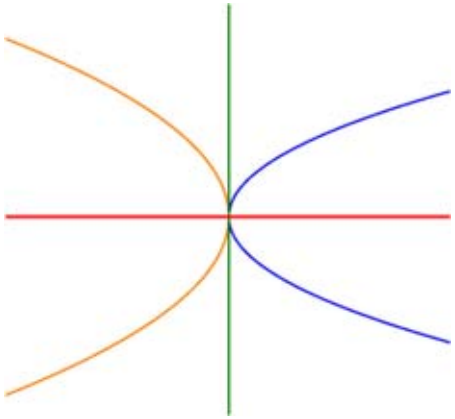
$$x^2 = 2py$$



$$x = t$$

$$y = \frac{1}{2p}t^2$$

$$y^2 = 2px$$



$$x = \frac{1}{2p}t^2$$

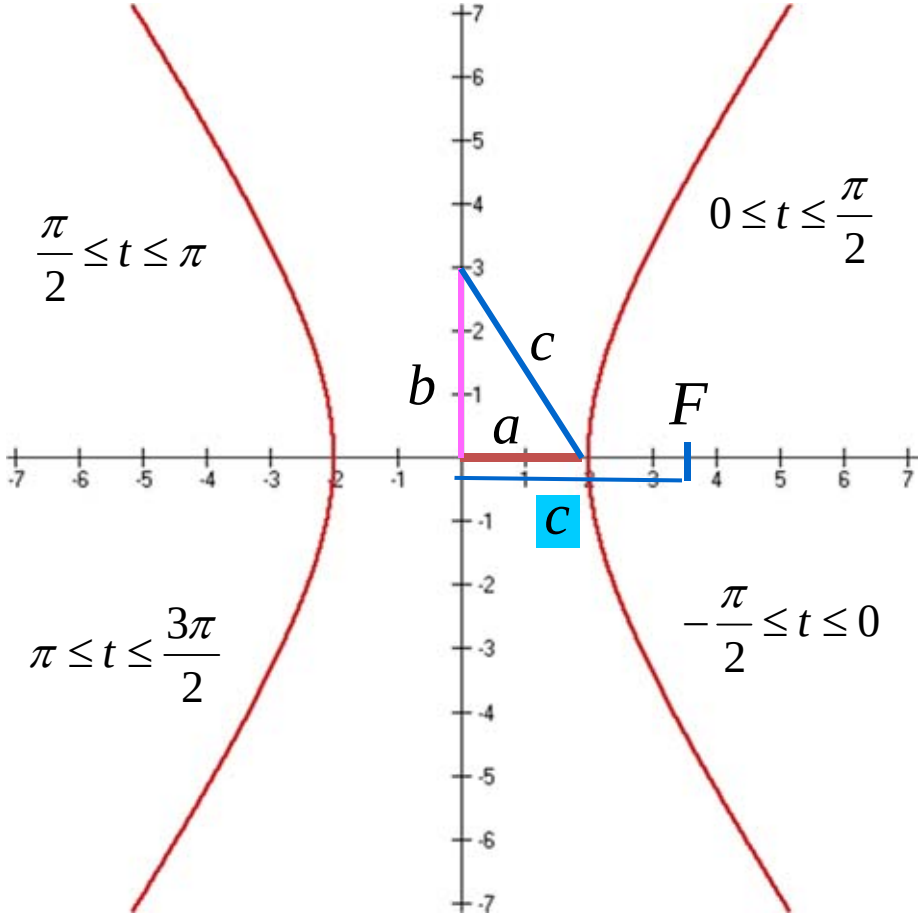
$$y = t$$

LINIJE (KRIVE) U RAVNI

Parametarske jednačine hiperbole

$$x = a\sec t \qquad -\frac{\pi}{2} \le t \le \frac{3\pi}{2}$$

$$y = b\tan t$$



LINIJE (KRIVE) U RAVNI

$$x = -a\cosh t$$
$$y = b\sinh t$$
$$t \in R$$

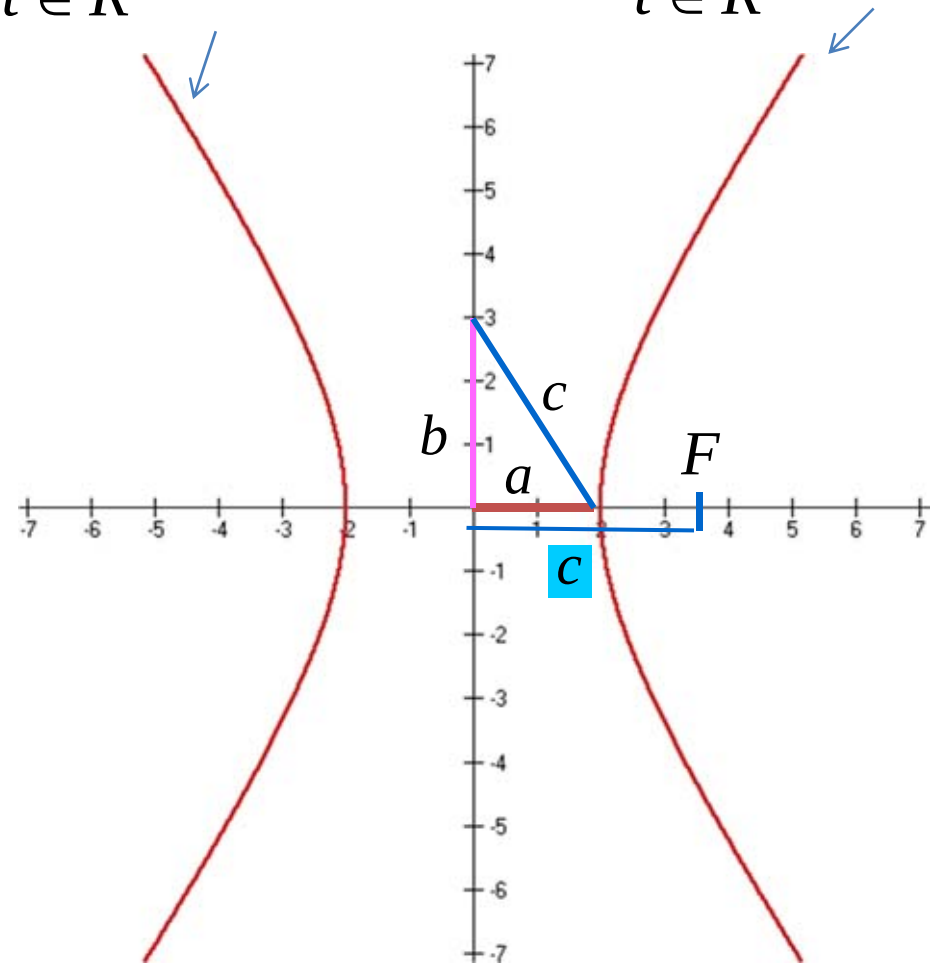
$$x = a\cosh t$$
$$y = b\sinh t$$
$$t \in R$$

Parametarske jednačine hiperbole

$$x = a\cosh t \qquad t \in R$$
$$y = b\sinh t$$

$$\sinh x = \frac{e^x - e^{-x}}{2} = \frac{e^{2x} - 1}{2e^x}$$

$$\cosh x = \frac{e^x + e^{-x}}{2} = \frac{e^{2x} + 1}{2e^x}$$



LINIJE (KRIVE) U RAVNI

$x = -2\cosh t$

$y = 3\sinh t$

$x = 2\cosh t$

$y = 3\sinh t$

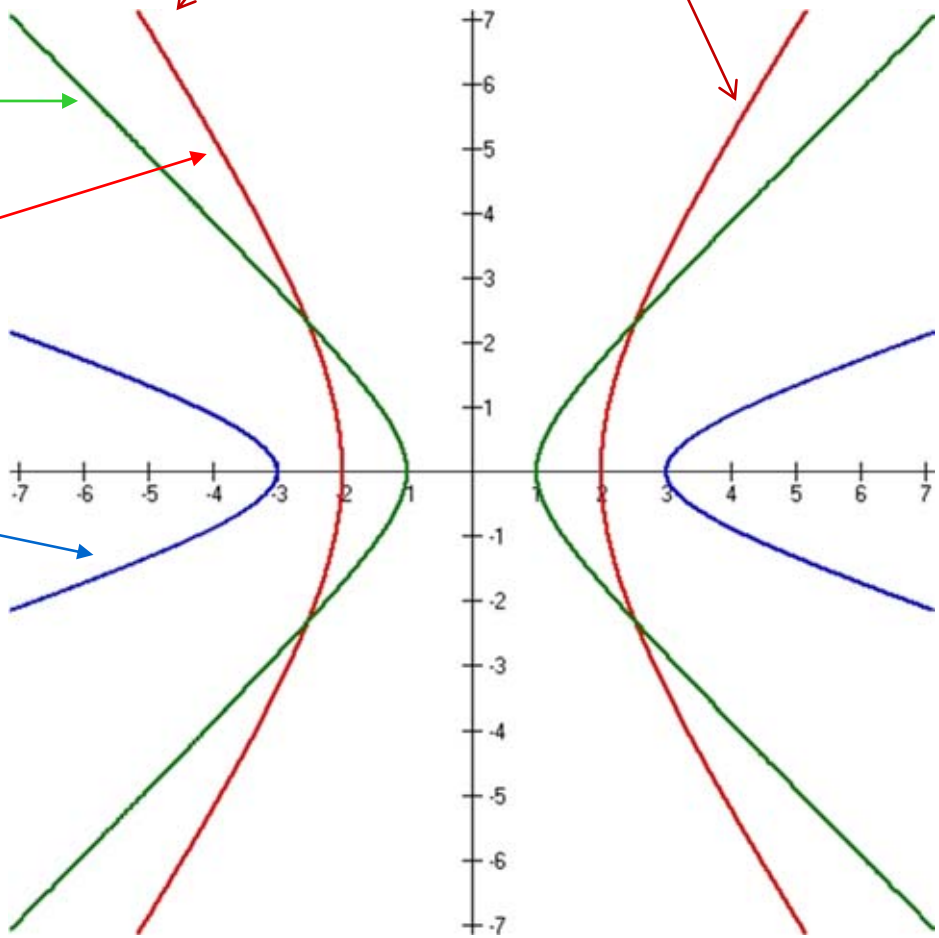
$x = a\sec t,\ y = b\tan t$

$x = \sec t,\ y = \tan t$

$x = 2\sec t,\ y = 3\tan t$

$x = 3\sec t,\ y = \tan t$

$$-\frac{\pi}{2} \le t \le \frac{3\pi}{2}$$



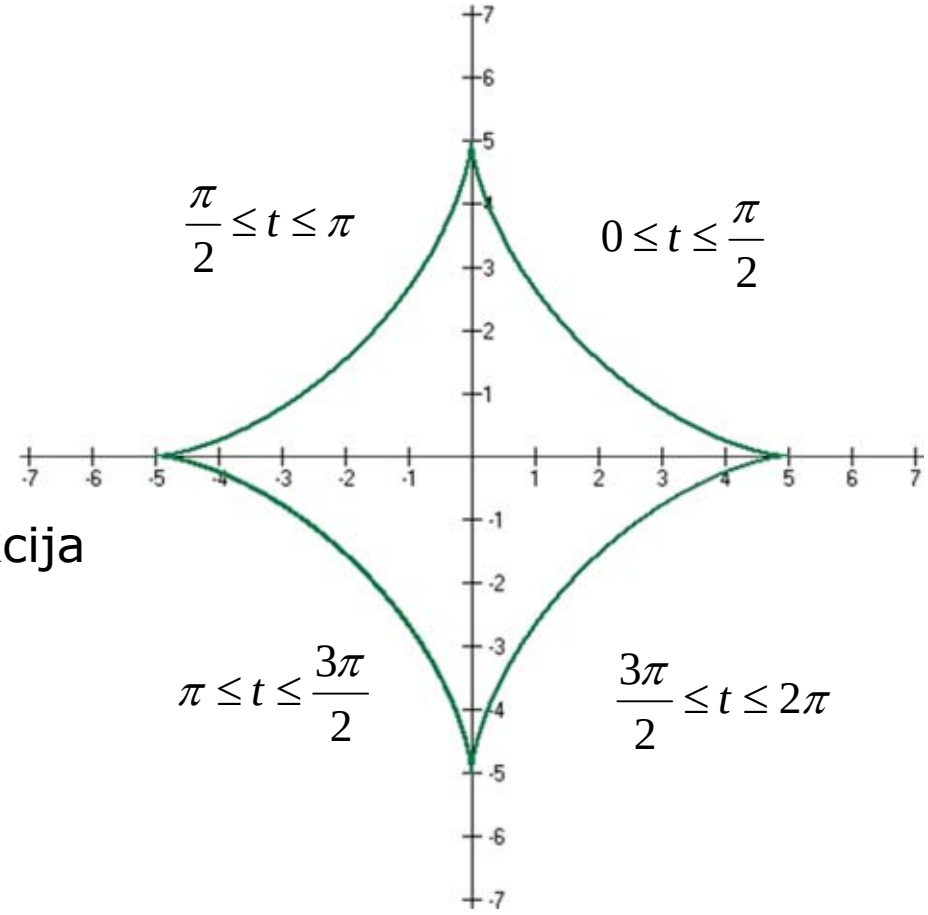
LINIJE (KRIVE) U RAVNI

Astroida - parametarske jednačine

$$x = a\cos^3 t$$

$$y = a\sin^3 t$$

$$0 \le t \le 2\pi$$



Astroida –implicitno zadata funkcija

$$x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^2$$

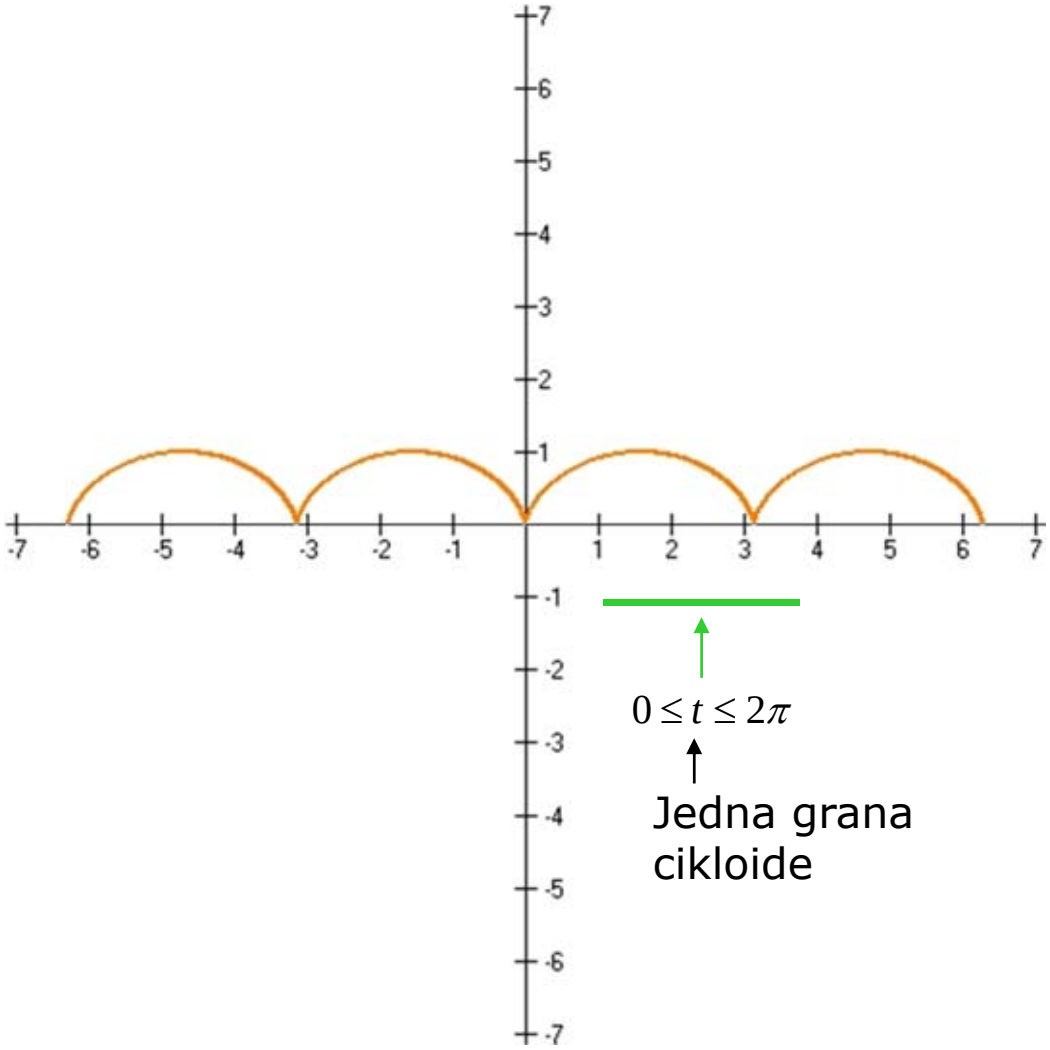
LINIJE (KRIVE) U RAVNI

Cikloida

$$x = a(t - \sin t)$$

$$y = a(1 - \cos t)$$

$$t \in R$$



LINIJE (KRIVE) U RAVNI

$y = t,\ x = t^2,\ -0.5 \le t \le 2.5$

$y = t^2,\ x = t,\ -2 \le t \le 2$

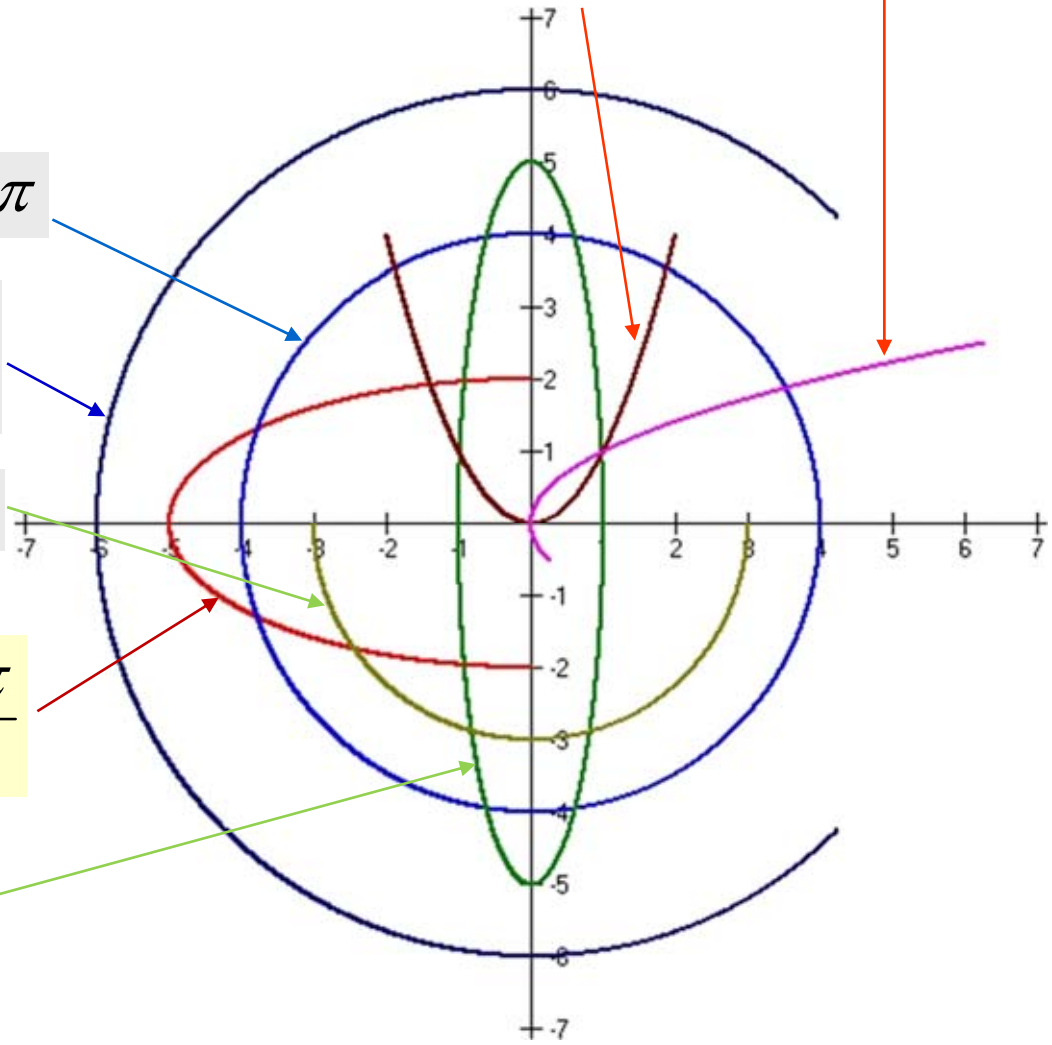
$x = 4\cos t,\ y = 4\sin t,\ 0 \le t \le 2\pi$

$x = 6\cos t,\ y = 6\sin t,\ \frac{\pi}{4} \le t \le \frac{7\pi}{4}$

$x = 3\cos t,\ y = 3\sin t,\ \pi \le t \le 2\pi$

$x = 5\cos t,\ y = 2\sin t,\ \frac{\pi}{2} \le t \le \frac{3\pi}{2}$

$x = \cos t,\ y = 5\sin t,\ 0 \le t \le 2\pi$



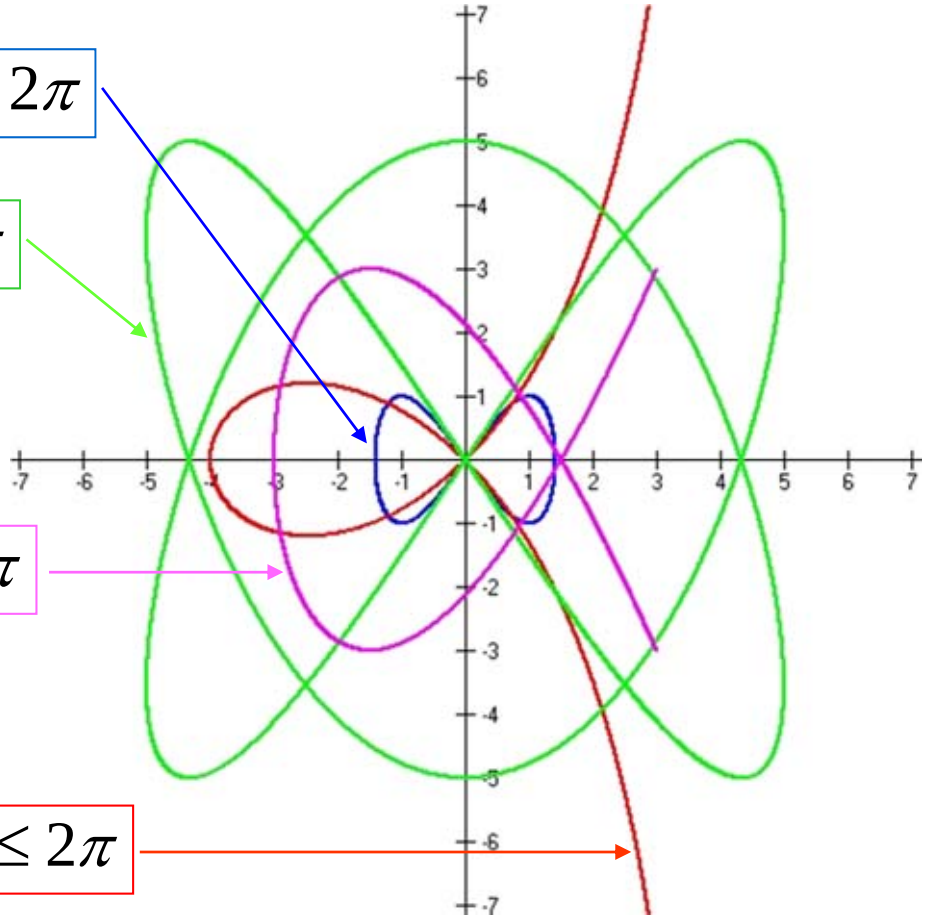
LINIJE (KRIVE) U RAVNI

$x = \sin t + \cos t,\ y = \cos 2t,\ 0 \le t \le 2\pi$

$x = 5\sin 2t,\ y = 5\sin 3t,\ 0 \le t \le 2\pi$

$x = 3\cos 2t,\ y = 3\cos 3t,\ 0 \le t \le 2\pi$

$x = 4\sin t,\ y = tg\, t(1+\sin t),\ 0 \le t \le 2\pi$



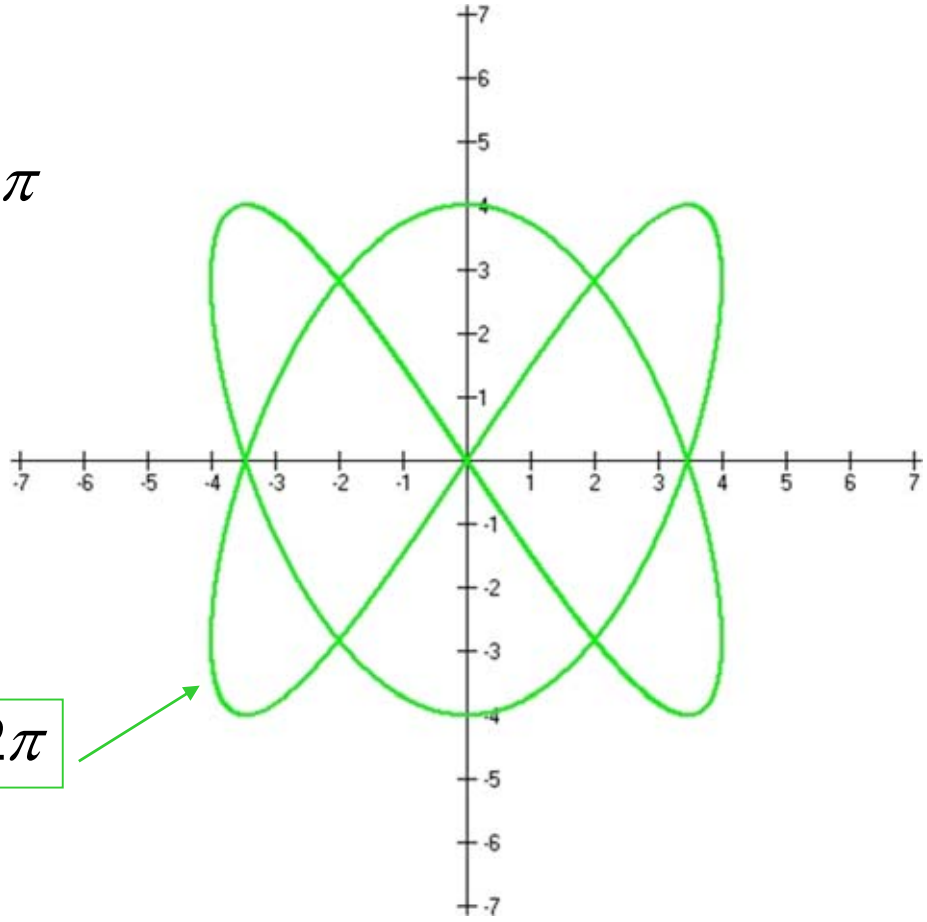
LINIJE (KRIVE) U RAVNI

$$x = a\sin 2t,\ y = b\sin 3t,\ 0 \le t \le 2\pi$$

$$a = b$$

$$a = b = 4$$

$$x = 4\sin 2t,\ y = 4\sin 3t,\ 0 \le t \le 2\pi$$



LINIJE (KRIVE) U RAVNI

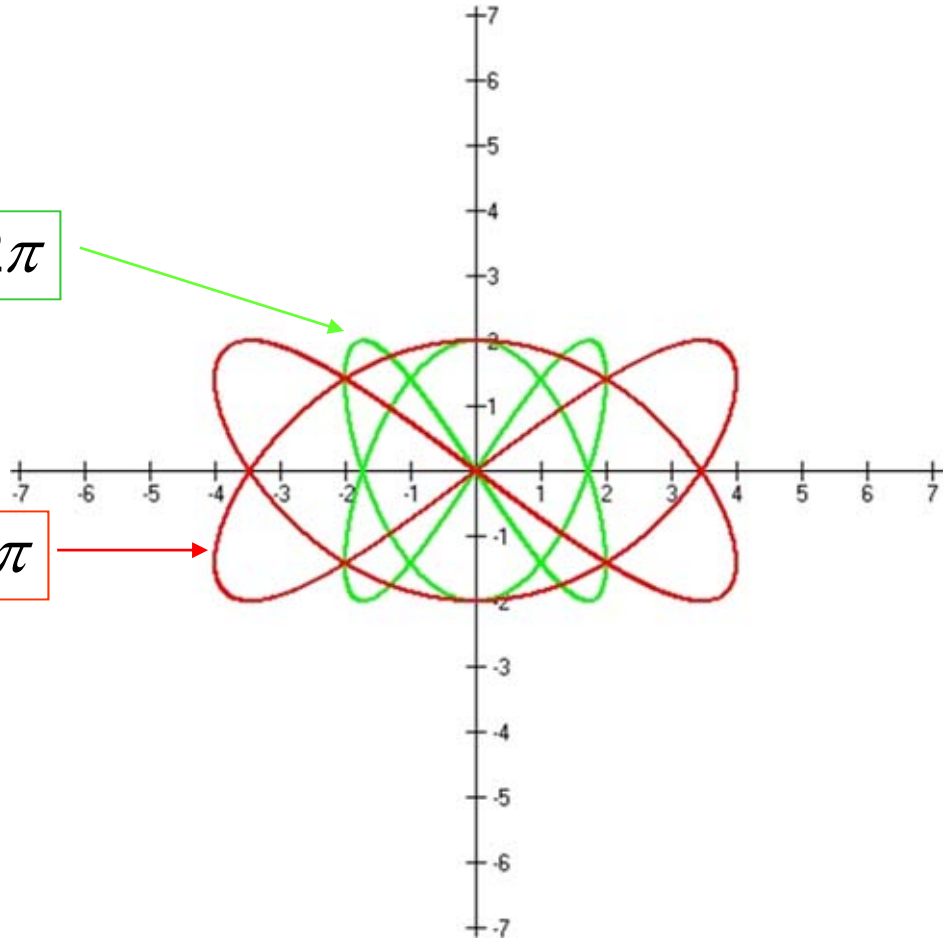
$$x = a\sin 2t,\ y = b\sin 3t,\ 0 \le t \le 2\pi$$

$$a = b = 2$$

$$x = 2\sin 2t,\ y = 2\sin 3t,\ 0 \le t \le 2\pi$$

$$a \neq b \qquad a = 4, \qquad b = 2$$

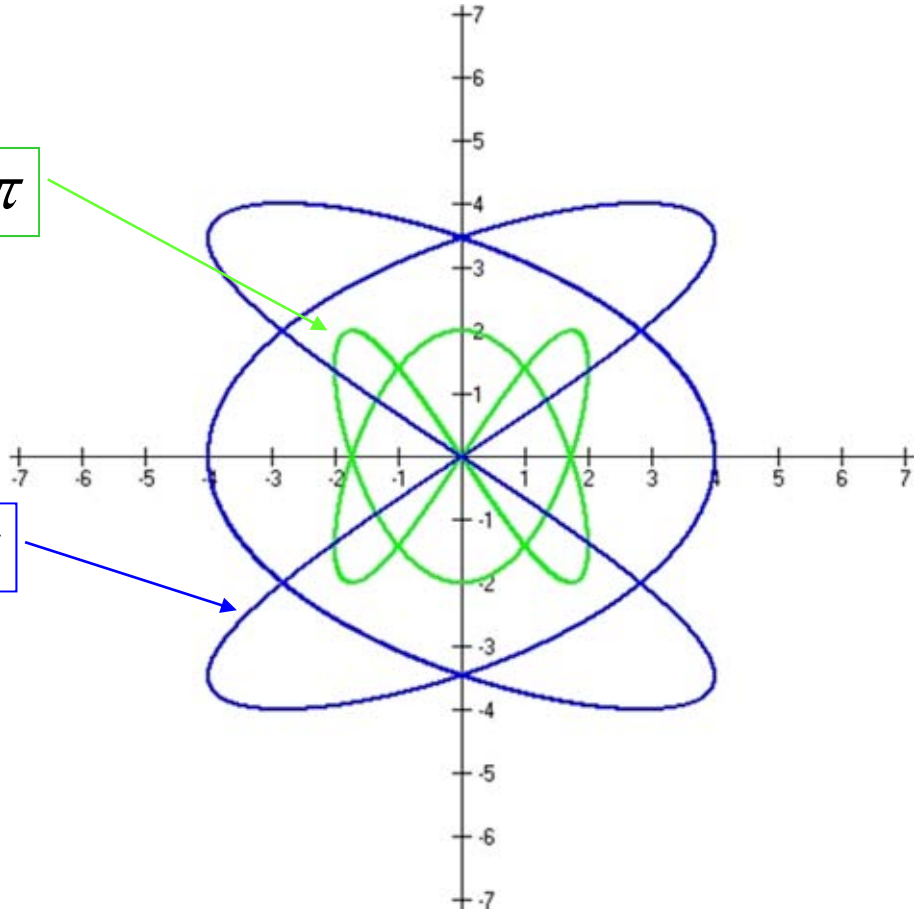
$$x = 4\sin 2t,\ y = 2\sin 3t,\ 0 \le t \le 2\pi$$



LINIJE (KRIVE) U RAVNI

$$x = 2\sin 2t,\ y = 2\sin 3t,\ 0 \le t \le 2\pi$$

$$x = 4\sin 3t,\ y = 4\sin 2t,\ 0 \le t \le 2\pi$$



LINIJE (KRIVE) U RAVNI

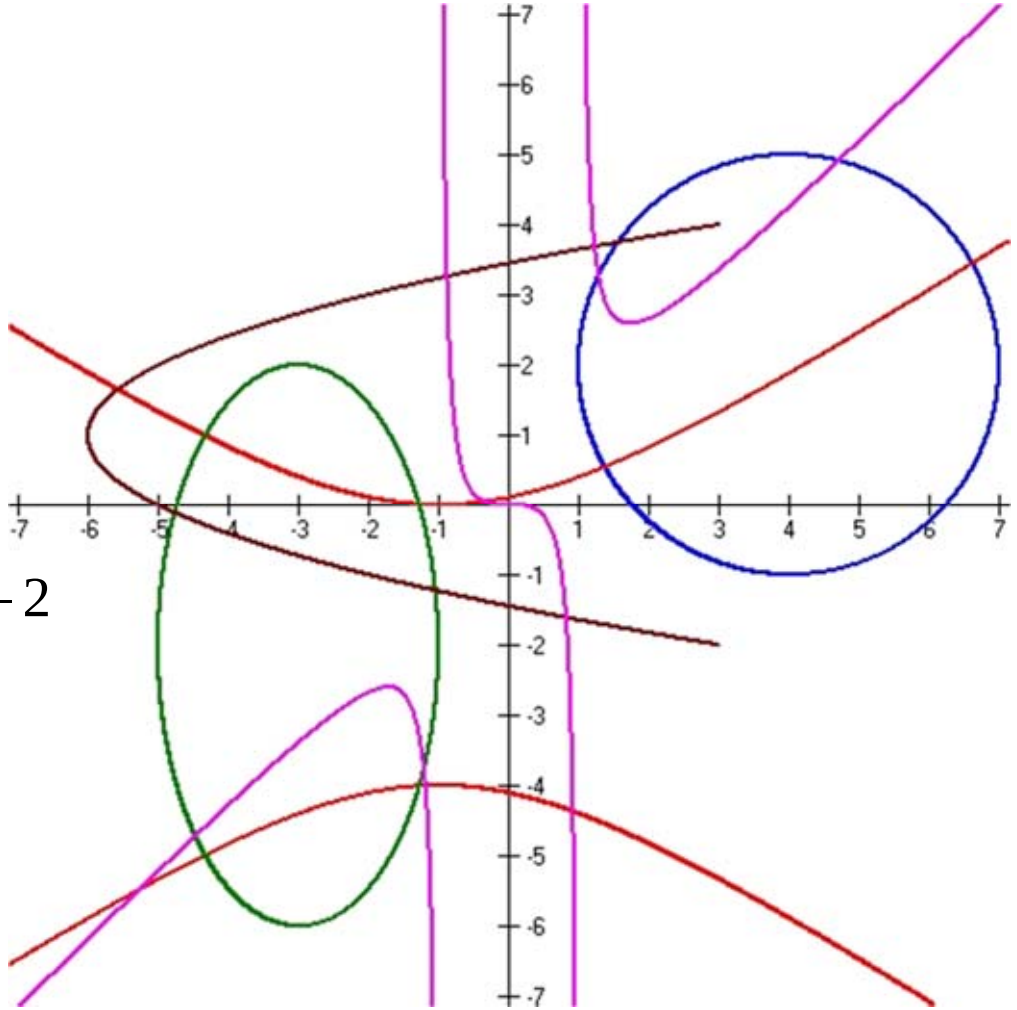
krug $x = 3\cos t + 4, y = 3\sin t + 2$

elipsa $x = 2\cos t - 3, y = 4\sin t - 2$

hiperbola $x = 3\tan t - 1, y = 2\sec t - 2$

parabola $x = t^2 - 6, y = t + 1$

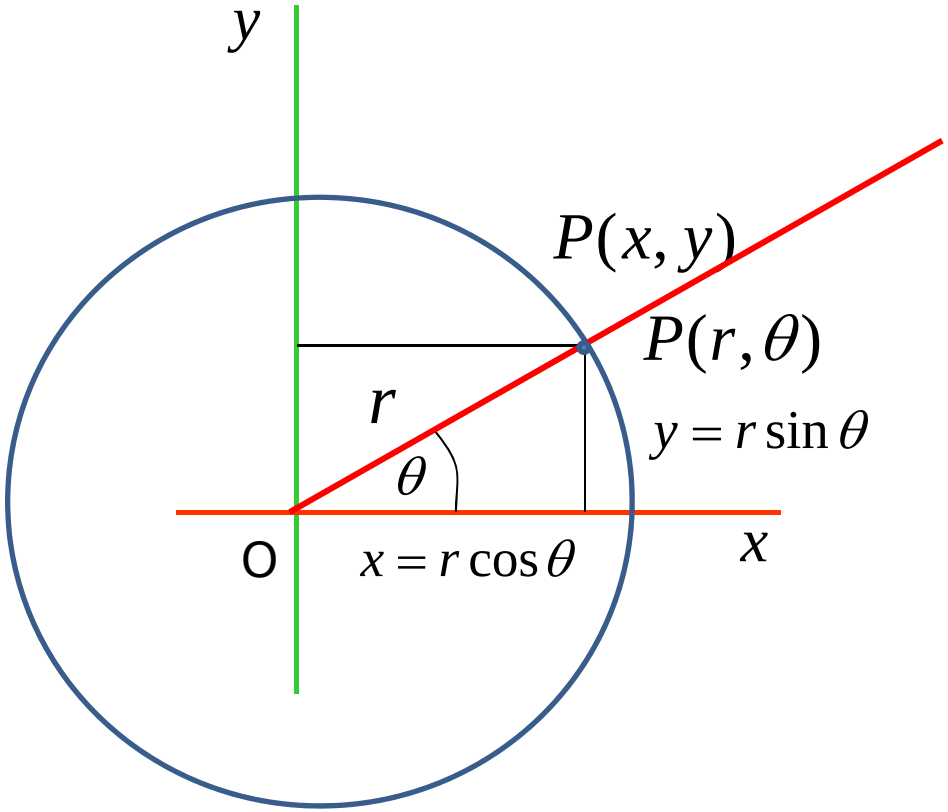
funkcija $y = \frac{x^3}{x^2 - 1}$



LINIJE (KRIVE) U RAVNI

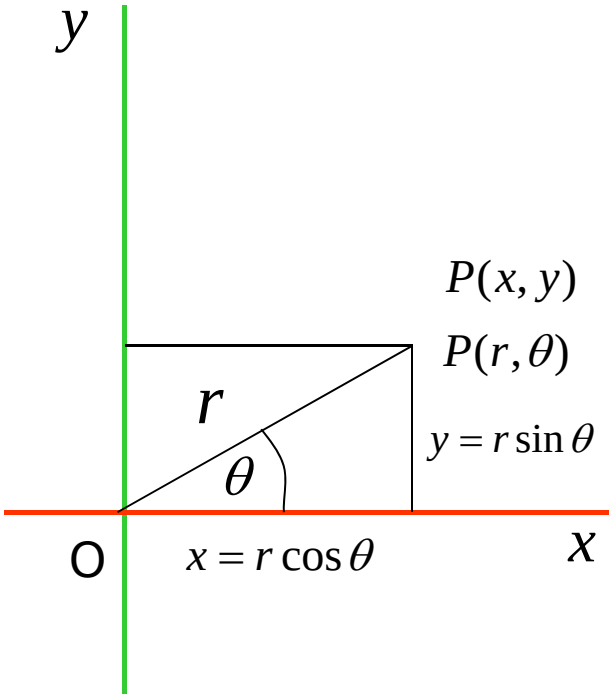
Polarni koordinatni sistem

$$x = r\cos\theta$$
$$y = r\sin\theta$$



LINIJE (KRIVE) U RAVNI

Polarni koordinatni sistem



Odnos

Dekartovih i polarnih koordinata

$$x = r\cos\theta$$

$$y = r\sin\theta$$

Kriva u polarnim koordinatama

$$r = \varphi(\theta)$$

Prelazak na parametarski oblik

$$x = r\cos\theta = \varphi(\theta)\cos\theta$$

$$y = r\sin\theta = \varphi(\theta)\sin\theta$$

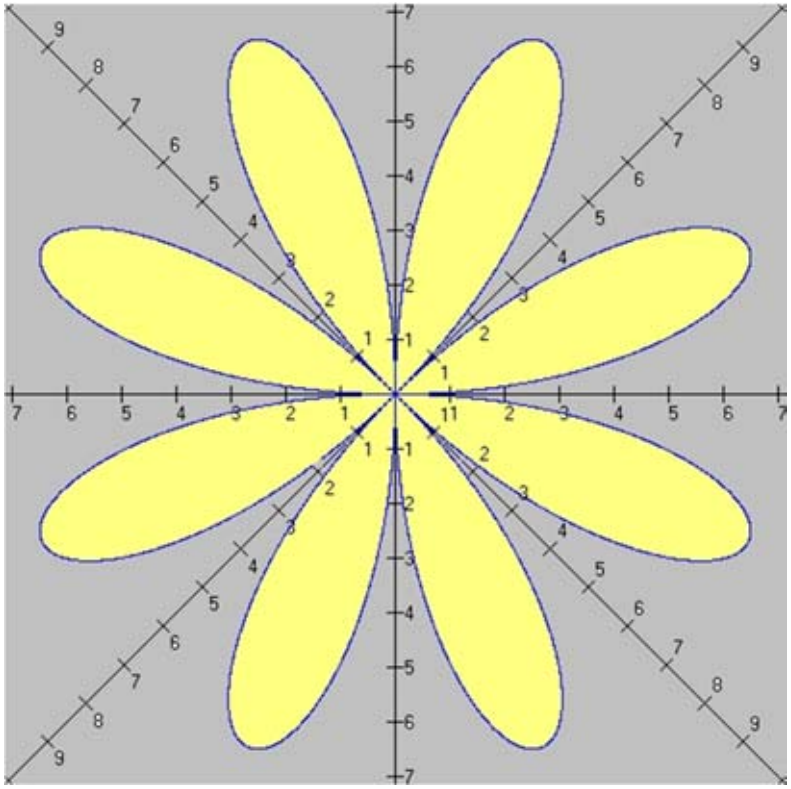
LINIJE (KRIVE) U RAVNI

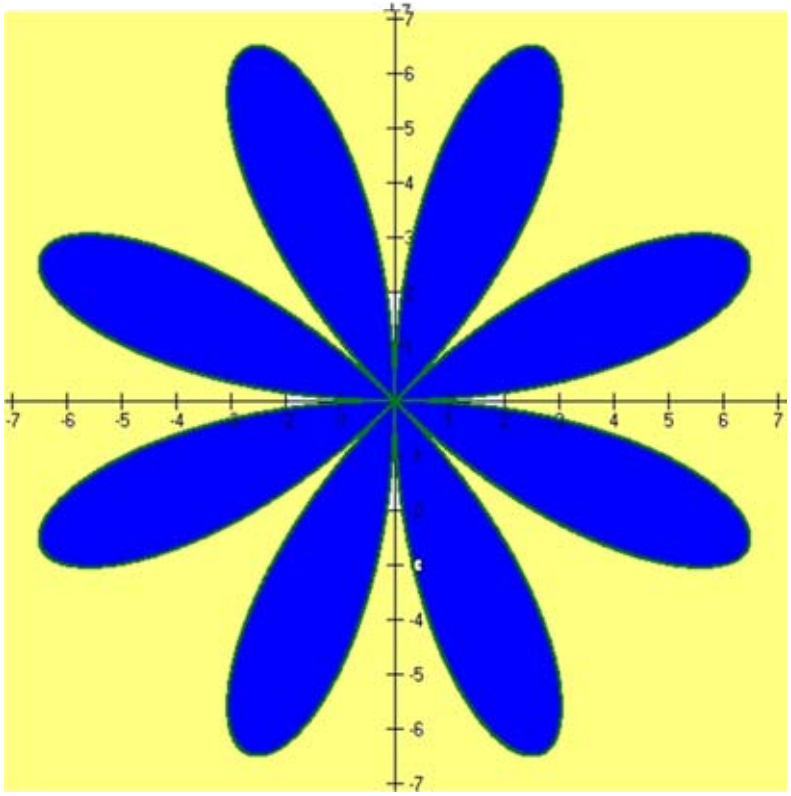
$$x = 7\sin 4\theta \cos\theta$$

$$y = 7\sin 4\theta \sin\theta$$

$$0 \le \theta \le 2\pi$$

$$r = 7\sin 4\theta,\ 0 \le \theta \le 2\pi$$





LINIJE (KRIVE) U RAVNI

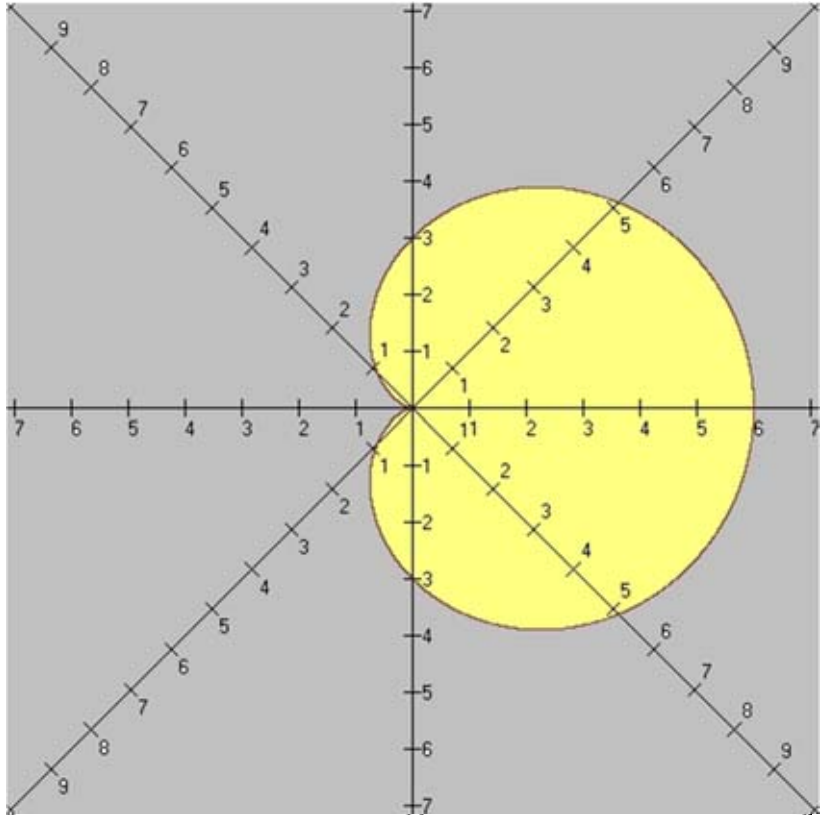
kardioida

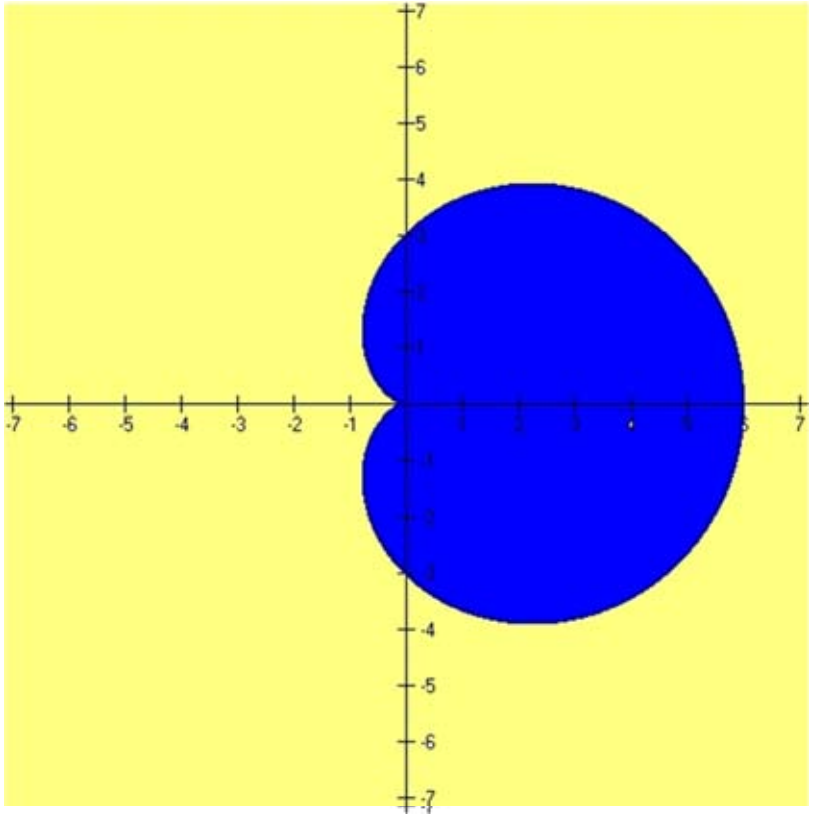
$$r = 3(1+\cos\theta),\ 0 \le \theta \le 2\pi$$

$$x = 3(1+\cos t)\cos t$$

$$y = 3(1+\cos t)\sin t$$

$$0 \le t \le 2\pi$$





LINIJE (KRIVE) U RAVNI

Arhimedova spirala

$$r = a\theta$$

$$x = at\cos t$$

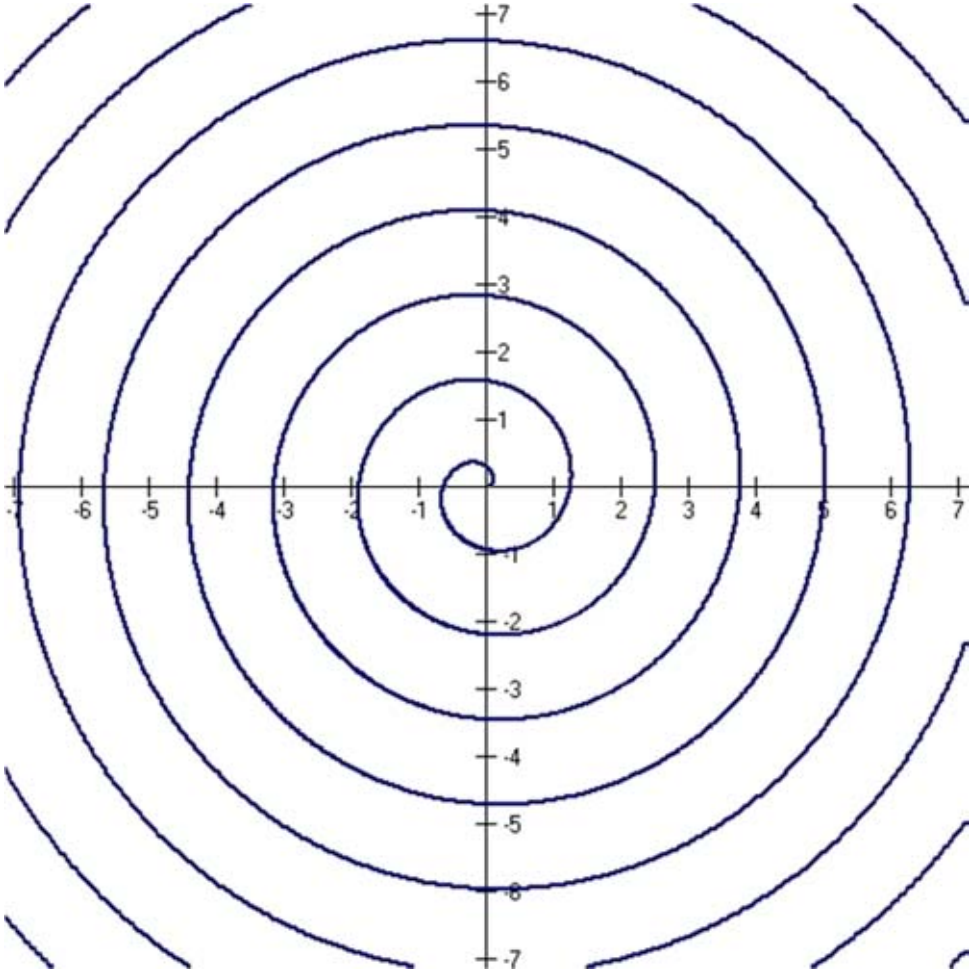
$$y = at\sin t$$

Grafički prikaz

$$x = 0.2t\cos t$$

$$y = 0.2t\sin t$$

$$0 \le t \le 20\pi$$



LINIJE (KRIVE) U RAVNI

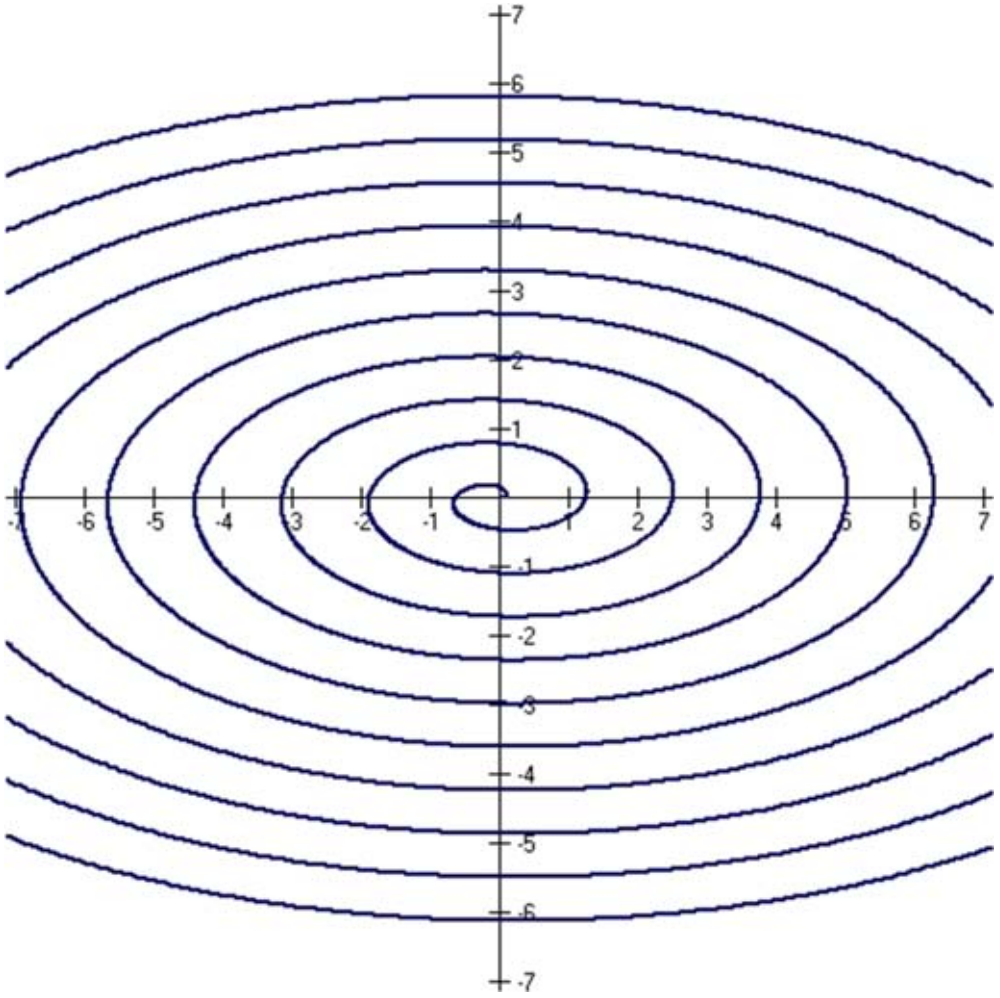
Modifikovana

Arhimedova spirala

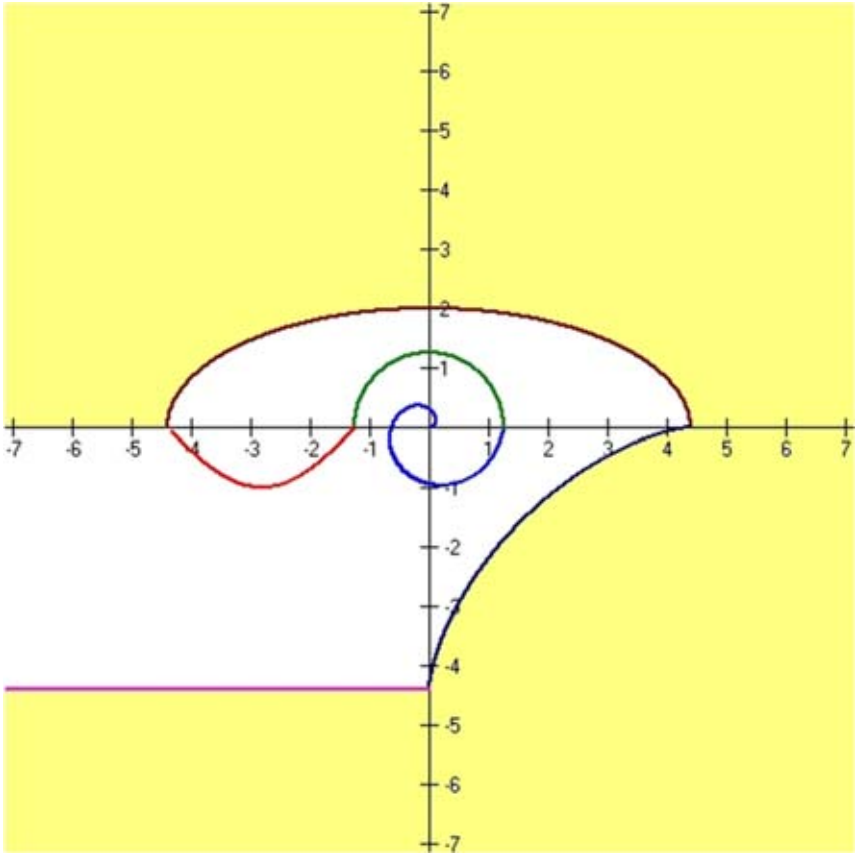
$$x = 0.2t\cos t$$

$$y = 0.1t\sin t$$

$$0 \le t \le 20\pi$$



LINIJE (KRIVE) U RAVNI



$$\begin{array}{l|l} x = 0.2t\cos t & x = 0.4\pi\cos t \\ y = 0.2t\sin t & y = 0.4\pi\sin t \\ 0 \le t \le 2\pi & 0 \le t \le \pi \end{array}$$

$$y = \sin(x + 0.4\pi), \quad -1.4\pi \le x \le -0.4\pi$$

$$\begin{array}{l|l} x = 1.4\pi\cos t & x = 1.4\pi\cos^3 t \\ y = 2\sin t & y = 1.4\pi\sin^3 t \\ 0 \le t \le \pi & \frac{3\pi}{2} \le t \le 2\pi \end{array}$$

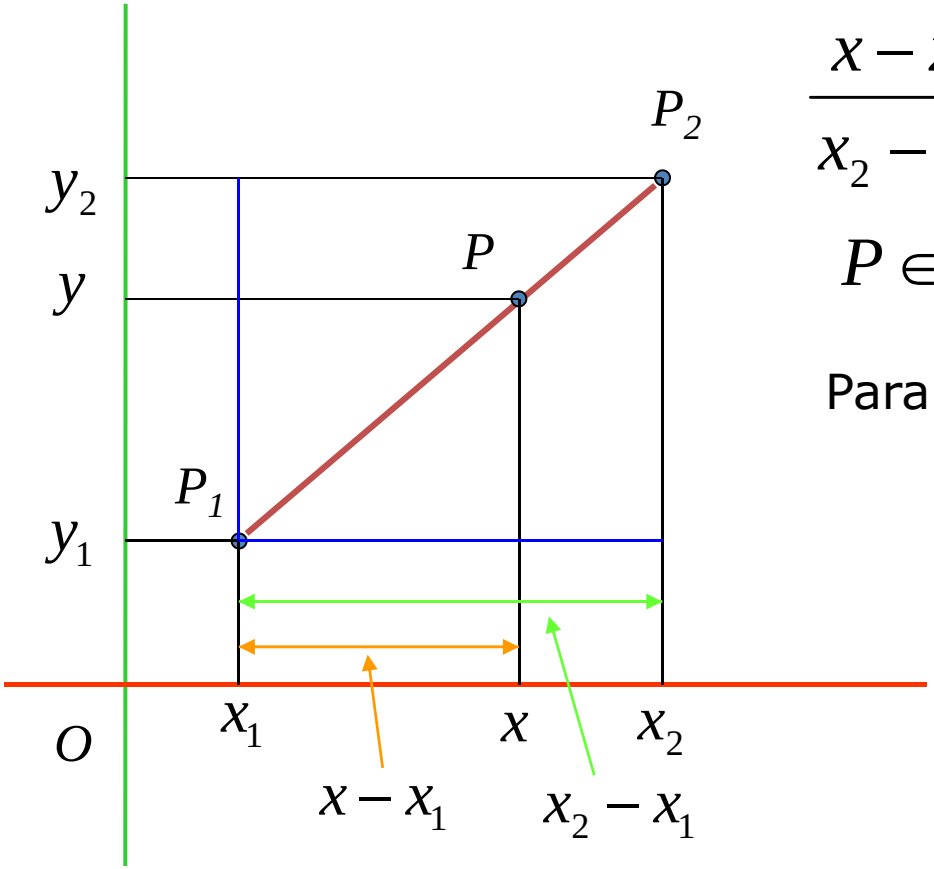
$$y = -1.4\pi, \; -8 \le x \le 0$$

LINIJE U RAVNI

prava linija

$$P_1(x_1, y_1)$$

$$P_2(x_2, y_2)$$



$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{d(P_1, P)}{d(P_1, P_2)} = t$$

$$P \in P_1P_2 \Leftrightarrow 0 \le t \le 1$$

Parametarske jednačine duži P_1P_2

$$x = x_1 + t(x_2 - x_1)$$

$$y = y_1 + t(y_2 - y_1)$$

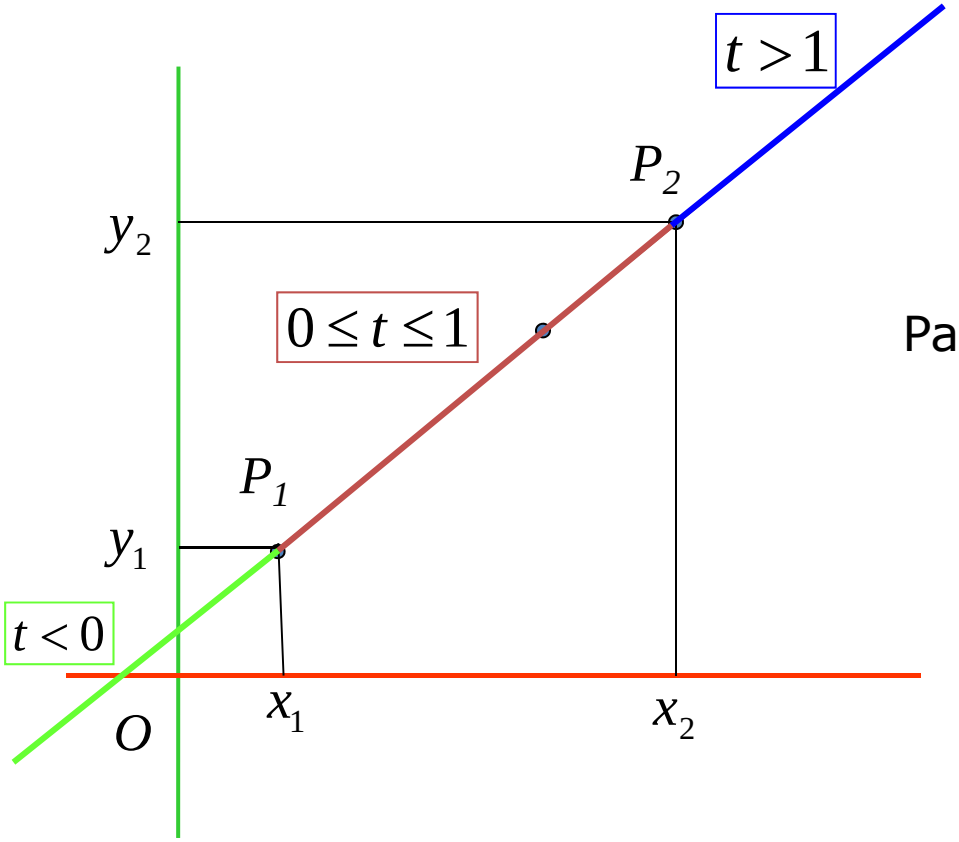
$$0 \le t \le 1$$

LINIJE U RAVNI

prava linija

$$P_1(x_1, y_1)$$

$$P_2(x_2, y_2)$$



$$\frac{d(P_1, P)}{d(P_1, P_2)} = t$$

Parametarske jednačine prave P_1P_2

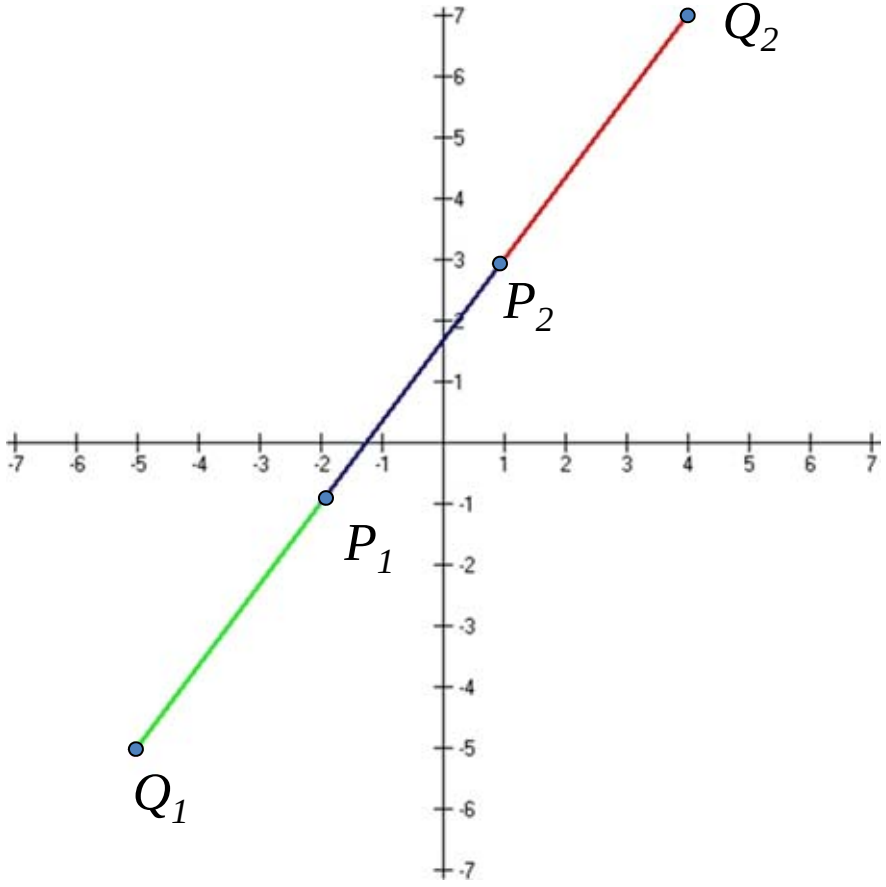
$$x = x_1 + t(x_2 - x_1)$$

$$y = y_1 + t(y_2 - y_1)$$

$$t \in R$$

LINIJE U RAVNI

prava linija



$$x = -2 + 3t$$

$$y = -1 + 4t$$

P_1P_2 — $0 \le t \le 1$

P_2Q_2 — $1 \le t \le 2$

Q_1P_1 — $-1 \le t \le 0$

P_1Q_2 — $0 \le t \le 2$

Q_1P_2 — $-1 \le t \le 1$

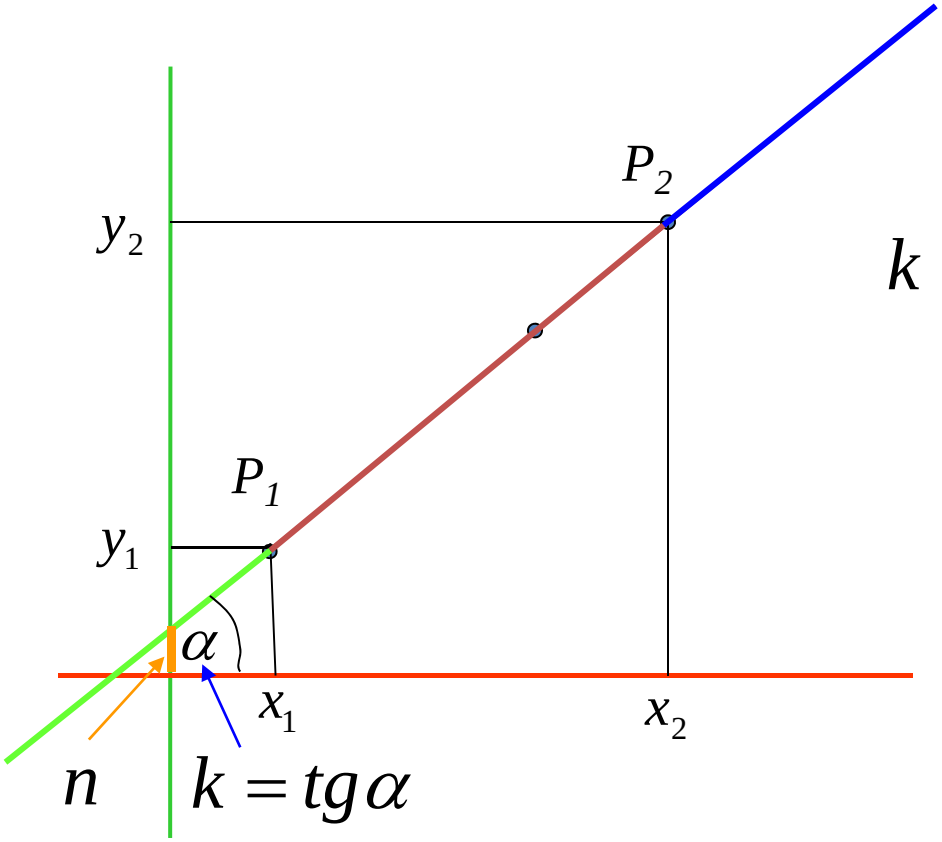
LINIJE U RAVNI

prava linija

$$P_1(x_1, y_1)$$

$$P_2(x_2, y_2)$$

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}$$



$$k = \frac{y_2 - y_1}{x_2 - x_1} \qquad n = y_1 - \frac{y_2 - y_1}{x_2 - x_1} x_1$$

$$y = k\,x + n$$

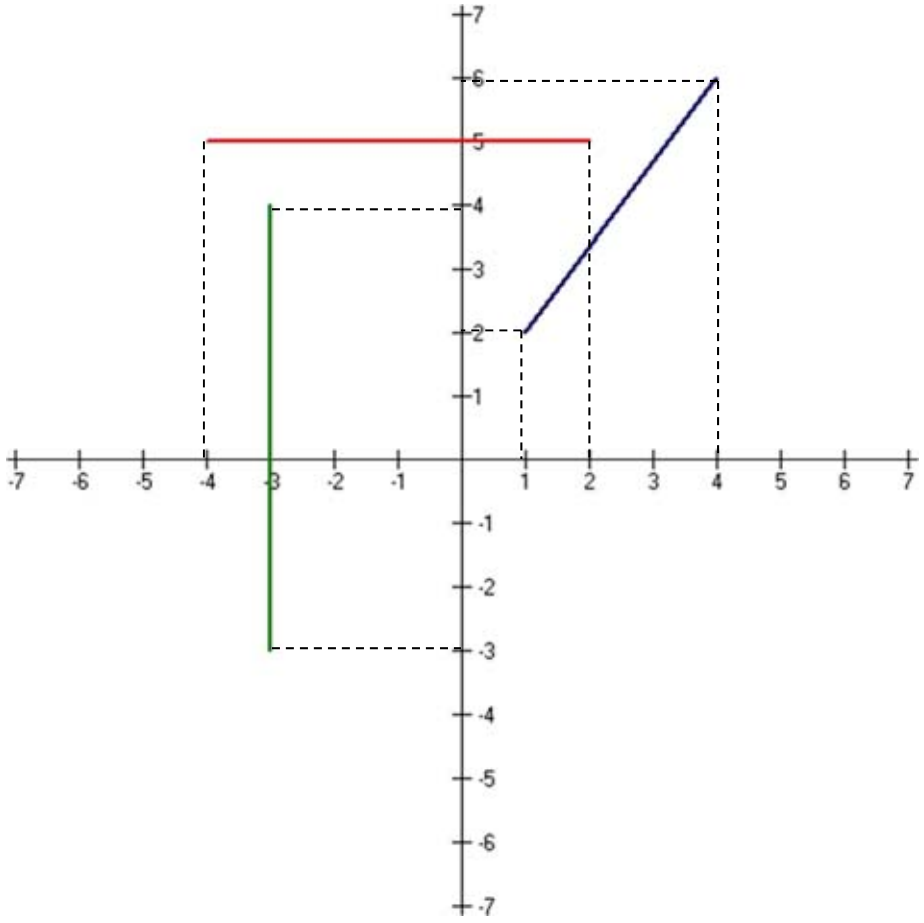
$x \in R$ — prava

$x_1 \le x \le x_2$ — duž P_1P_2

LINIJE U RAVNI

prava linija

$$x = x_1 + t(x_2 - x_1)$$
$$y = y_1 + t(y_2 - y_1)$$
$$0 \le t \le 1$$



(1,2), (4,6)
$$x = 1 + 3t$$
$$y = 2 + 4t$$
$$0 \le t \le 1$$

(–4,5), (2,5)
$$x = -4 + 6t$$
$$y = 5$$
$$0 \le t \le 1$$

(–3,–3), (–3,4)
$$x = -3$$
$$y = -3 + 7t$$
$$0 \le t \le 1$$