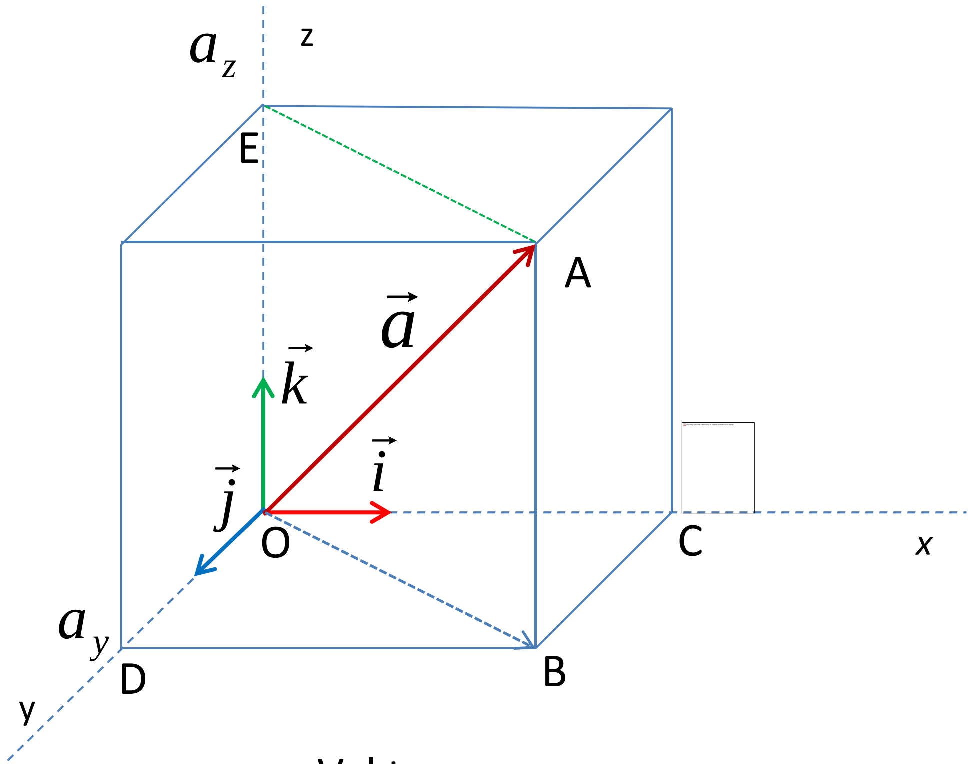
ANALITIČKA GEOMETRIJA U PROSTORU

VEKTOR POLOŽAJA TAČKE

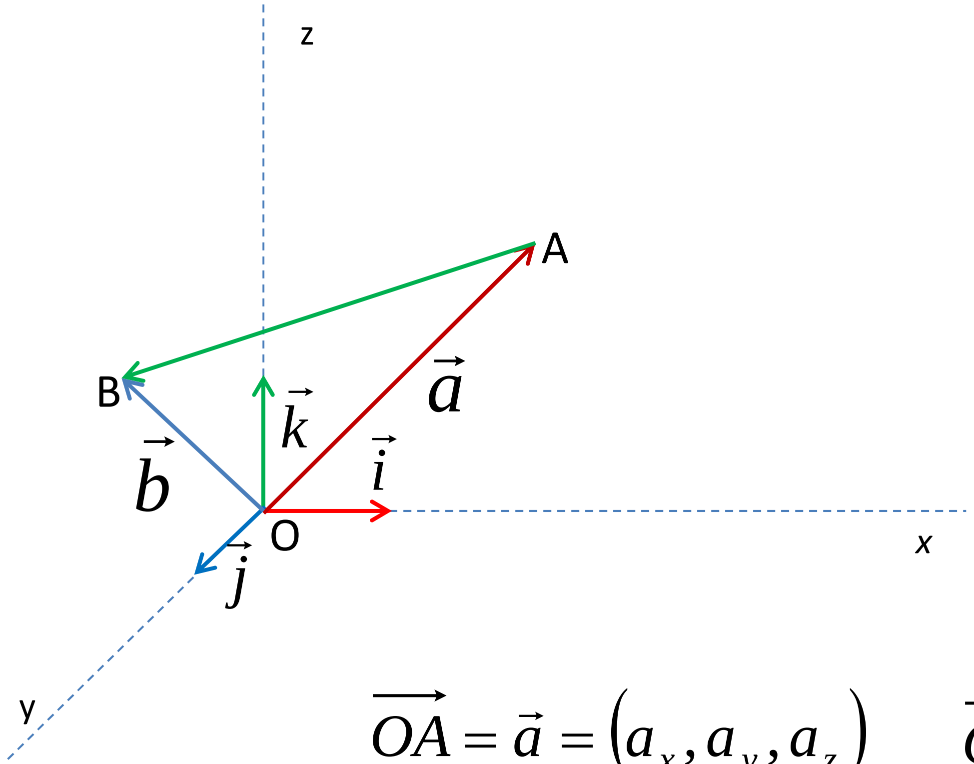


Vektor

$$\vec{a} = \overrightarrow{OA} = (a_x, a_y, a_z) = a_x\vec{i} + a_y\vec{j} + a_z\vec{k}$$

je vektor položaja tačke $A(a_x, a_y, a_z)$.

KOORDINATE VEKTORA ODREDJENOG POČETNOM I ZAVRŠNOM TAČKOM

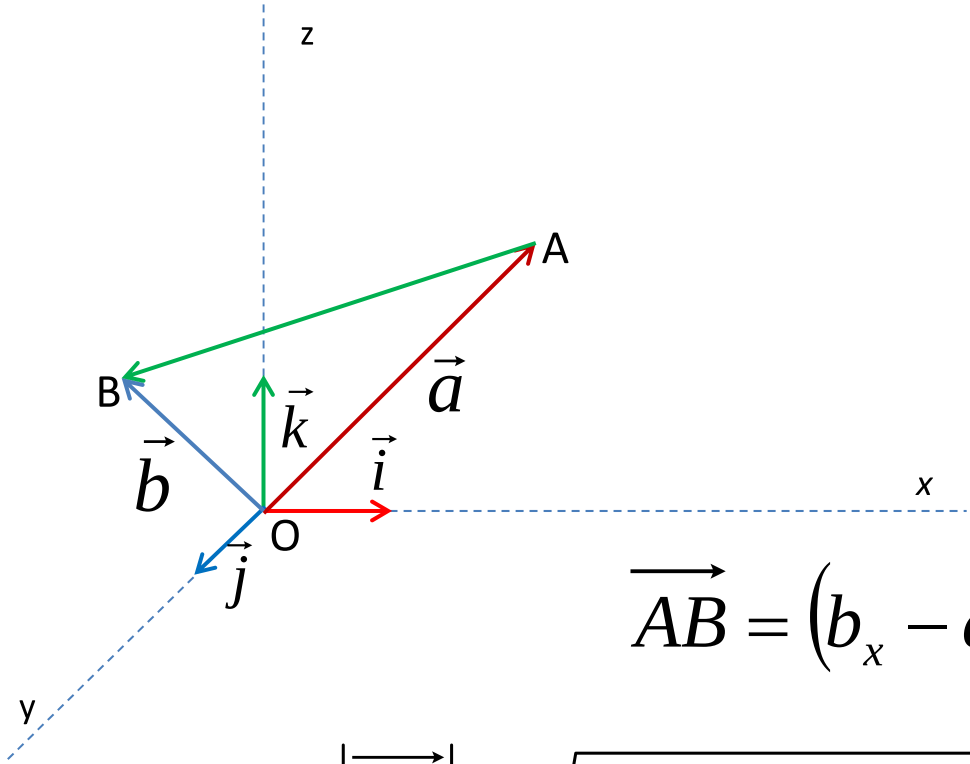


$$\overrightarrow{OA} = \vec{a} = (a_x, a_y, a_z) \qquad \overrightarrow{OB} = \vec{b} = (b_x, b_y, b_z)$$

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \vec{b} - \vec{a} = (b_x, b_y, b_z) - (a_x, a_y, a_z) = (b_x - a_x, b_y - a_y, b_z - a_z)$$

$$\overrightarrow{AB} = (b_x - a_x, b_y - a_y, b_z - a_z)$$

RASTOJANJE IZMEDJU DVE TAČKE



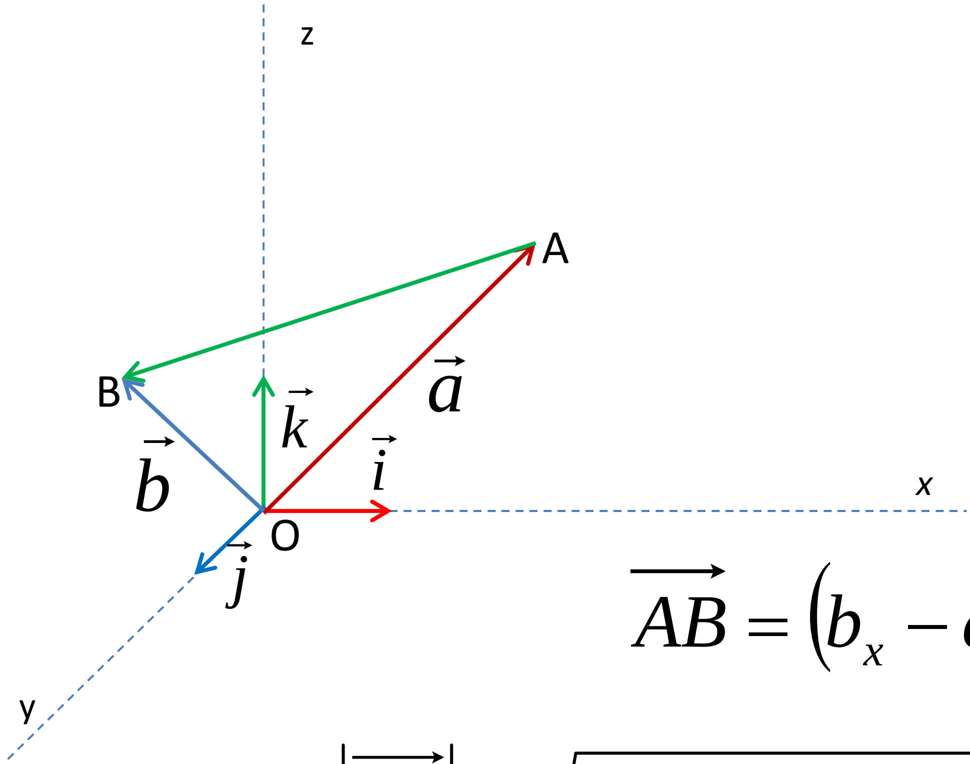
$$A(a_x, a_y, a_z)$$

$$\overrightarrow{AB} = (b_x - a_x, b_y - a_y, b_z - a_z)$$

$$d(A,B) = \left|\overrightarrow{AB}\right| = \sqrt{(b_x - a_x)^2 + (b_y - a_y)^2 + (b_z - a_z)^2}$$

Rastojanje izmedju dve tačke *A* i *B* je intezitet vektora $\overrightarrow{AB}$.

RASTOJANJE IZMEDJU DVE TAČKE



$$A(a_x, a_y, a_z)$$

$$B(b_x, b_y, b_z)$$

$$\overrightarrow{AB} = (b_x - a_x, b_y - a_y, b_z - a_z)$$

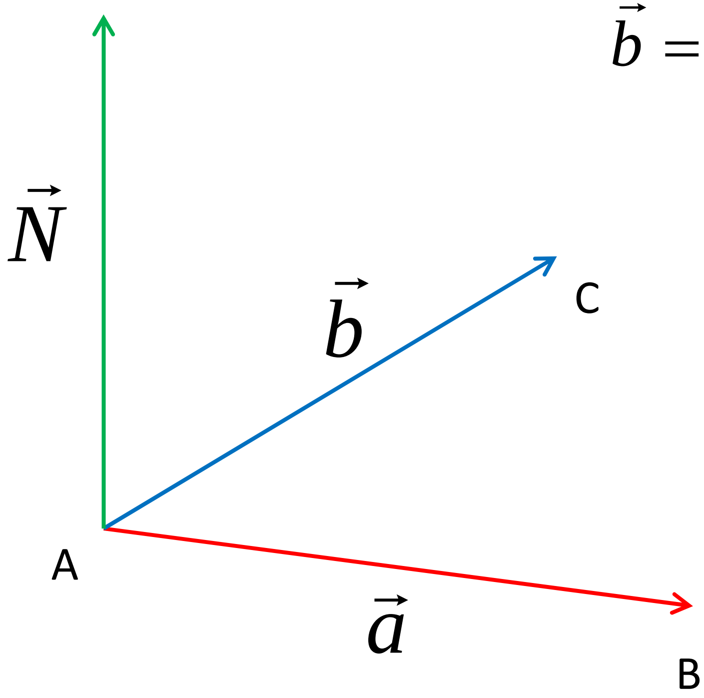
$$d(A,B) = \left|\overrightarrow{AB}\right| = \sqrt{(b_x - a_x)^2 + (b_y - a_y)^2 + (b_z - a_z)^2}$$

Rastojanje izmedju dve tačke *A* i *B* je intezitet vektora $\overrightarrow{AB}$.

PRIMER

Date su tačke $A=(1,-1,2)$ $B=(4,0,-2)$ $C=(-2,1,-1)$

Odrediti vektor normalan na ravan koju odredjuju te tri tačke.



$$\vec{a}=\overrightarrow{AB}=(4-1,0-(-1),-2-2)=(3,1,-4)$$

$$\vec{b}=\overrightarrow{AC}=(-2-1,1-(-1),-1-2)=(-3,2,-3)$$

$$\vec{N}=\vec{a}\times\vec{b}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 3 & 1 & -4\\ -3 & 2 & -3\end{vmatrix}$$

$$\vec{N}=\vec{i}\begin{vmatrix}1 & -4\\ 2 & -3\end{vmatrix}-\vec{j}\begin{vmatrix}3 & -4\\ -3 & -3\end{vmatrix}+\vec{k}\begin{vmatrix}3 & 1\\ -3 & 2\end{vmatrix}$$

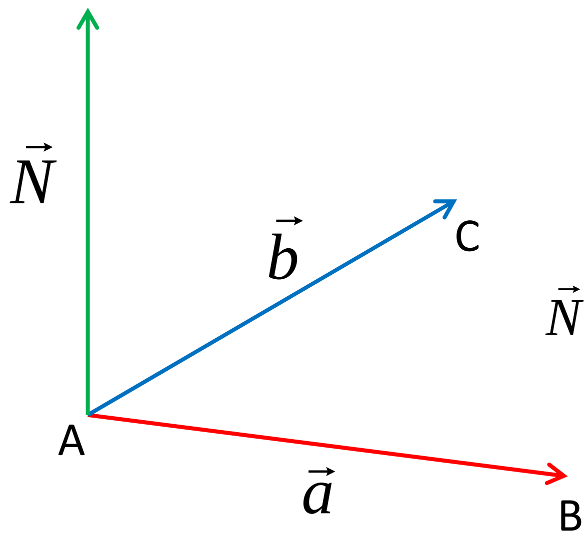
$$\vec{N}=5\vec{i}+21\vec{j}+9\vec{k}$$

$$\vec{N}=(5,21,9)$$

PRIMER

Date su tačke $A=(0,2,0)$ $B=(2,0,0)$ $C=(2,2,2)$

Odrediti vektor normalan na ravan koju odredjuju te tri tačke.



$$\vec{a}=\overrightarrow{AB}=(2-0,0-2,0-0)=(2,-2,0)$$

$$\vec{b}=\overrightarrow{AC}=(2-0,2-2,2-0)=(2,0,2)$$

$$\vec{N}=\vec{a}\times\vec{b}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 2 & -2 & 0\\ 2 & 0 & 2\end{vmatrix}=\vec{i}\begin{vmatrix}-2 & 0\\ 0 & 2\end{vmatrix}-\vec{j}\begin{vmatrix}2 & 0\\ 2 & 2\end{vmatrix}+\vec{k}\begin{vmatrix}2 & -2\\ 2 & 0\end{vmatrix}$$

$$\vec{N}=-4\vec{i}-4\vec{j}+4\vec{k} \qquad \vec{N}=(-4,-4,4)$$

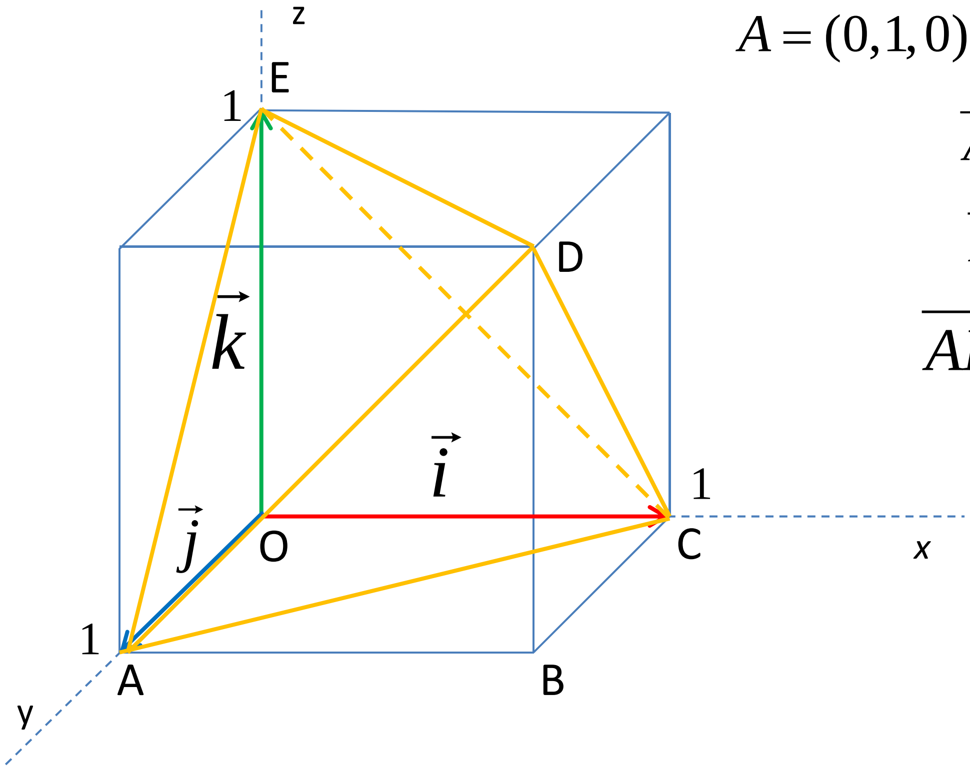
$$\lambda\vec{N}=(-4\lambda,-4\lambda,4\lambda) \qquad \lambda=\frac{1}{4} \qquad \frac{1}{4}\vec{N}=\frac{\vec{N}}{4}=(-1,-1,1)$$

$$ort\,\vec{N}=\left(-\frac{1}{\sqrt{3}},-\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)$$

Vektori normalni na ravan

PRIMER

Data je kocka osnovne ivice $a=1$ sa jednim temenom u koordinatnom početku O i sa ivicama koje polaze iz tog temena i leze na pozitivnim delovima koordinatnih osa. U tu kocku upisan je tetraedar ACDE prikazan na slici cije su ivice dijagonale strana date kocke. Odredititi ugao izmedju vektora $\overrightarrow{AD}$ i $\overrightarrow{AE}$.



$$A=(0,1,0)\quad C=(1,0,0)\quad D=(1,1,1)\quad E=(0,0,1)$$

$$\overrightarrow{AE}=(0-0,0-1,1-0)=(0,-1,1)$$

$$\overrightarrow{AD}=(1-0,1-1,1-0)=(1,0,1)$$

$$\overrightarrow{AD}\cdot\overrightarrow{AE}=0\cdot 1+(-1)\cdot 0+1\cdot 1=1$$

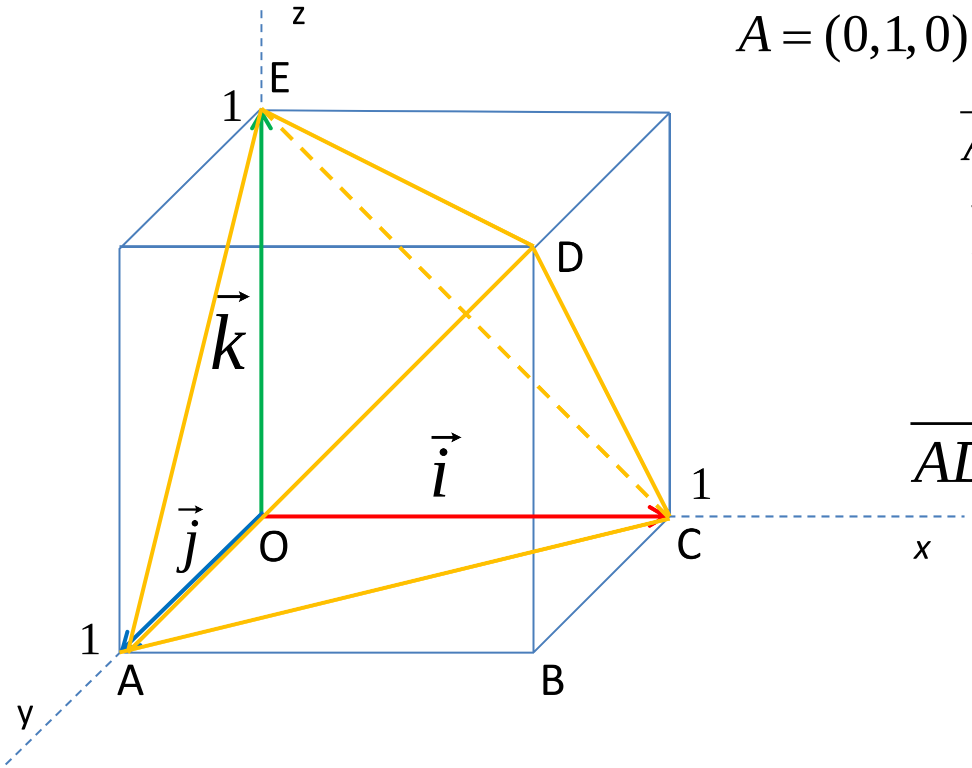
$$\left|\overrightarrow{AD}\right|=\left|\overrightarrow{AE}\right|=\sqrt{2}$$

$$\cos\varphi=\frac{\overrightarrow{AD}\cdot\overrightarrow{AE}}{\left|\overrightarrow{AD}\right|\cdot\left|\overrightarrow{AE}\right|}=\frac{1}{\sqrt{2}\cdot\sqrt{2}}=\frac{1}{2}$$

$$\varphi=\frac{\pi}{3}=60^0$$

PRIMER

Data je kocka osnovne ivice $a=1$ sa jednim temenom u koordinatnom početku O i sa ivicama koje polaze iz tog temena i leze na pozitivnim delovima koordinatnih osa. U tu kocku upisan je tetraedar ACDE prikazan na slici cije su ivice dijagonale strana date kocke. Odrediti $\overrightarrow{AD}\times\overrightarrow{AE}$ i ort tog vektora.



$$A=(0,1,0)\quad C=(1,0,0)\quad D=(1,1,1)\quad E=(0,0,1)$$

$$\overrightarrow{AE}=(0-0,0-1,1-0)=(0,-1,1)$$

$$\overrightarrow{AD}=(1-0,1-1,1-0)=(1,0,1)$$

$$\overrightarrow{AD}\times\overrightarrow{AE}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 1 & 0 & 1\\ 0 & -1 & 1\end{vmatrix}=\vec{i}-\vec{j}-\vec{k}$$

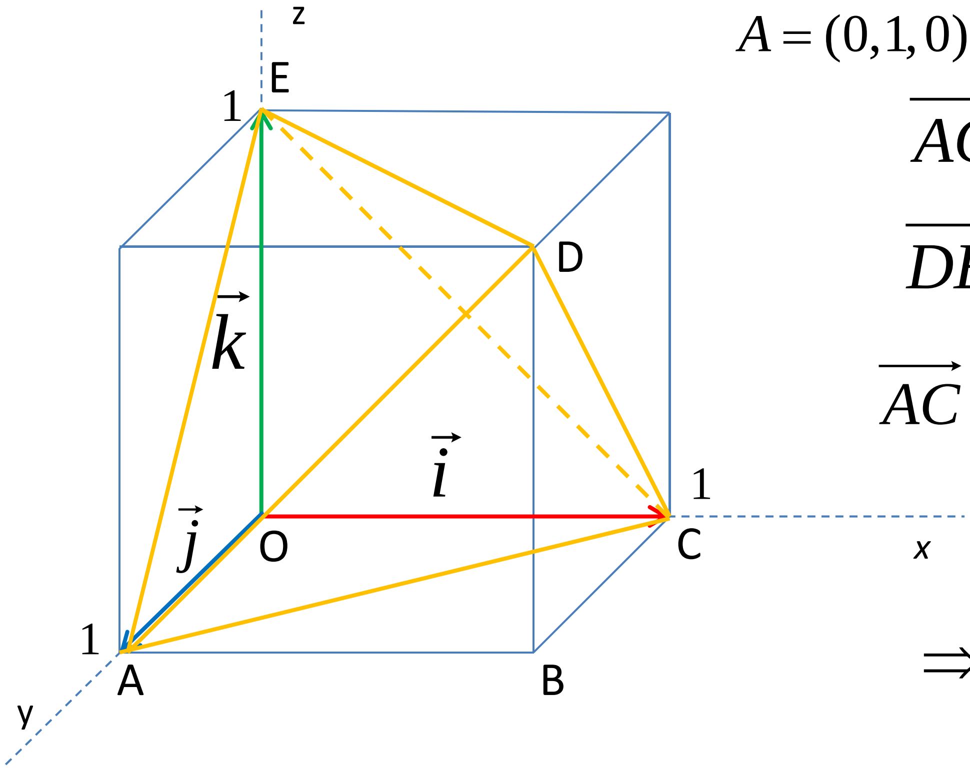
$$\overrightarrow{AD}\times\overrightarrow{AE}=(1,-1,-1)$$

$$\left|\overrightarrow{AD}\times\overrightarrow{AE}\right|=\sqrt{1^2+(-1)^2+(-1)^2}=\sqrt{3}$$

$$ort\left(\overrightarrow{AD}\times\overrightarrow{AE}\right)=\left(\frac{1}{\sqrt{3}},\frac{-1}{\sqrt{3}},\frac{-1}{\sqrt{3}}\right)$$

PRIMER

Data je kocka osnovne ivice $a = 1$ sa jednim temenom u koordinatnom početku O i sa ivicama koje polaze iz tog temena i leze na pozitivnim delovima koordinatnih osa. U tu kocku upisan je tetraedar ACDE prikazan na slici cije su ivice dijagonale strana date kocke. Izračunati $\overrightarrow{AC} \cdot \overrightarrow{DE}$



$A = (0,1,0)$ $C = (1,0,0)$ $D = (1,1,1)$ $E = (0,0,1)$

$$\overrightarrow{AC} = (1-0, 0-1, 0-0) = (1,-1,0)$$

$$\overrightarrow{DE} = (0-1, 0-1, 1-1) = (-1,-1,0)$$

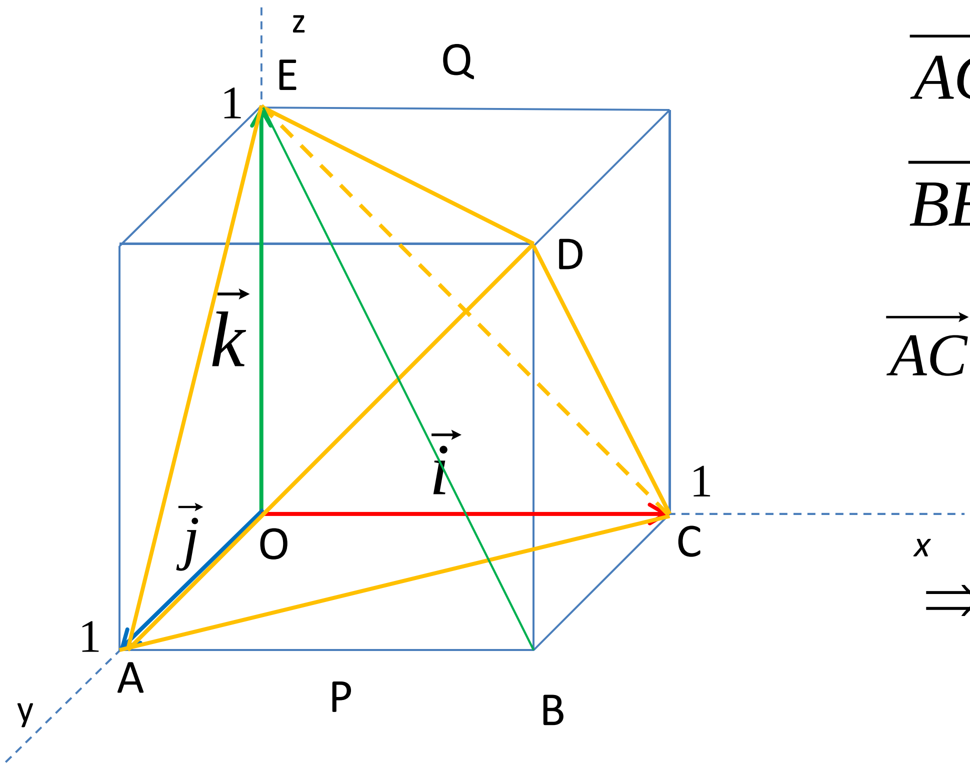
$$\overrightarrow{AC} \cdot \overrightarrow{DE} = 1 \cdot (-1) + (-1) \cdot (-1) + 0 \cdot 0 = 0$$

$\Rightarrow$ $\overrightarrow{AC}$ i $\overrightarrow{DE}$ Uzajamno ortogonalni

PRIMER

Data je kocka osnovne ivice $a=1$ sa jednim temenom u koordinatnom početku O i sa ivicama koje polaze iz tog temena koje leze na pozitivnim delovima koordinatnih osa. U tu kocku upisan je tetraedar ACDE prikazan na slici cije su ivice dijagonale strana date kocke. Izračunati $\overrightarrow{AC}\cdot\overrightarrow{BE}$.

$$A=(0,1,0)\quad B=(1,1,0)\quad C=(1,0,0)\quad D=(1,1,1)\quad E=(0,0,1)$$



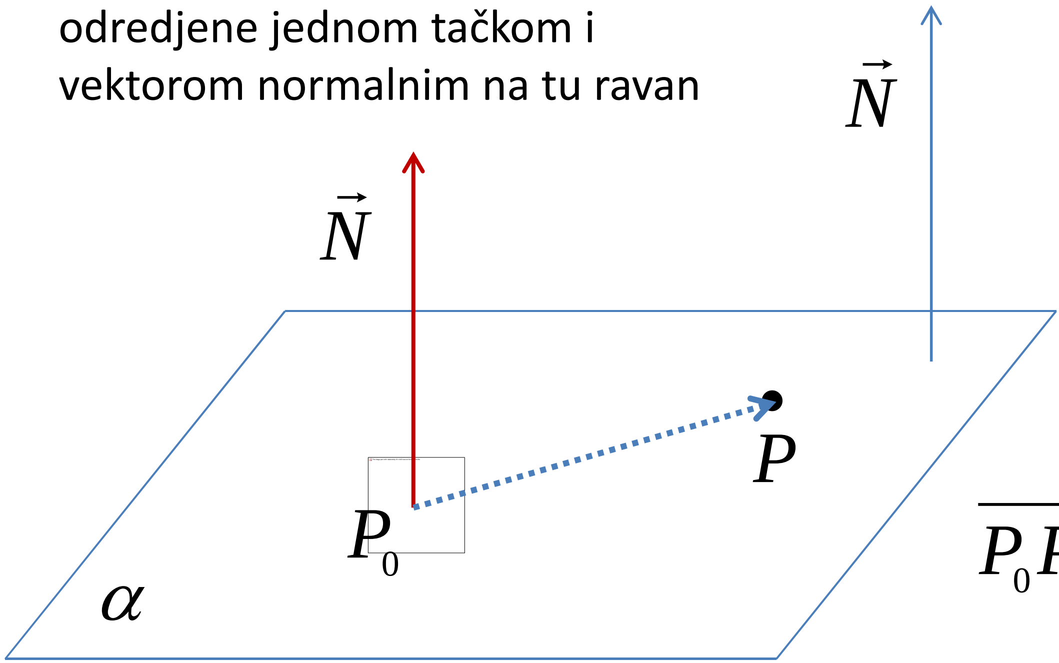
$$\overrightarrow{AC}=(1-0,0-1,0-0)=(1,-1,0)$$

$$\overrightarrow{BE}=(0-1,0-1,1-0)=(-1,-1,1)$$

$$\overrightarrow{AC}\cdot\overrightarrow{BE}=1\cdot(-1)+(-1)\cdot(-1)+0\cdot 1=0$$

$\Rightarrow$ $\overrightarrow{AC}$ i $\overrightarrow{BE}$ Uzajamno ortogonalni

JEDNAČINA RAVNI
odredjene jednom tačkom i
vektorom normalnim na tu ravan



$\alpha:$

$P_0(x_0, y_0, z_0)$

$\vec{N} = (A, B, C)$

$P(x, y, z) \in \alpha$

$\overrightarrow{P_0P} = (x - x_0, y - y_0, z - z_0)$

Tačka $P(x, y, z)$ pripada ravni α ako i samo su vektori $\overrightarrow{P_0P}$ i $\vec{N}$

uzajamno ortogonalni. Iz uslova ortogonalnosti $\vec{N} \cdot \overrightarrow{P_0P} = 0$
sledi jednačina ravni:

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$$

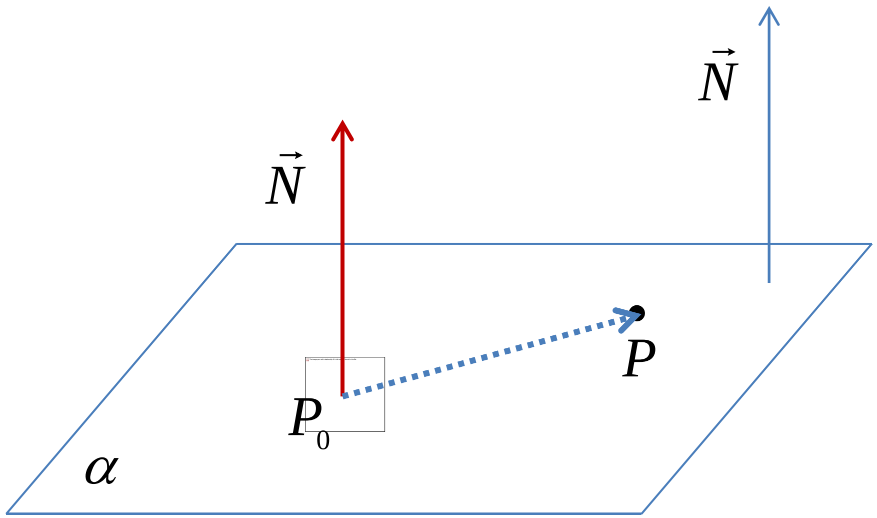
PRIMER

Napisati jednačinu ravni α
koja sadrži tačku

$P_0(2,-5,-6)$

i normalna je na vektor

$\vec{N}=(-1,2,-3)$.



$$A(x-x_0)+B(y-y_0)+C(z-z_0)=0$$

$$-(x-2)+2(y+5)-3(z+6)=0$$

$$-x+2y-3z-6=0$$

JEDNAČINA RAVNI

Vektor normalan na ravan

$$\vec{N} = \left(A, B, C\right)$$

$$P_0\left(x_0, y_0, z_0\right)$$

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$$

$$Ax + By + Cz + D = 0$$

$$D = -Ax_0 - By_0) - Cz_0$$

$$Ax + By + Cz + D = 0 \Rightarrow \vec{N} = (A, B, C)$$

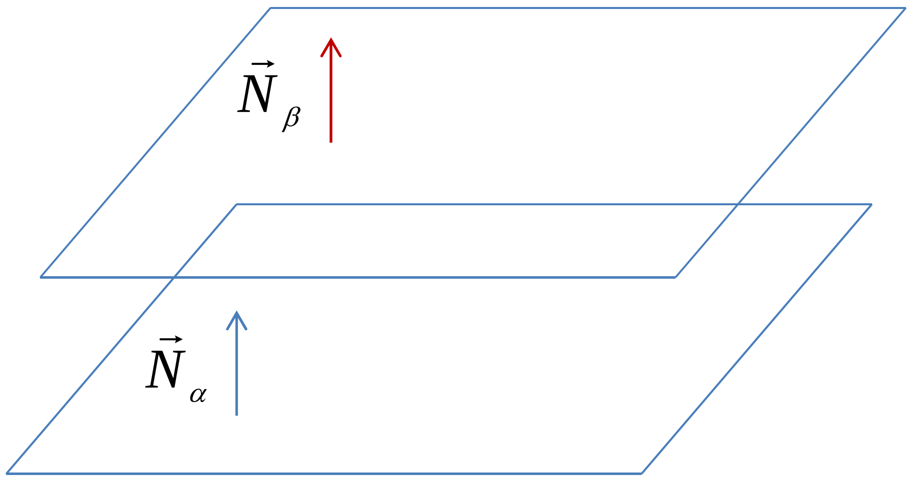
PRIMER

Napisati jednačinu ravni β
koja sadrži tačku

$$P_0(-2,5,3)$$

i paralelna je sa ravni

$$\alpha: \quad -x+2y-3z-6=0$$



$$\vec{N}_\beta = \vec{N}_\alpha = (-1,2,-3)$$

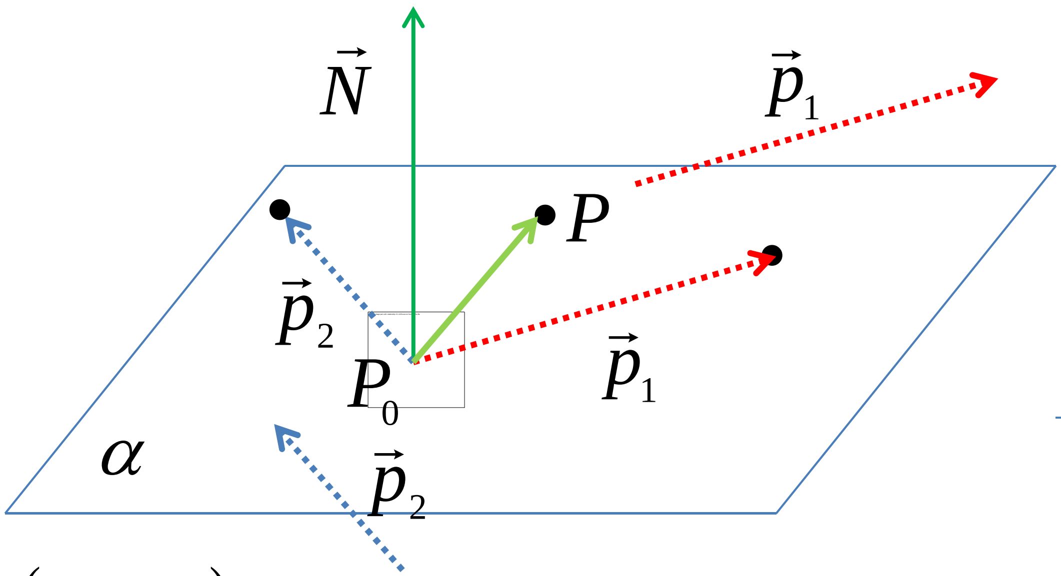
$$\beta: \quad P_0(-2,5,3) \qquad \vec{N}_\beta = (-1,2,-3)$$

$$A(x-x_0)+B(y-y_0)+C(z-z_0)=0$$

$$-(x+2)+2(y-5)-3(z-3)=0$$

$$-x+2y-3z-3=0$$

JEDNAČINA RAVNI
odredjene jednom tačkom i dva
vektora paralelna sa tom ravni



$$\alpha:$$

$$P_0(x_0, y_0, z_0)$$

$$\vec{p}_1 = (a_1, b_1, c_1)$$

$$\vec{p}_2 = (a_2, b_2, c_2)$$

$$\vec{N} = \vec{p}_1 \times \vec{p}_2 = (A, B, C)$$

$$P(x, y, z) \in \alpha$$

$$\overrightarrow{P_0P} = (x - x_0, y - y_0, z - z_0)$$

$$\alpha:$$

$$P_0(x_0, y_0, z_0)$$

$$\vec{N} = \vec{p}_1 \times \vec{p}_2 = (A, B, C)$$

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$$

PRIMER

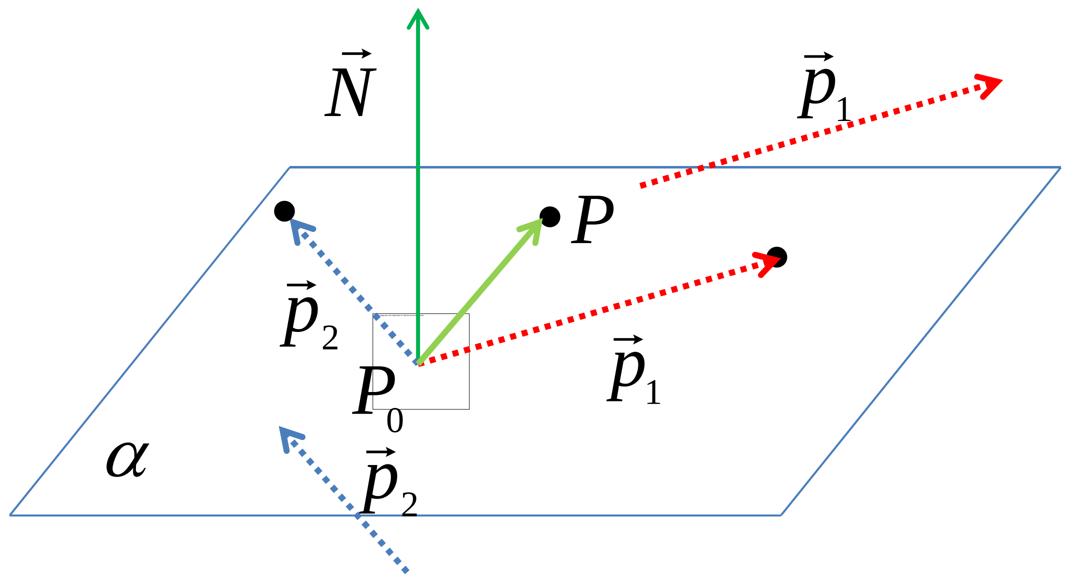
Napisati jednačinu ravni α koja sadrži tačku

$P_0(2,-5,-6)$

i paralelna je vektorima

$\vec{p}_1 = (1,-3,-2)$

$\vec{p}_2 = (1,0,-1)$

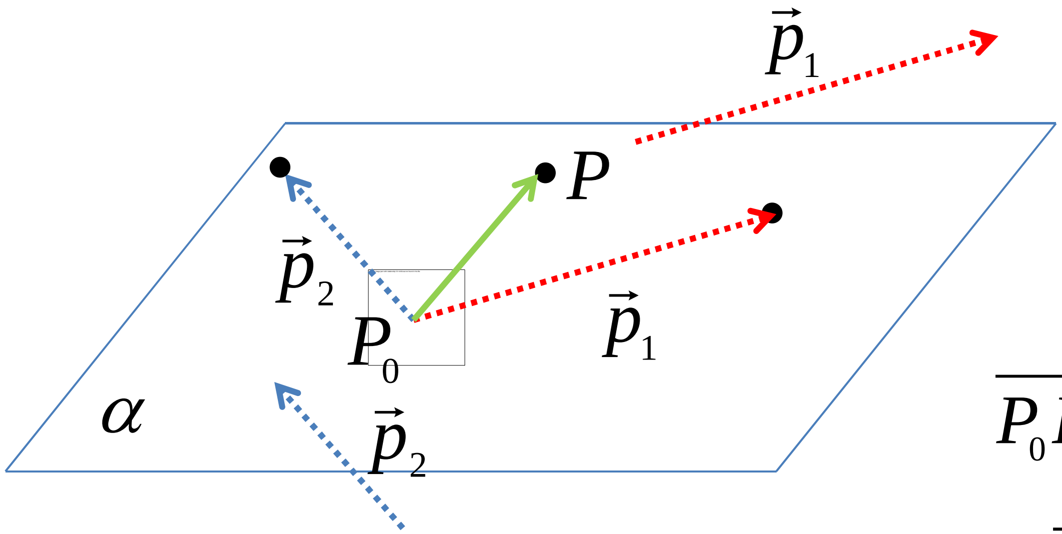


$$\vec{N} = \vec{p}_1 \times \vec{p}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -3 & -2 \\ 1 & 0 & -1 \end{vmatrix} = \vec{i}\begin{vmatrix} -3 & -2 \\ 0 & -1 \end{vmatrix} - \vec{j}\begin{vmatrix} 1 & -2 \\ 1 & -1 \end{vmatrix} + \vec{k}\begin{vmatrix} 1 & -3 \\ 1 & 0 \end{vmatrix} = 3\vec{i} - \vec{j} + 3\vec{k}$$

$$\alpha: \quad P_0(2,-5,-6) \quad \vec{N} = (3,-1,3)$$

$$3(x-2) - 1(y+5) + 3(z+6) = 0 \qquad 3x - y + 3z + 7 = 0$$

JEDNAČINA RAVNI
odredjene jednom tačkom i dva vektora paralelna sa tom ravni



$$\alpha:$$
$$P_0(x_0, y_0, z_0)$$
$$\vec{p}_1 = (a_1, b_1, c_1)$$
$$\vec{p}_2 = (a_2, b_2, c_2)$$

$$P(x, y, z) \in \alpha$$
$$\overrightarrow{P_0P} = (x - x_0, y - y_0, z - z_0)$$

Tačka $P(x, y, z) \in \alpha$ ako i samo ako su vektori $\overrightarrow{P_0P},\ \vec{p}_1,\ \vec{p}_2$ komplanarni.

$$\begin{vmatrix} x - x_0 & y - y_0 & z - z_0 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

PRIMER

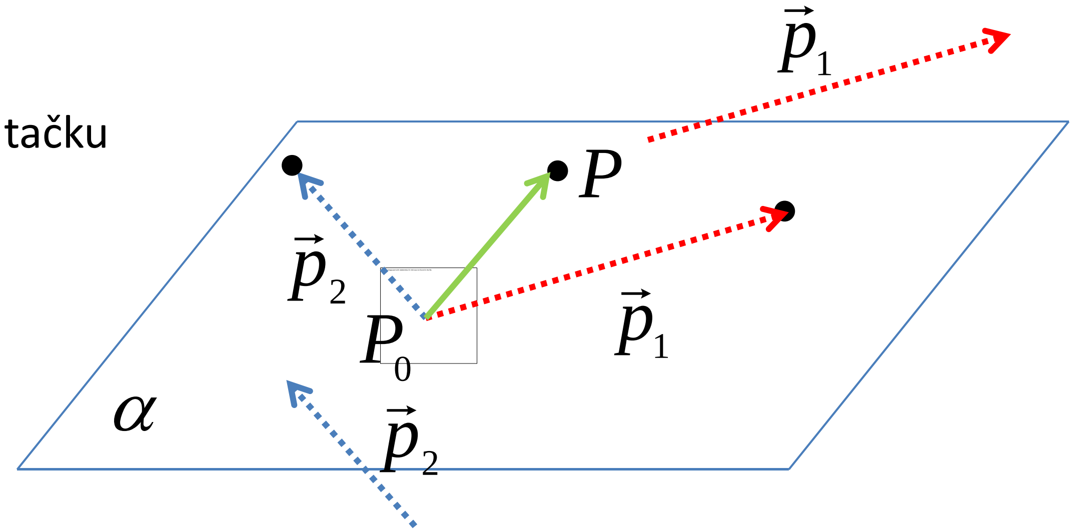
Napisati jednačinu ravni koja sadrži tačku

$$P_0(-2,1,3)$$

i paralelna je sa vektorima

$$\vec{p}_1 = (1,-3,-2)$$

$$\vec{p}_2 = (1,0,-1)$$



$$\alpha: \quad P_0(-2,1,3) \quad \vec{p}_1 = (1,-3,-2) \quad \vec{p}_2 = (1,0,-1)$$

$$\begin{vmatrix} x+2 & y-1 & z-3 \\ 1 & -3 & -2 \\ 1 & 0 & -1 \end{vmatrix} = 0$$

$$(x+2)\begin{vmatrix} -3 & -2 \\ 0 & -1 \end{vmatrix} - (y-1)\begin{vmatrix} 1 & -2 \\ 1 & -1 \end{vmatrix} + (z-3)\begin{vmatrix} 1 & -3 \\ 1 & 0 \end{vmatrix} = 0$$

$$3(x+2) - (y-1) + 3(z-3) = 0 \qquad 3x - y + 3z - 2 = 0$$

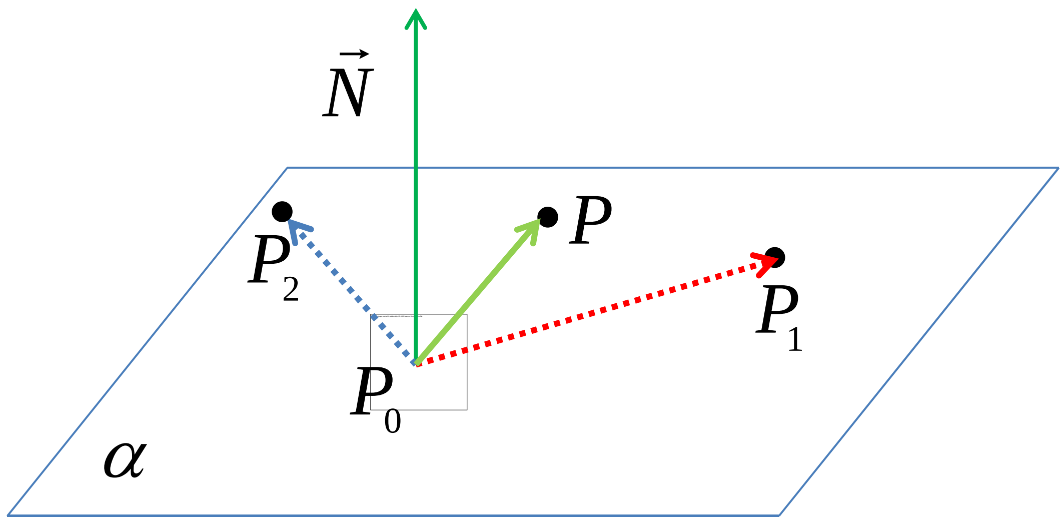
JEDNAČINA RAVNI
odredjene sa tri tačke

$\alpha :$

$P_0(x_0, y_0, z_0)$

$P_1(x_1, y_1, z_1)$

$P_2(x_2, y_2, z_2)$



$P(x, y, z) \in \alpha$

$$\overrightarrow{P_0P_1} = (x_1 - x_0, y_1 - y_0, z_1 - z_0)$$

$$\overrightarrow{P_0P_2} = (x_2 - x_0, y_2 - y_0, z_2 - z_0)$$

$$\overrightarrow{P_0P} = (x - x_0, y - y_0, z - z_0)$$

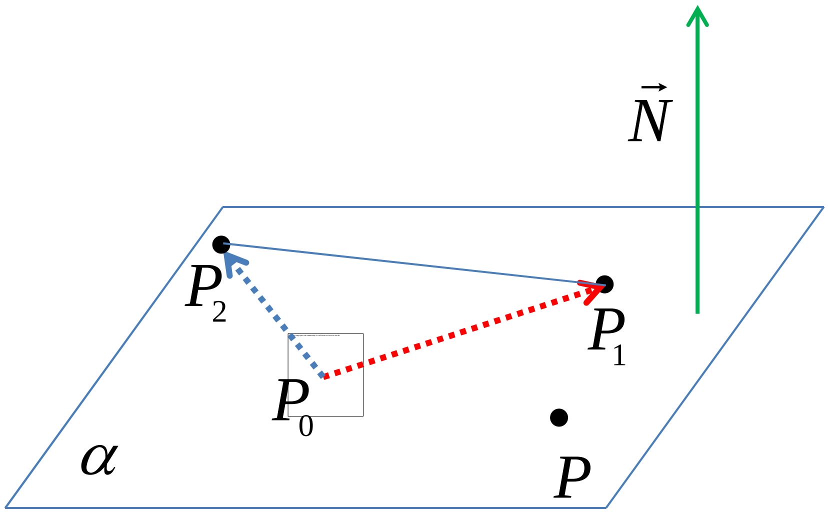
$$\vec{N} = \overrightarrow{P_0P_1} \times \overrightarrow{P_0P_2} = (A, B, C)$$

$\alpha : \quad P_0(x_0, y_0, z_0)$

$$\vec{N} = (A, B, C)$$

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$$

JEDNAČINA RAVNI
odredjene sa tri tačke



$$\vec{N} = \overrightarrow{P_0P_1} \times \overrightarrow{P_0P_2}$$

$$\vec{N} = \overrightarrow{P_1P_0} \times \overrightarrow{P_1P_2}$$

$$\vec{N} = \overrightarrow{P_2P_0} \times \overrightarrow{P_2P_1}$$

$$\vec{N} = (A, B, C)$$

$$\alpha :$$

$$P_0(x_0, y_0, z_0)$$
$$P_1(x_1, y_1, z_1)$$
$$P_2(x_2, y_2, z_2)$$

$$P(x, y, z) \in \alpha$$

$$\alpha : \quad P_0(x_0, y_0, z_0)$$
$$\vec{N} = (A, B, C)$$
$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$$

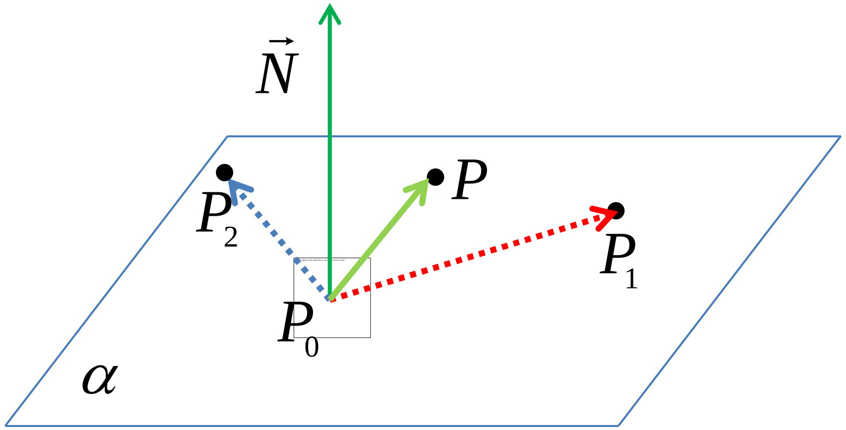
$$\alpha : \quad P_1(x_1, y_1, z_1)$$
$$\vec{N} = (A, B, C)$$
$$A(x - x_1) + B(y - y_1) + C(z - z_1) = 0$$

$$\alpha : \quad P_2(x_2, y_2, z_2)$$
$$\vec{N} = (A, B, C)$$
$$A(x - x_2) + B(y - y_2) + C(z - z_2) = 0$$

PRIMER

Napisati jednačinu ravni
koja sadrži tačke

$P_0(1,2,2)$ $P_1(2,0,0)$ $P_2(2,2,3)$



$$\vec{a} = \overrightarrow{P_0P_1} = (2-1, 0-2, 0-2) = (1,-2,-2)$$

$$\vec{b} = \overrightarrow{P_0P_2} = (2-1, 2-2, 3-2) = (1,0,1)$$

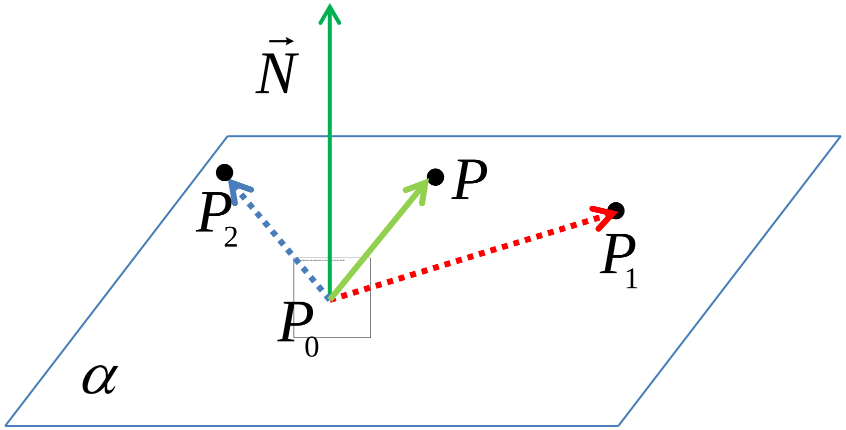
$$\vec{N} = \vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -2 & -2 \\ 1 & 0 & 1 \end{vmatrix} = \vec{i}\begin{vmatrix} -2 & -2 \\ 0 & 1 \end{vmatrix} - \vec{j}\begin{vmatrix} 1 & -2 \\ 1 & 1 \end{vmatrix} + \vec{k}\begin{vmatrix} 1 & -2 \\ 1 & 0 \end{vmatrix} = -2\vec{i} - 3\vec{j} + 2\vec{k}$$

$$\vec{N} = (-2,-3,2)$$

$$-2(x-1) - 3(y-2) + 2(z-2) = 0$$

$$-2x - 3y + 2z + 4 = 0$$

PRIMER



Napisati jednačinu ravni
koja sadrži tačke

$P_0(1,2,2)$ $P_1(2,0,0)$ $P_2(2,2,3)$

$$\vec{a}=\overrightarrow{P_0P_1}=(2-1,0-2,0-2)=(1,-2,-2)$$

$$\vec{b}=\overrightarrow{P_0P_2}=(2-1,2-2,3-2)=(1,0,1)$$

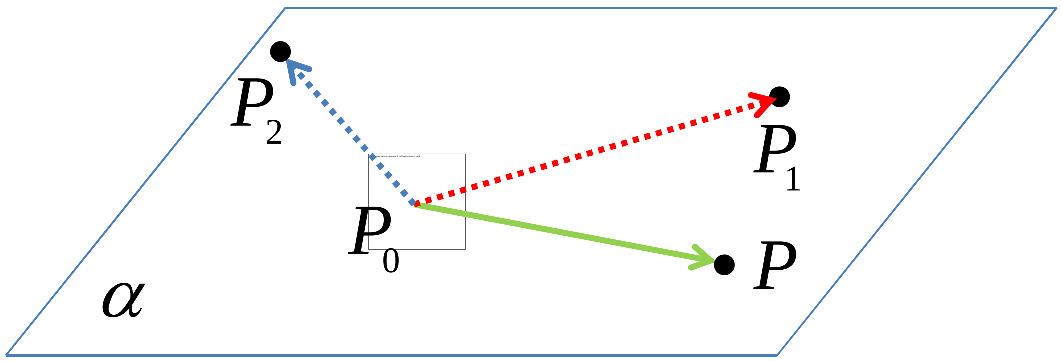
$$\vec{N}=\vec{a}\times\vec{b}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 1 & -2 & -2\\ 1 & 0 & 1\end{vmatrix}=\vec{i}\begin{vmatrix}-2 & -2\\ 0 & 1\end{vmatrix}-\vec{j}\begin{vmatrix}1 & -2\\ 1 & 1\end{vmatrix}+\vec{k}\begin{vmatrix}1 & -2\\ 1 & 0\end{vmatrix}=-2\vec{i}-3\vec{j}+2\vec{k}$$

$$\vec{N}=(-2,-3,2)$$

$$\boxed{-2(x-2)-3(y-0)+2(z-0)=0}$$

$$-2x-3y+2z+4=0$$

JEDNAČINA RAVNI
Odredjene sa tri tačke



$$\alpha:$$

$$P_0(x_0, y_0, z_0)$$

$$P_1(x_1, y_1, z_1)$$

$$P_2(x_2, y_2, z_2)$$

$$P(x, y, z) \in \alpha$$

$$\overrightarrow{P_0P} = (x - x_0, y - y_0, z - z_0)$$

$$\overrightarrow{P_0P_1} = (x_1 - x_0, y_1 - y_0, z_1 - z_0)$$

$$\overrightarrow{P_0P_2} = (x_2 - x_0, y_2 - y_0, z_2 - z_0)$$

Tačka $P(x, y, z)$ pripada ravni α ako i samo ako su ovi vektori komplanarni.

$$\left[\overrightarrow{P_0P}, \overrightarrow{P_0P_1}, \overrightarrow{P_0P_2}\right] = \left(\overrightarrow{P_0P} \times \overrightarrow{P_0P_1}\right) \cdot \overrightarrow{P_0P_2} = 0$$

Jednačina ravni:

$$\begin{vmatrix} x - x_0 & y - y_0 & z - z_0 \\ x_1 - x_0 & y_1 - y_0 & z_1 - z_0 \\ x_2 - x_0 & y_2 - y_0 & z_2 - z_0 \end{vmatrix} = 0$$

PRIMER

Napisati jednačinu ravni koja sadrži tačke $P_0(1,2,2)$ $P_1(2,0,0)$ $P_2(2,2,3)$

$$P(x,y,z)\in\alpha$$

$$\overrightarrow{P_0P}=(x-1,y-2,z-2)$$

$$\overrightarrow{P_0P_1}=(2-1,0-2,0-2)=(1,-2,-2)$$

$$\overrightarrow{P_0P_2}=(2-1,2-2,3-2)=(1,0,1)$$

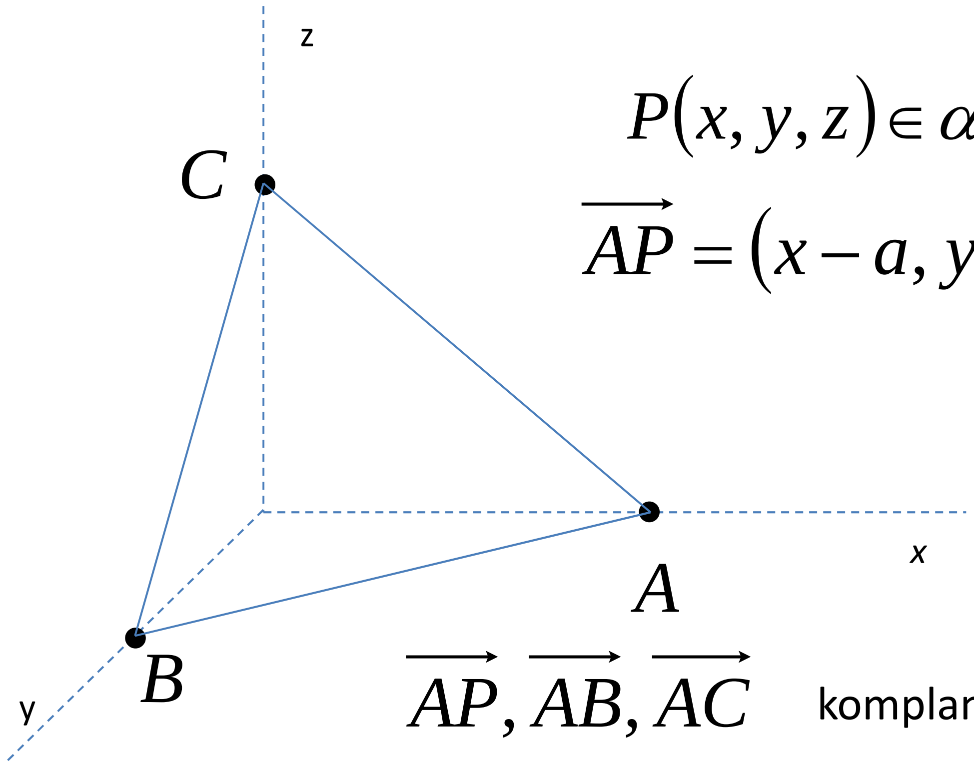
$$\begin{vmatrix} x-1 & y-2 & z-2 \\ 1 & -2 & -2 \\ 1 & 0 & 1 \end{vmatrix}=0$$

$$(x-1)\begin{vmatrix} -2 & -2 \\ 0 & 1 \end{vmatrix}-(y-2)\begin{vmatrix} 1 & -2 \\ 1 & 1 \end{vmatrix}+(z-2)\begin{vmatrix} 1 & -2 \\ 1 & 0 \end{vmatrix}=0$$

$$-2(x-1)-3(y-2)+2(z-2)=0$$

$$-2x-3y+2z+4=0$$

JEDNAČINA RAVNI – SEGMENTNI OBLIK



$$P(x,y,z)\in\alpha$$

$$\overrightarrow{AP}=(x-a,y,z)$$

$$\alpha:$$

$$A(a,0,0)$$

$$B(0,b,0)$$

$$C(0,0,c)$$

$$\overrightarrow{AB}=(-a,b,0)$$

$$\overrightarrow{AC}=(-a,0,c)$$

$\overrightarrow{AP},\overrightarrow{AB},\overrightarrow{AC}$ komplanarni $\Leftrightarrow \left[\overrightarrow{AP},\overrightarrow{AB},\overrightarrow{AC}\right]=0$

$$\begin{vmatrix} x-a & y & z \\ -a & b & 0 \\ -a & 0 & c \end{vmatrix}=0 \quad\Rightarrow\quad \frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

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Napisati jednačinu ravni koja na osama x,z,y redom odseca

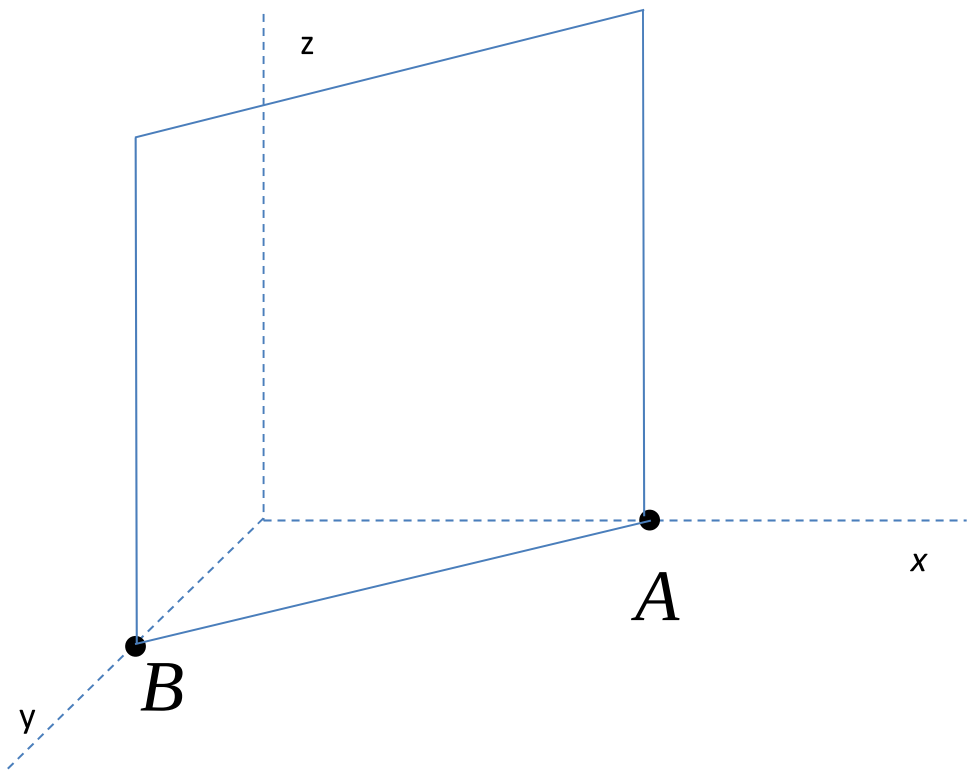
$$a = 2,\ b = -3,\ \ c = 5$$

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

$$\frac{x}{2} - \frac{y}{3} + \frac{z}{5} = 1 \ \Big|\cdot 30$$

$$15x - 10y + 6z - 30 = 0$$

JEDNAČINA RAVNI – SEGMENTNI OBLIK



$\alpha:$

$A(a,0,0)$

$B(0,b,0)$

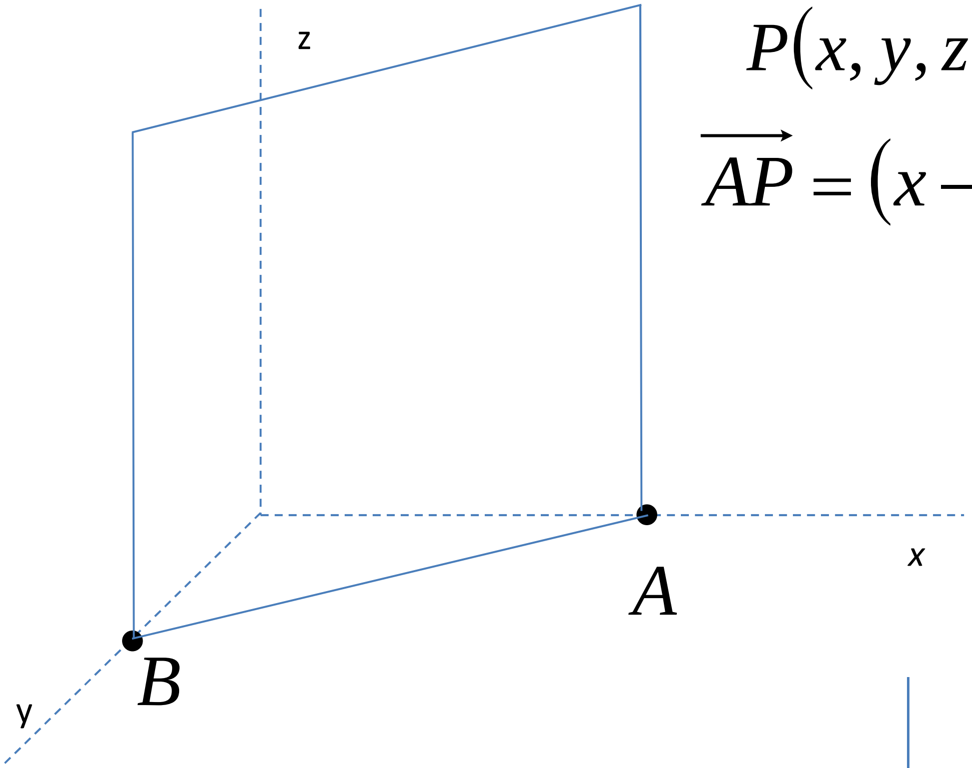
Paralelna z-osi

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

$$c \to \infty$$

$$\frac{x}{a}+\frac{y}{b}=1$$

JEDNAČINA RAVNI – SEGMENTNI OBLIK



$$P(x, y, z) \in \alpha$$

$$\overrightarrow{AP} = (x - a, y, z)$$

α :

$A(a,0,0)$

$B(0,b,0)$

Paralelna z-osi

$$\overrightarrow{AB} = (-a, b, 0)$$

$$\vec{k} = (0,0,1)$$

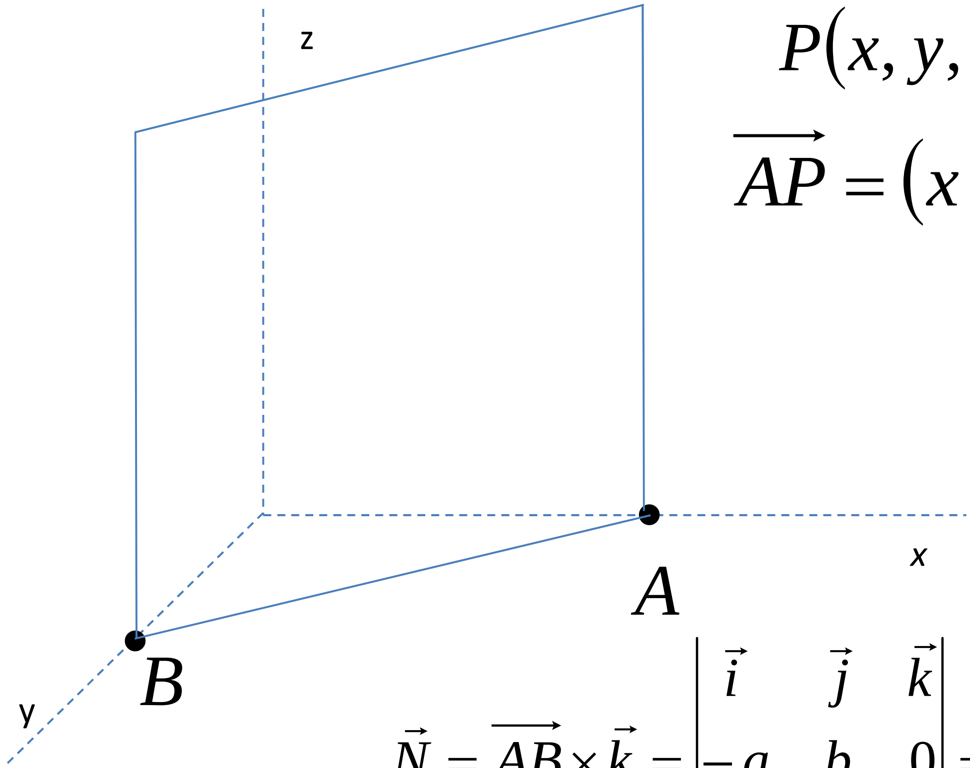
$$\begin{vmatrix} x-a & y & z \\ -a & b & 0 \\ 0 & 0 & 1 \end{vmatrix} = 0 \Rightarrow$$

$$b(x-a) + a\,y = 0$$

$$bx + a\,y = ab$$

$$\frac{x}{a} + \frac{y}{b} = 1$$

JEDNAČINA RAVNI – SEGMENTNI OBLIK



$$P(x,y,z)\in\alpha$$

$$\overrightarrow{AP}=(x-a,y,z)$$

α :

$A(a,0,0)$

$B(0,b,0)$

Paralelna z-osi

$$\overrightarrow{AB}=(-a,b,0)$$

$$\vec{k}=(0,0,1)$$

$$\vec{N}=\overrightarrow{AB}\times\vec{k}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ -a & b & 0\\ 0 & 0 & 1\end{vmatrix}=\vec{i}\begin{vmatrix}b & 0\\ 0 & 1\end{vmatrix}-\vec{j}\begin{vmatrix}-a & 0\\ 0 & 1\end{vmatrix}+\vec{k}\begin{vmatrix}-a & b\\ 0 & 0\end{vmatrix}=b\vec{i}+a\vec{j}$$

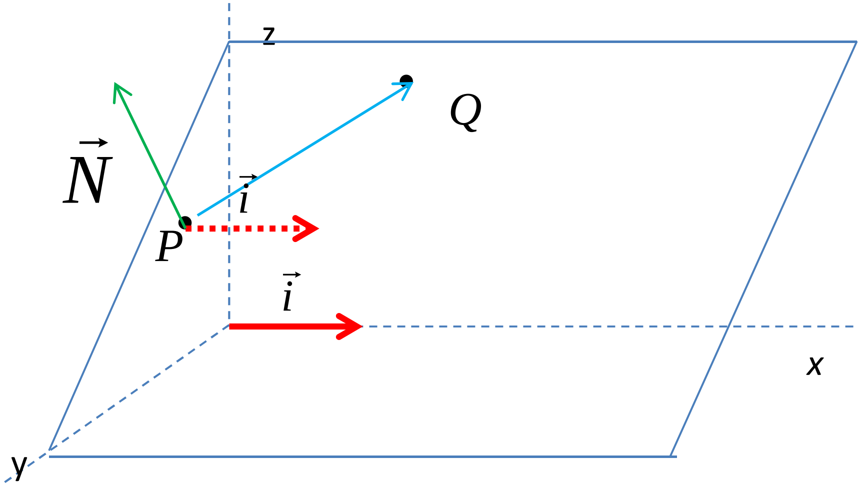
$$\vec{N}=(b,a,0)\ \ b(x-a)+a\,y=0\Rightarrow bx+a\,y=ab\Rightarrow\ \frac{x}{a}+\frac{y}{b}=1$$

PRIMER

Napisati jednačinu ravni koja sadrži tačke

$P(1,2,2) \qquad Q(3,1,4)$

i paralelna je x-osi.



$$\overrightarrow{PQ} = (3-1, 1-2, 4-2) = (2,-1,2)$$

$$\vec{i} = (1, 0, 0)$$

$$\vec{N} = \overrightarrow{PQ} \times \vec{i} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & 2 \\ 1 & 0 & 0 \end{vmatrix} = \vec{i}\begin{vmatrix} -1 & 2 \\ 0 & 0 \end{vmatrix} - \vec{j}\begin{vmatrix} 2 & 2 \\ 1 & 0 \end{vmatrix} + \vec{k}\begin{vmatrix} 2 & -1 \\ 1 & 0 \end{vmatrix} = 2\vec{j} + \vec{k}$$

$\alpha:$

$$P(1,2,2) \qquad \vec{N} = (0,2,1)$$

$$0 \cdot (x-1) + 2(y-2) + (z-2) = 0$$

$$2y + z - 6 = 0$$