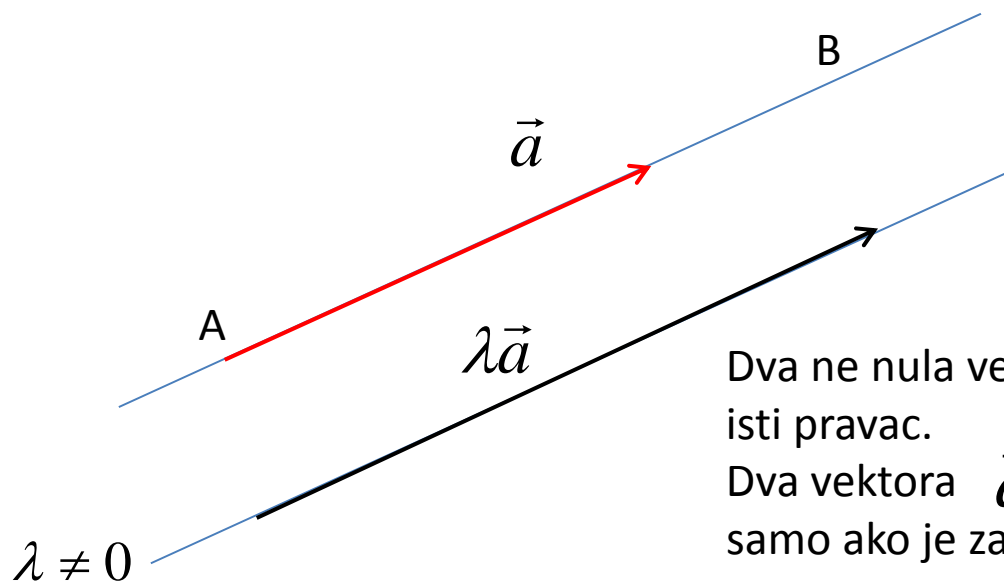


KOLINEARNOST VEKTORA ZADATIH POMOĆU KOORDINATA



Dva ne nula vektora su kolinearni ako imaju isti pravac.

Dva vektora \vec{a} i \vec{b} su kolinearni ako i samo ako je za neko

$$\lambda \in \mathbb{R}, \quad \lambda \neq 0, \quad \vec{b} = \lambda\vec{a}$$

$$\vec{a} = (a_x, a_y, a_z)$$

$$\vec{b} = (b_x, b_y, b_z) \quad \vec{b} = \lambda\vec{a} \quad (b_x, b_y, b_z) = (\lambda a_x, \lambda a_y, \lambda a_z)$$

$$\frac{b_x}{a_x} = \frac{b_y}{a_y} = \frac{b_z}{a_z}$$

Uslov kolinearnosti dva vektora

PRIMER

Vektori

$$\vec{a} = (-1, 2, -3) \quad \vec{b} = (2, -4, 6)$$

Su kolinearni zato što je

$$\frac{2}{-1} = \frac{-4}{2} = \frac{6}{-3} = -2$$

Vektori

$$\vec{a} = (-1, 2, -3) \quad \vec{b} = (2, -4, 3)$$

nisu kolinearni zato što je

$$\frac{2}{-1} = \frac{-4}{2} \neq \frac{3}{-3}$$

PRIMER

Odrediti vrednosti parametre λ i μ tako da su vektori

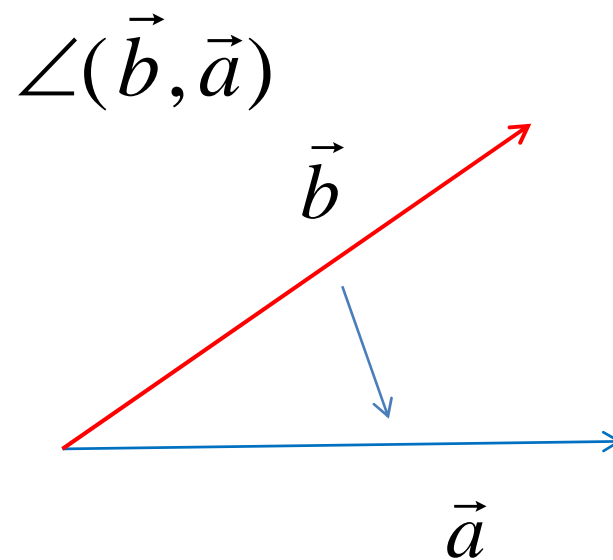
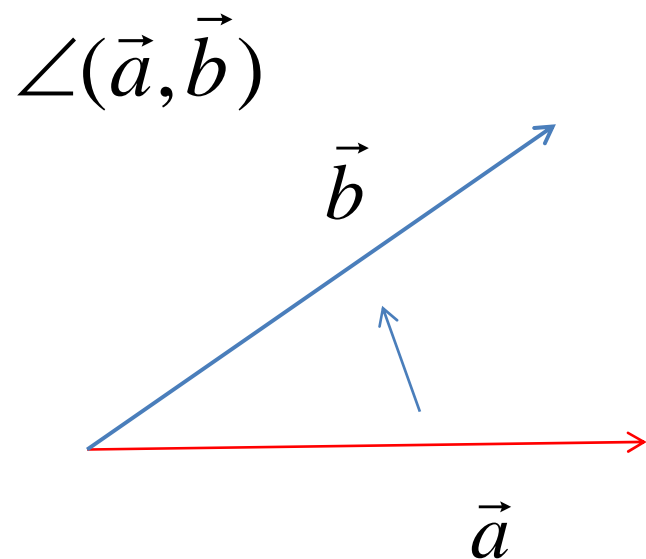
$$\vec{a} = (\lambda, 3, 2) \quad \text{i} \quad \vec{b} = (4, 6, \mu)$$

kolinearni.

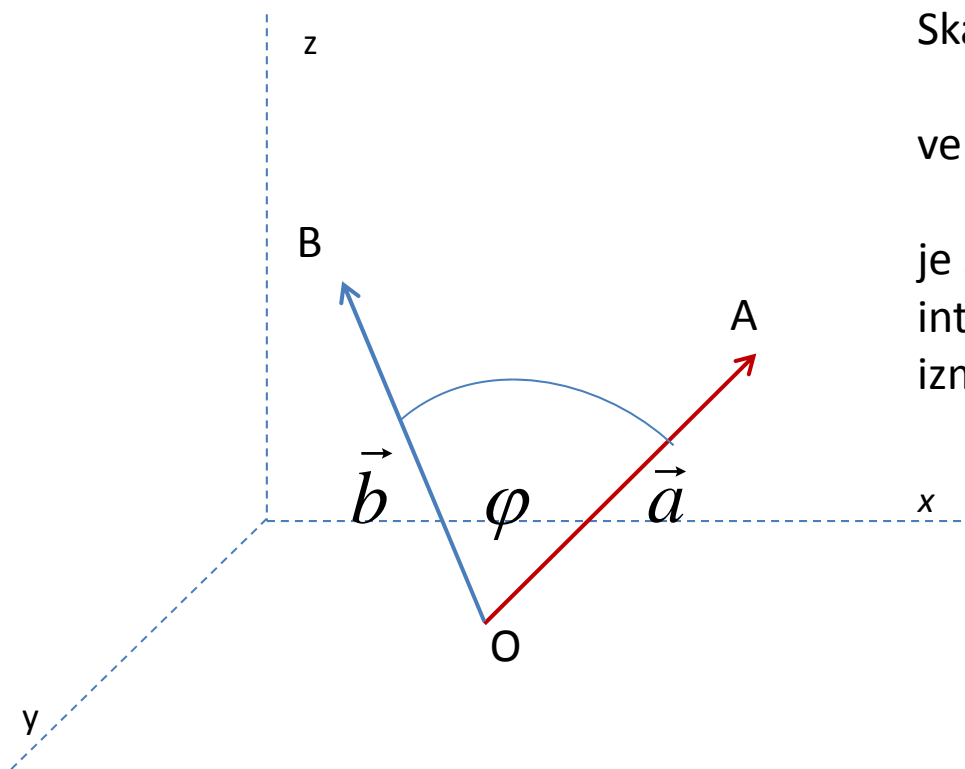
$$\frac{4}{\lambda} = \frac{6}{3} = \frac{\mu}{2} \quad \Rightarrow \quad \lambda = 2, \mu = 4$$

UGAO IZMEDJU DVA VEKTORA

Ugao izmedju dva vektora je ugao za koji treba zarotirati jedan od njih da bi se poklopio po pravcu i smeru sa drugim vektorom.



SKALARNI PROIZVOD VEKTORA



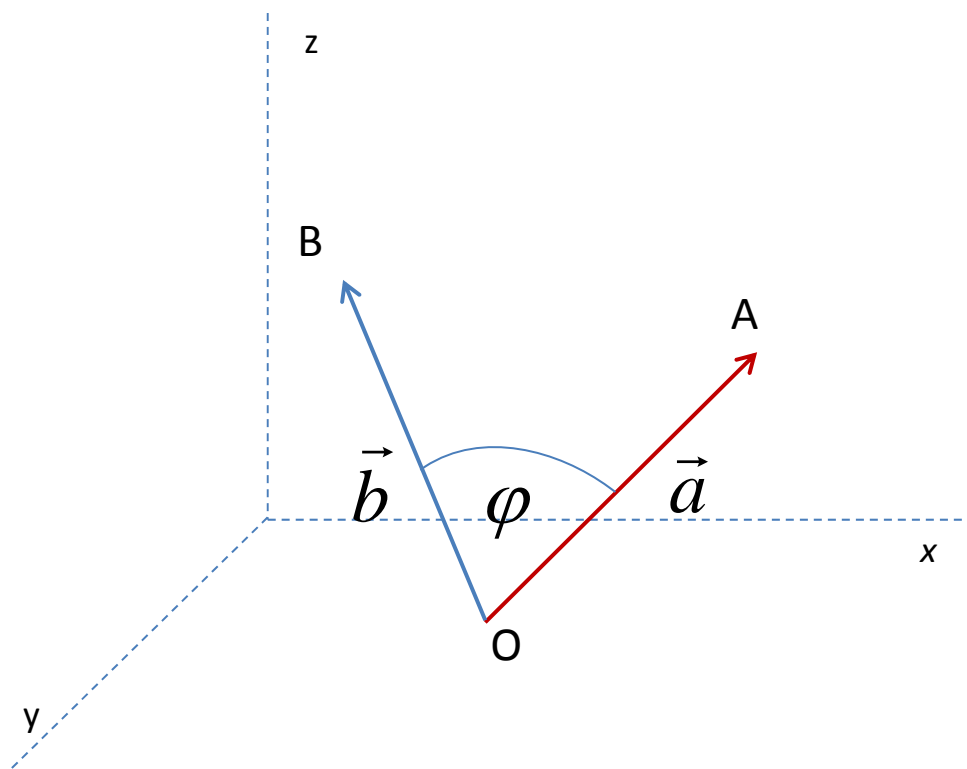
Skalarni proizvod $\vec{a} \cdot \vec{b}$ dva vektora \vec{a} i \vec{b}

je skalar, koji je jednak proizvodu intenziteta vektora i kosinusa ugla izmedju njih.

$$\vec{a} \cdot \vec{b}$$

$$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos \varphi$$

SKALARNI PROIZVOD VEKTORA



$$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos \varphi$$

Osobine skalarnog proizvoda

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$$

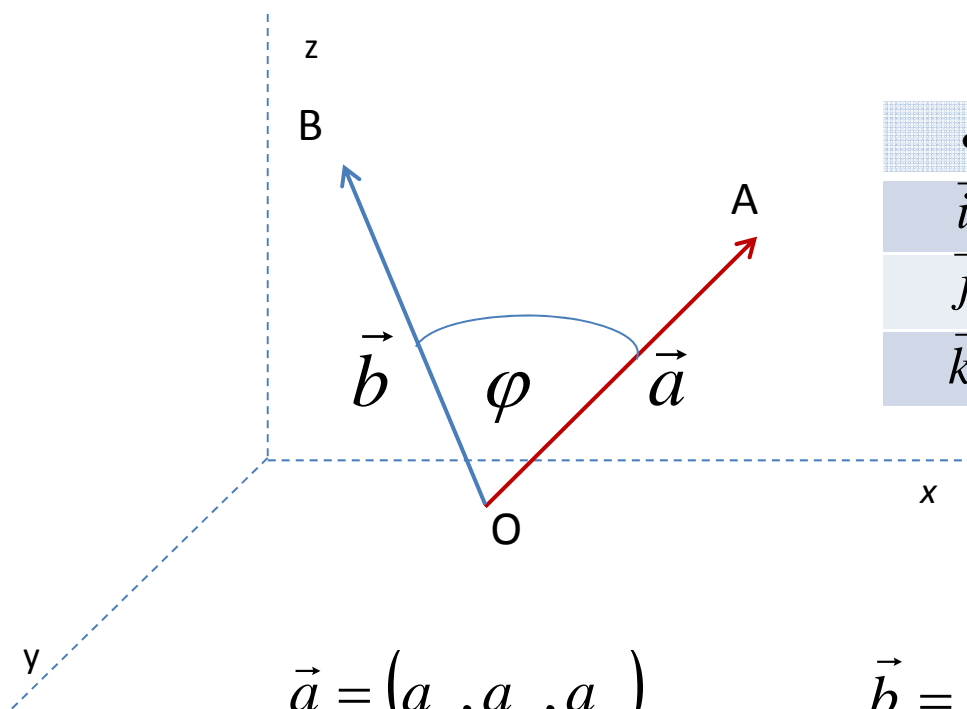
$$(\lambda \vec{a}) \cdot \vec{b} = \vec{a} \cdot (\lambda \vec{b}) = \lambda (\vec{a} \cdot \vec{b})$$

$$\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$$

$$\vec{a} \cdot \vec{a} = |\vec{a}|^2 = |\vec{a}|^2 \geq 0$$

$$\vec{a} \cdot \vec{a} = 0 \Leftrightarrow \vec{a} = \vec{0}$$

SKALARNI PROIZVOD VEKTORA ZADATIH POMOĆU KOORDINATA



\cdot	\vec{i}	\vec{j}	\vec{k}
\vec{i}	1	0	0
\vec{j}	0	1	0
\vec{k}	0	0	1

$$\vec{a} = (a_x, a_y, a_z) \quad \vec{b} = (b_x, b_y, b_z)$$

$$\vec{a} = a_x \vec{i} + a_y \vec{j} + a_z \vec{k} \quad \vec{b} = b_x \vec{i} + b_y \vec{j} + b_z \vec{k}$$

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$$

PRIMER

Izračunati skalarni proizvod datih vektora:

$$\vec{a} = (1, 1, -2) \quad \vec{b} = (1, -1, 4)$$

$$\vec{a} \cdot \vec{b} = 1 \cdot 1 + 1 \cdot (-1) + (-2) \cdot 4 = -8$$

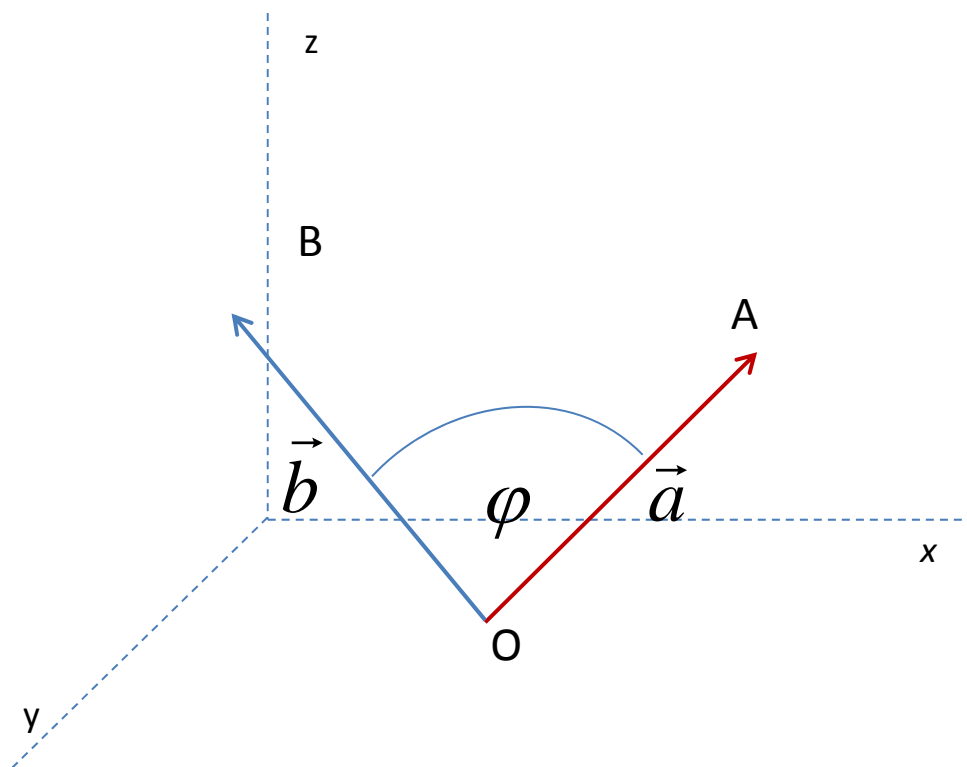
$$\vec{a} = (1, 5, -2) \quad \vec{b} = (1, -1, 4)$$

$$\vec{a} \cdot \vec{b} = 1 \cdot 1 + 5 \cdot (-1) + (-2) \cdot 4 = -12$$

$$\vec{a} = (1, -2, -2) \quad \vec{b} = (1, -1, 3)$$

$$\vec{a} \cdot \vec{b} = 1 \cdot 1 + (-2) \cdot (-1) + (-2) \cdot 3 = -3$$

SKALARNI PROIZVOD VEKTORA - USLOV ORTOGONALNOSTI DVA VEKTORA



$$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos \varphi$$

Dva nenula vektora su ortogonalni ako i samo ako je njihov skalarni proizvod jednak nuli.

$$\vec{a} \cdot \vec{b} = 0 \Leftrightarrow \varphi = 90^0$$

Uslov ortogonalnosti vektora zadatih pomoću koordinata

$$\vec{a} = (a_x, a_y, a_z) \quad \vec{b} = (b_x, b_y, b_z)$$

$$\vec{a} \cdot \vec{b} = 0 \Leftrightarrow a_x b_x + a_y b_y + a_z b_z = 0$$

PRIMER

Vektori

$$\vec{a} = (-3, 3, 2) \quad \vec{b} = (4, 6, -3)$$

Su ortogonalni zato sto je

$$\vec{a} \cdot \vec{b} = (-3) \cdot 4 + 3 \cdot 6 + 2 \cdot (-3) = -12 + 18 - 6 = 0$$

Vektori

$$\vec{a} = (-3, 3, 2) \quad \vec{b} = (4, 6, -2)$$

nisu ortogonalni zato sto je

$$\vec{a} \cdot \vec{b} = (-3) \cdot 4 + 3 \cdot 6 + 2 \cdot (-2) = -12 + 18 - 4 = 2 \neq 0$$

PRIMER

Odrediti vrednost parametra λ tako da su vektori $\vec{a} = (\lambda, 3, 2)$ i $\vec{b} = (4, 6, \lambda)$ ortogonalni.

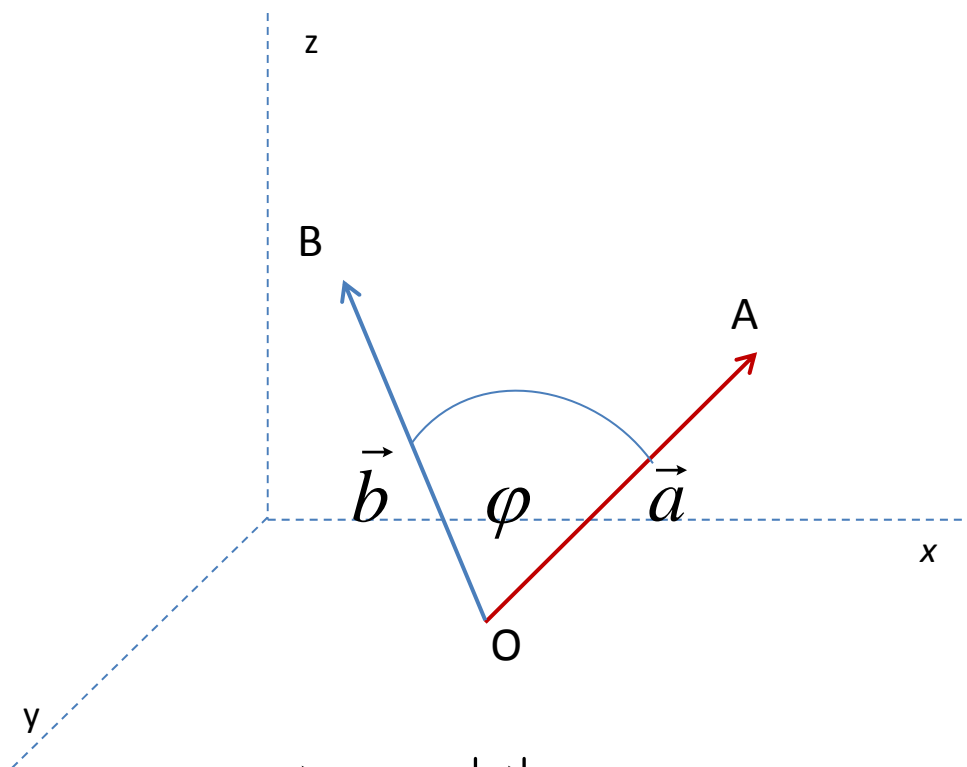
$$\vec{a} \perp \vec{b} \Leftrightarrow \vec{a} \cdot \vec{b} = 0$$

$$\vec{a} \cdot \vec{b} = \lambda \cdot 4 + 3 \cdot 6 + 2 \cdot \lambda = 0$$

$$6\lambda + 18 = 0$$

$$\lambda = -3$$

SKALARNI PROIZVOD VEKTORA - UGAO IZMEDJU DVA VEKTORA



$$\vec{a} = (a_x, a_y, a_z)$$

$$\vec{b} = (b_x, b_y, b_z)$$

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$$

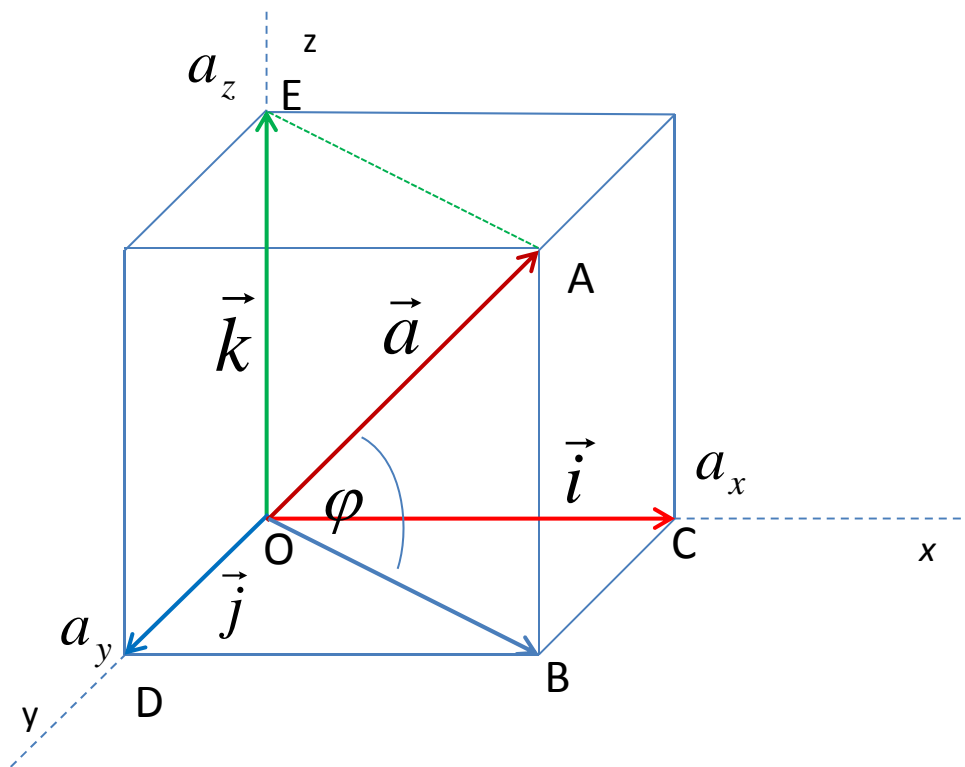
$$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos \varphi \quad \cos \varphi = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|}$$

$$\cos \varphi = \frac{a_x b_x + a_y b_y + a_z b_z}{\sqrt{a_x^2 + a_y^2 + a_z^2} \sqrt{b_x^2 + b_y^2 + b_z^2}}$$

PRIMER

Data je kocka osnovne ivice $a = 1$ sa jednim temenom u koordinatnom početku O i sa ivicama koje polaze iz tog temena koje leze na pozitivnim delovima koordinatnih osa. OA je dijagonala te kocke. OB je dijagonala donje osnove te kocke.

Odrediti kosinus ugla izmedju vektora \overrightarrow{OA} i \overrightarrow{OB}



$$\vec{a} = \overrightarrow{OA} = (1, 1, 1)$$

$$\vec{b} = \overrightarrow{OB} = (1, 1, 0)$$

$$|\vec{a}| = \sqrt{3} \quad |\vec{b}| = \sqrt{2}$$

$$\cos \varphi = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|}$$

$$\cos \varphi = \frac{1 \cdot 1 + 1 \cdot 1 + 1 \cdot 0}{\sqrt{2} \cdot \sqrt{3}} = \frac{\sqrt{6}}{3}$$

PRIMER

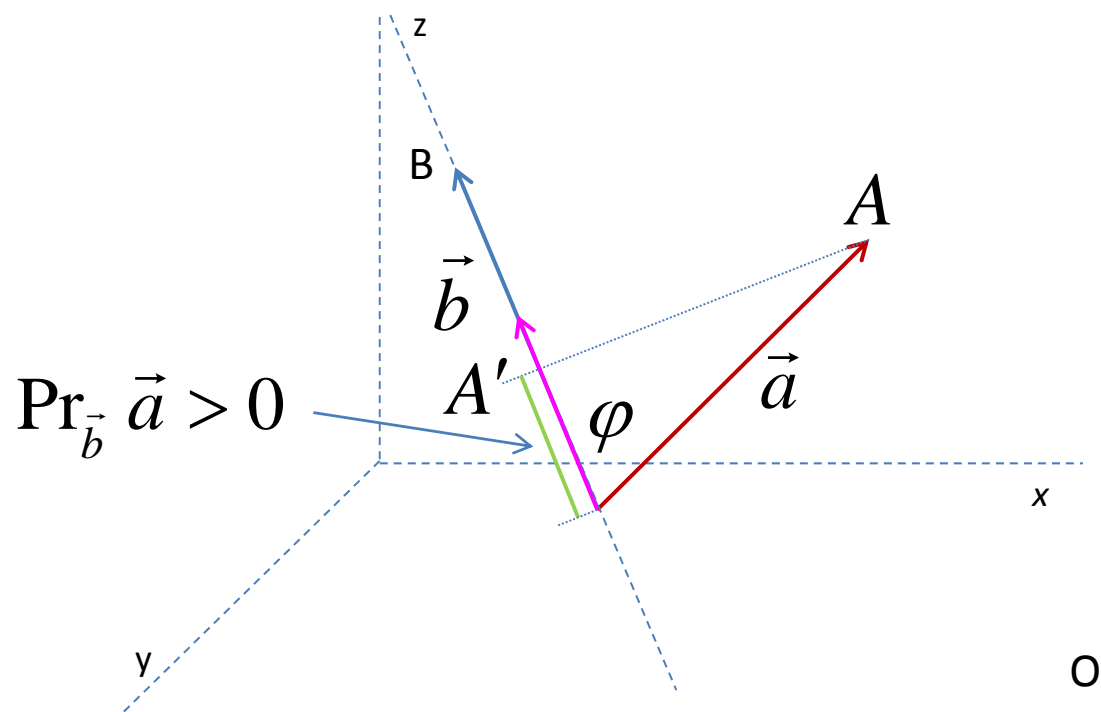
Odrediti ugao između vektora $\vec{a} = (1, 0, 1)$ i $\vec{b} = (0, -1, 1)$.

$$\cos \varphi = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|}$$

$$\cos \varphi = \frac{1 \cdot 0 + 0 \cdot (-1) + 1 \cdot 1}{\sqrt{1^2 + 0^2 + 1^2} \cdot \sqrt{0^2 + (-1)^2 + 1^2}} = \frac{1}{\sqrt{2} \cdot \sqrt{2}} = \frac{1}{2}$$

$$\varphi = \frac{\pi}{3} = 60^\circ$$

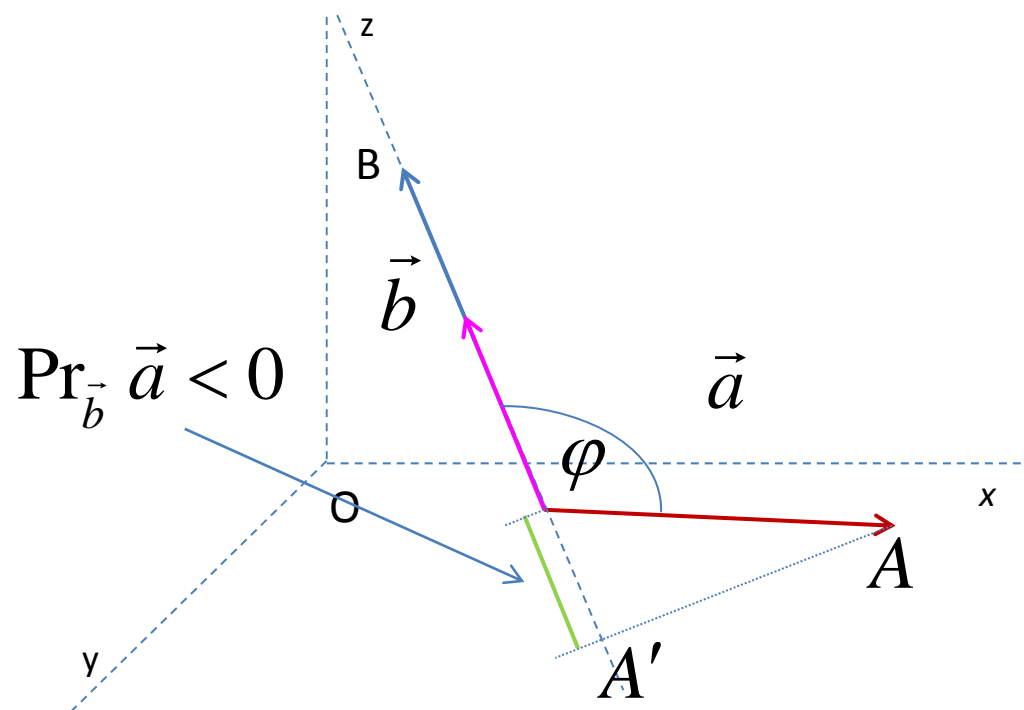
ORTOGONALNA ALGEBARSKA PROJEKCIJA VEKTORA



$$\text{ort } \vec{b} = \frac{\vec{b}}{|\vec{b}|}$$

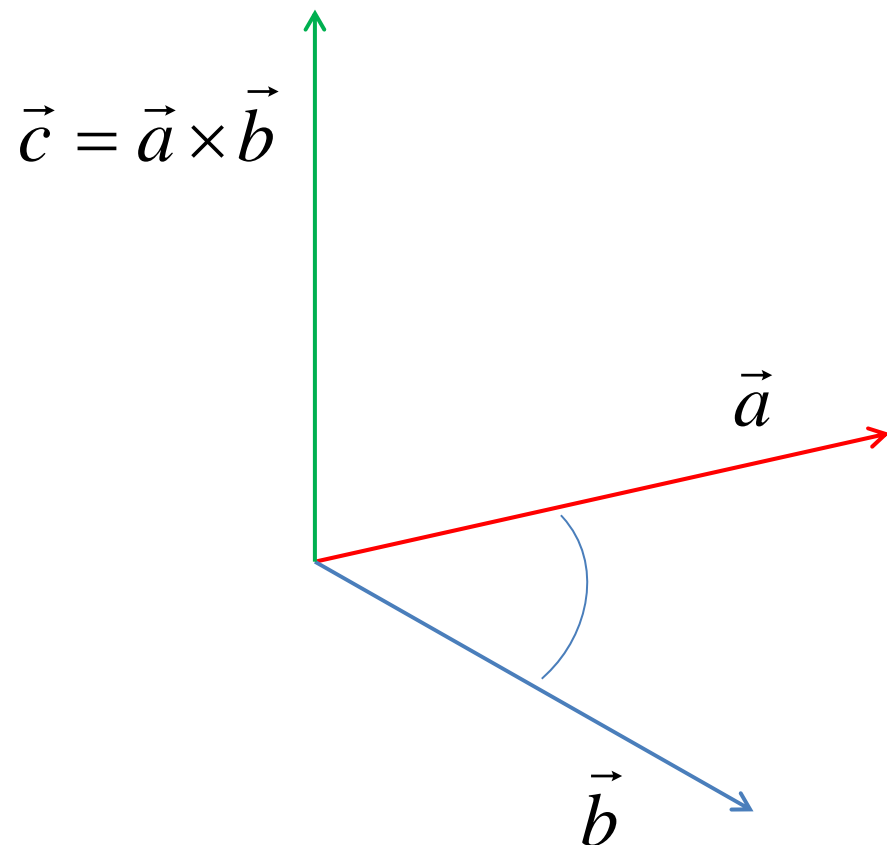
$$\text{Pr}_{\vec{b}} \vec{a} = |\vec{a}| \cdot \cos \varphi = \vec{a} \cdot \text{ort } \vec{b}$$

ORTOGONALNA ALGEBARSKA PROJEKCIJA VEKTORA



$$\text{ort } \vec{b} = \frac{\vec{b}}{|\vec{b}|} \quad \text{Pr}_{\vec{b}} \vec{a} = |\vec{a}| \cdot \cos \varphi = \vec{a} \cdot \text{ort } \vec{b}$$

VEKTORSKI PROIZVOD



Vektorski proizvod vektora \vec{a} i \vec{b} je vektor

$$\vec{c} = \vec{a} \times \vec{b}$$

čiji je intezitet jednak površini paralelograma konstruisanog nad

vektorima \vec{a} i \vec{b} ,

pravac normalan na ravan odredjenu tim vektorima,

a smer je takav da vektori

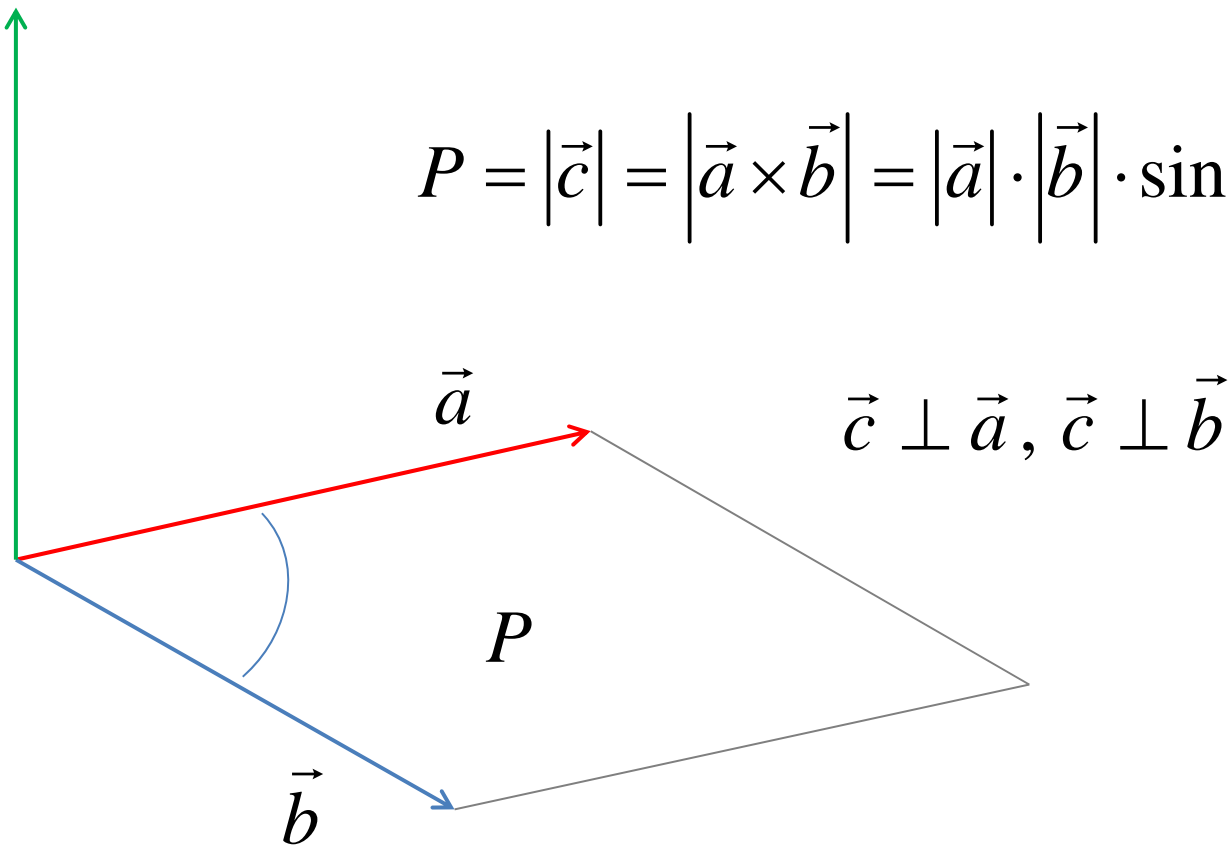
$$\vec{a}, \vec{b}, \vec{c}$$

čine trijedar iste orijentacije kao i koordinatni sistem.

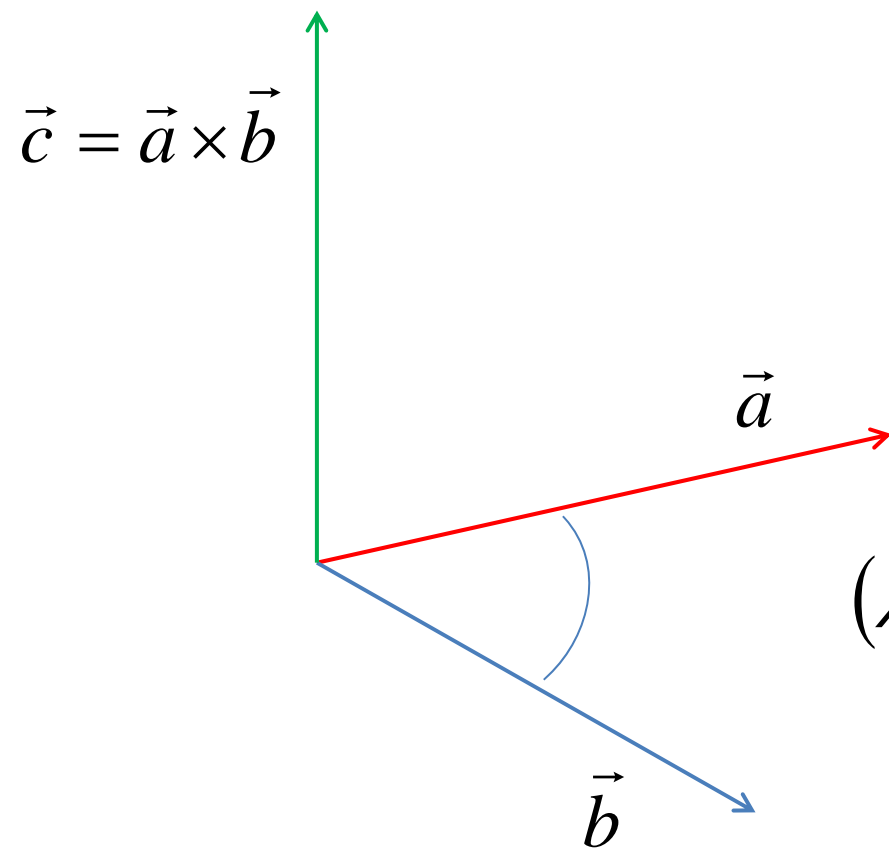
VEKTORSKI PROIZVOD

$$\vec{c} = \vec{a} \times \vec{b}$$

$$P = |\vec{c}| = |\vec{a} \times \vec{b}| = |\vec{a}| \cdot |\vec{b}| \cdot \sin \angle(\vec{a}, \vec{b})$$



VEKTORSKI PROIZVOD - OSOBINE

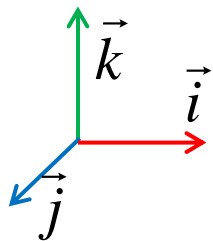


$$\vec{b} \times \vec{a} = -\vec{a} \times \vec{b}$$

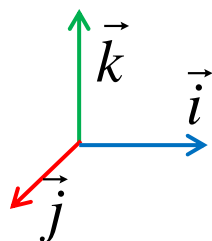
$$(\lambda \vec{a}) \times \vec{b} = \vec{a} \times (\lambda \vec{b}) = \lambda (\vec{a} \times \vec{b})$$

$$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$$

VEKTORSKI PROIZVOD - IZRAČUNAVANJE



\times	\vec{i}	\vec{j}	\vec{k}
\vec{i}	0	\vec{k}	$-\vec{j}$
\vec{j}	$-\vec{k}$	0	\vec{i}
\vec{k}	\vec{j}	$-\vec{i}$	0



$$\vec{a} = (a_x, a_y, a_z) \quad \vec{a} = a_x \vec{i} + a_y \vec{j} + a_z \vec{k}$$

$$\vec{b} = (b_x, b_y, b_z) \quad \vec{b} = b_x \vec{i} + b_y \vec{j} + b_z \vec{k}$$

$$\vec{a} \times \vec{b} = (a_x \vec{i} + a_y \vec{j} + a_z \vec{k}) \times (b_x \vec{i} + b_y \vec{j} + b_z \vec{k})$$

$$\vec{a} \times \vec{b} = \vec{i}(a_y b_z - a_z b_y) - \vec{j}(a_x b_z - a_z b_x) + \vec{k}(a_x b_y - a_y b_x)$$

VEKTORSKI PROIZVOD - IZRAČUNAVANJE

$$\vec{a} \times \vec{b} = \vec{i} \begin{vmatrix} a_y & a_z \\ b_y & b_z \end{vmatrix} - \vec{j} \begin{vmatrix} a_x & a_z \\ b_x & b_z \end{vmatrix} + \vec{k} \begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \vec{i} \begin{vmatrix} a_y & a_z \\ b_y & b_z \end{vmatrix} - \vec{j} \begin{vmatrix} a_x & a_z \\ b_x & b_z \end{vmatrix} + \vec{k} \begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix}$$

DETERMINANTE DRUGOG I TREĆEG REDA

Izračunati determinantu, drugog I treceg reda

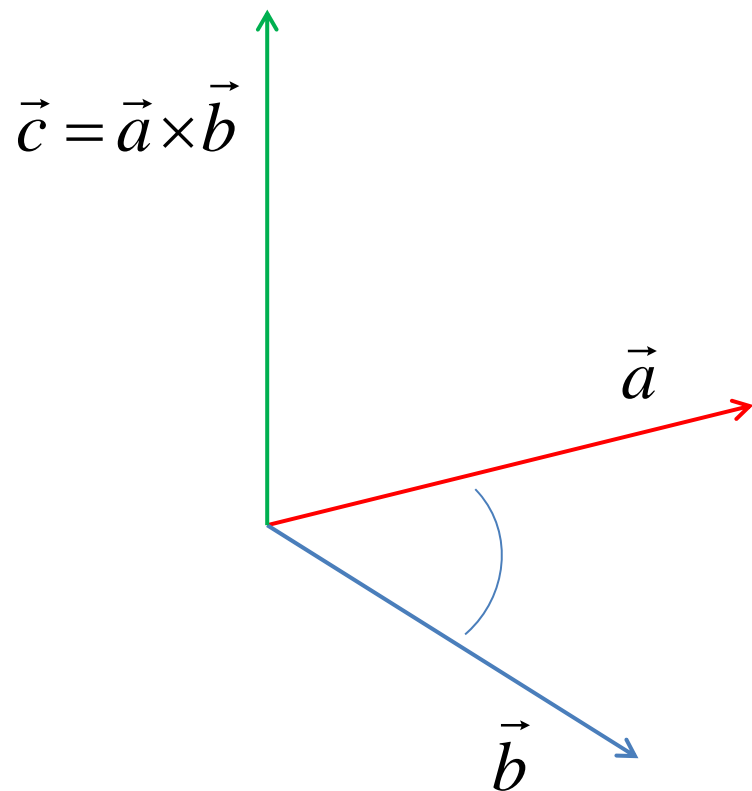
$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11} \cdot a_{22} - a_{12} \cdot a_{21}$$

$$\begin{vmatrix} -1 & 3 \\ -3 & 4 \end{vmatrix} = (-1) \cdot 4 - 3 \cdot (-3) = 5$$

$$\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix} \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$\begin{vmatrix} 2 & 1 & -2 \\ -1 & 3 & -2 \\ -3 & 4 & 1 \end{vmatrix} = 2 \cdot \begin{vmatrix} 3 & -2 \\ 4 & 1 \end{vmatrix} - \begin{vmatrix} -1 & -2 \\ -3 & 1 \end{vmatrix} - 2 \begin{vmatrix} -1 & 3 \\ -3 & 4 \end{vmatrix} = 22 + 7 - 10 = 19$$

PRIMER



Izračunati vektorski proizvod vektora

$$\vec{a} = (1, -1, 2)$$

$$\vec{b} = (-1, 1, 0)$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 2 \\ -1 & 1 & 0 \end{vmatrix} = \vec{i} \begin{vmatrix} -1 & 2 \\ 1 & 0 \end{vmatrix} - \vec{j} \begin{vmatrix} 1 & 2 \\ -1 & 0 \end{vmatrix} + \vec{k} \begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix}$$

$$\vec{a} \times \vec{b} = -2\vec{i} - 2\vec{j}$$

$$\vec{a} \times \vec{b} = (-2, -2, 0)$$

PRIMER

Za date vektore $\vec{a} = (1, 1, -2)$ i $\vec{b} = (1, 0, 2)$

Izračunati $\vec{a} \times \vec{b}$ i $\vec{b} \times \vec{a}$.

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & -2 \\ 1 & 0 & 2 \end{vmatrix} = \vec{i} \begin{vmatrix} 1 & -2 \\ 0 & 2 \end{vmatrix} - \vec{j} \begin{vmatrix} 1 & -2 \\ 1 & 2 \end{vmatrix} + \vec{k} \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix}$$

$$\vec{a} \times \vec{b} = 2\vec{i} - 4\vec{j} - \vec{k}$$

$$\vec{a} \times \vec{b} = (2, -4, -1)$$

$$\vec{b} \times \vec{a} = -\vec{a} \times \vec{b} = -(2, -4, -1) = (-2, 4, 1)$$

PRIMER

Izračunati površinu paralelograma konstruisanog nad datim vektorima:

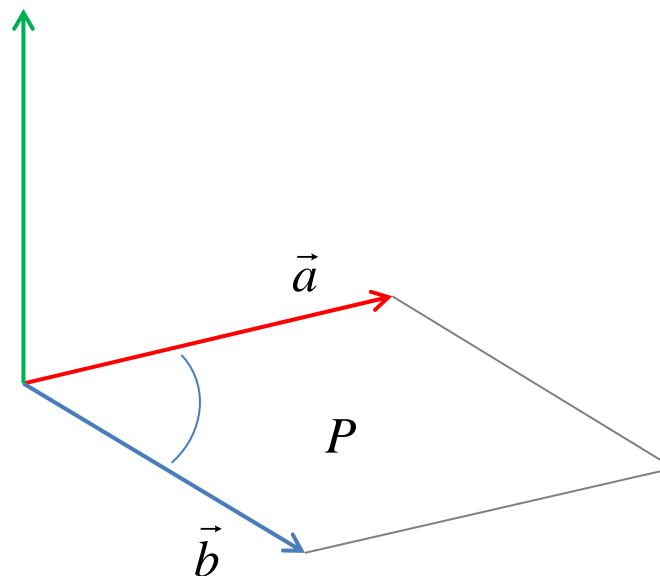
$$\vec{a} = (1, 5, -2) \quad \vec{b} = (1, 0, -2)$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 5 & -2 \\ 1 & 0 & -2 \end{vmatrix}$$

$$\vec{a} \times \vec{b} = \vec{i} \begin{vmatrix} 5 & -2 \\ 0 & 2 \end{vmatrix} - \vec{j} \begin{vmatrix} 1 & -2 \\ 1 & -2 \end{vmatrix} + \vec{k} \begin{vmatrix} 1 & 5 \\ 1 & 0 \end{vmatrix}$$

$$\vec{a} \times \vec{b} = 10\vec{i} - 5\vec{k} \quad \vec{a} \times \vec{b} = (10, 0, -5)$$

$$P = |\vec{a} \times \vec{b}| = \sqrt{10^2 + 0^2 + (-5)^2} = \sqrt{125} = 5\sqrt{5}$$



PRIMER

Izračunati površinu trougla konstruisanog nad datim vektorima:

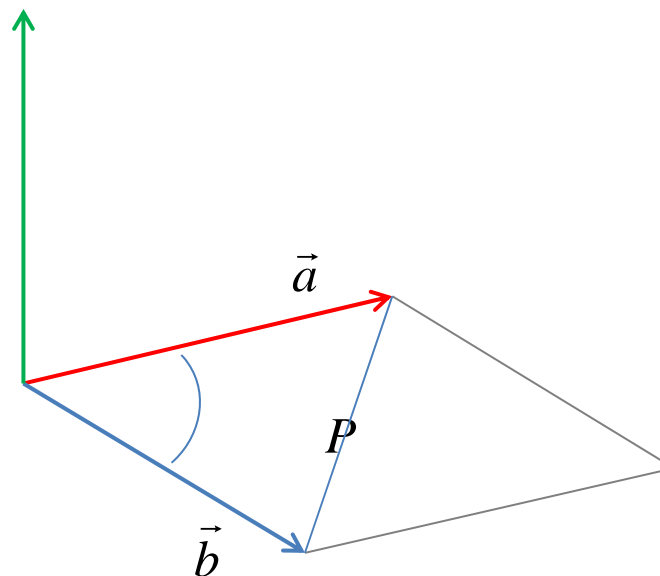
$$\vec{a} = (1, 5, -2) \quad \vec{b} = (1, 0, -2)$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 5 & -2 \\ 1 & 0 & -2 \end{vmatrix}$$

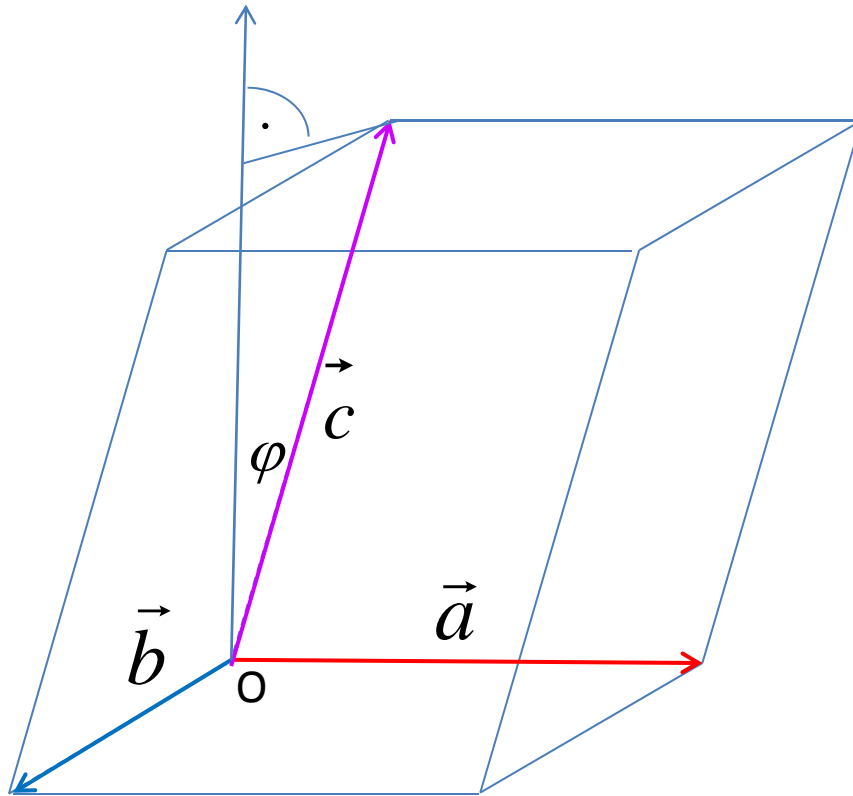
$$\vec{a} \times \vec{b} = \vec{i} \begin{vmatrix} 5 & -2 \\ 0 & 2 \end{vmatrix} - \vec{j} \begin{vmatrix} 1 & -2 \\ 1 & -2 \end{vmatrix} + \vec{k} \begin{vmatrix} 1 & 5 \\ 1 & 0 \end{vmatrix}$$

$$\vec{a} \times \vec{b} = 10\vec{i} - 5\vec{k} \quad \vec{a} \times \vec{b} = (10, 0, -5)$$

$$P = \frac{1}{2} |\vec{a} \times \vec{b}| = \frac{1}{2} \sqrt{10^2 + 0^2 + (-5)^2} = \frac{1}{2} \sqrt{125} = \frac{5\sqrt{5}}{2}$$



MEŠOVITI PROIZVOD VEKTORA



Mašoviti proizvod tri vektora

$$\vec{a}, \vec{b}, \vec{c}$$

je skalarni proizvod

vektora $\vec{a} \times \vec{b}$ i vektora \vec{c} :

$$[\vec{a}, \vec{b}, \vec{c}] = (\vec{a} \times \vec{b}) \cdot \vec{c}$$

$$[\vec{a}, \vec{b}, \vec{c}] = (\vec{a} \times \vec{b}) \cdot \vec{c} = |\vec{a} \times \vec{b}| \cdot |\vec{c}| \cdot \cos \varphi = B \cdot (\pm H) = \pm V$$

MEŠOVITI PROIZVOD VEKTORA

Mešoviti proizvod tri vektora $\vec{a}, \vec{b}, \vec{c}$ je skalar, čija je apsolutna vrednost jednaka zapremini paralelopipeda konstruisanog nad tim vektorima.

Znak mešovitog proizvoda označava orijentaciju trijedra koji grade ti vektori u odnosu na orijentaciju koordinatnog sistema. Pozitivan rezultat označava da je trijedar orijentisan isto kao i koordinatni sistem, a negativan rezultat označava da je njegova orijentacija suprotna od orijentacije koordinatnog sistema.

MEŠOVITI PROIZVOD VEKTORA

Cikličkom permutacijom vektora, vrednost mešovitog proizvoda se ne menja.

$$[\vec{a}, \vec{b}, \vec{c}] = [\vec{b}, \vec{c}, \vec{a}] = [\vec{c}, \vec{a}, \vec{b}]$$

$$(\vec{a} \times \vec{b}) \cdot \vec{c} = (\vec{b} \times \vec{c}) \cdot \vec{a} = (\vec{c} \times \vec{a}) \cdot \vec{b}$$

MEŠOVITI PROIZVOD VEKTORA

$$\vec{a} = (a_x, a_y, a_z)$$

$$\vec{b} = (b_x, b_y, b_z)$$

$$\vec{c} = (c_x, c_y, c_z)$$

$$[\vec{a}, \vec{b}, \vec{c}] = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}$$

MEŠOVITI PROIZVOD VEKTORA – USLOV KOMPLANARNOSTI

Tri vektora $\vec{a}, \vec{b}, \vec{c}$ su komplanarni ako i samo ako je njihov mešoviti proizvod jednak nuli.

USLOV KOMPLANARNOSTI TRI VEKTORA

$$\vec{a} = (a_x, a_y, a_z)$$

$$\vec{b} = (b_x, b_y, b_z)$$

$$\vec{c} = (c_x, c_y, c_z)$$

$$[\vec{a}, \vec{b}, \vec{c}] = (\vec{a} \times \vec{b}) \cdot \vec{c} = 0$$

$$[\vec{a}, \vec{b}, \vec{c}] = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix} = 0$$

PRIMER

Vektori $\vec{a} = (1, -2, 1)$ $\vec{b} = (2, 2, -2)$ $\vec{c} = (0, 2, -1)$ $[\vec{a}, \vec{b}, \vec{c}] = 2$

su nekomplanarni zato sto je

$$[\vec{a}, \vec{b}, \vec{c}] = \begin{vmatrix} 1 & -2 & 1 \\ 2 & 2 & -2 \\ 0 & 2 & -1 \end{vmatrix} = -2 \cdot \begin{vmatrix} 1 & 1 \\ 2 & -2 \end{vmatrix} - \begin{vmatrix} 1 & -2 \\ 2 & 2 \end{vmatrix} = 8 - 6 = 2 \neq 0$$

Vektori $\vec{a} = (1, -2, 3)$ $\vec{b} = (2, 3, -2)$ $\vec{c} = (3, 1, 1)$

su komplanarni zato sto je

$$[\vec{a}, \vec{b}, \vec{c}] = (\vec{a} \times \vec{b}) \cdot \vec{c} = \begin{vmatrix} 1 & -2 & 3 \\ 2 & 3 & -2 \\ 3 & 1 & 1 \end{vmatrix} = 1 \cdot \begin{vmatrix} 3 & -2 \\ 1 & 1 \end{vmatrix} + 2 \cdot \begin{vmatrix} 2 & -2 \\ 3 & 1 \end{vmatrix} + 3 \cdot \begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix}$$

$$[\vec{a}, \vec{b}, \vec{c}] = 5 + 16 - 21 = 0$$

PRIMER

Izračunati zapreminu paralelepipeda konstruisanog nad vektorima

$$\vec{a} = (1, -1, 2) \quad \vec{b} = (-1, 1, 0) \quad \vec{c} = (1, 2, 0)$$

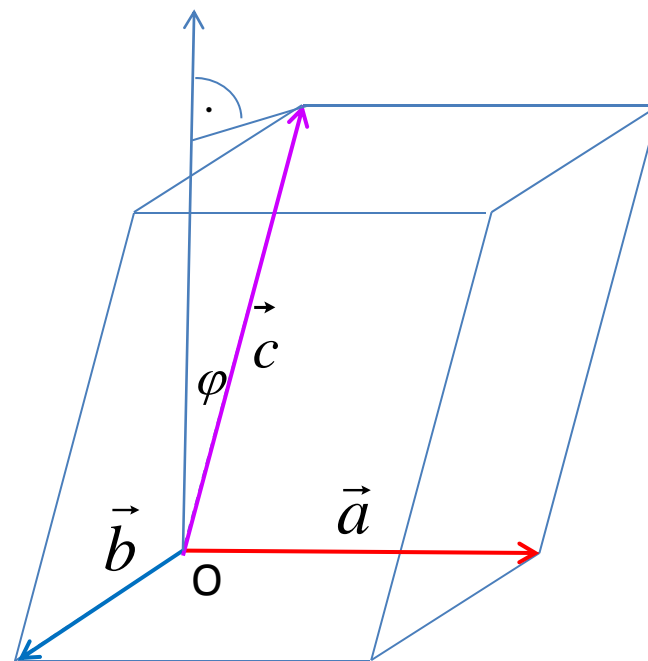
i odrediti orijentaciju u odnosu na orijentaciju koordinatnog sistema.

$$[\vec{a}, \vec{b}, \vec{c}] = (\vec{a} \times \vec{b}) \cdot \vec{c} = \begin{vmatrix} 1 & -1 & 2 \\ -1 & 1 & 0 \\ 1 & 2 & 0 \end{vmatrix}$$

$$[\vec{a}, \vec{b}, \vec{c}] = 2 \cdot \begin{vmatrix} -1 & 1 \\ 1 & 2 \end{vmatrix} = 2 \cdot (-3) = -6$$

$$V = |[\vec{a}, \vec{b}, \vec{c}]| = |-6| = 6$$

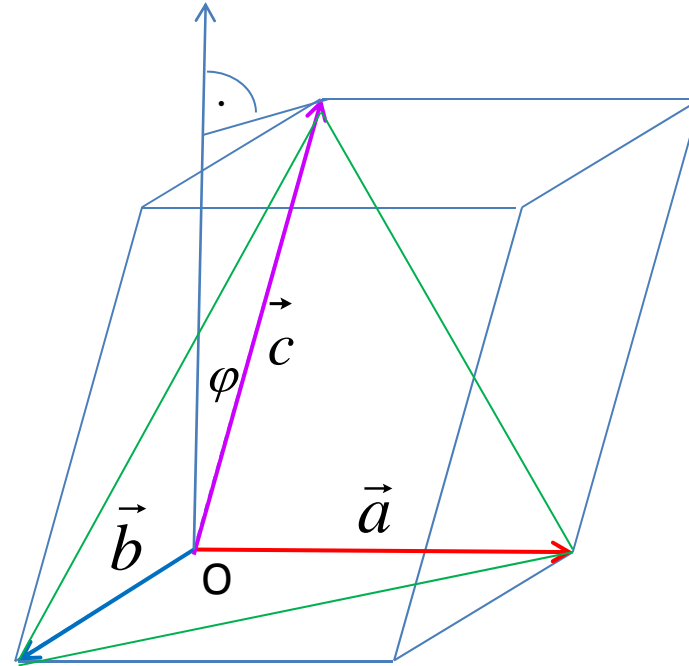
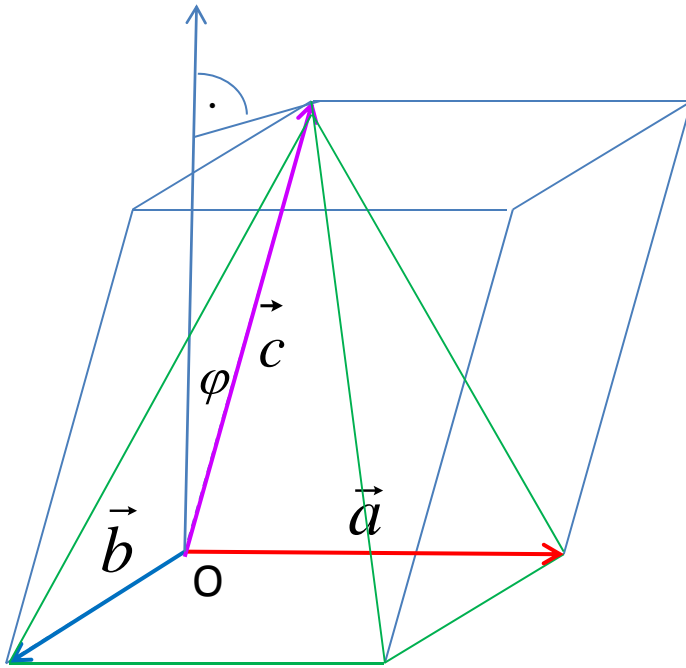
$[\vec{a}, \vec{b}, \vec{c}] = -6 < 0$ Trijedar $\vec{a}, \vec{b}, \vec{c}$ suprotne orijentacije u odnosu na koordinatni sistem.



PRIMER

$$V_{\text{paraleloipeda}} = |[\vec{a}, \vec{b}, \vec{c}]|$$

$$V_{\text{paraleloipeda}} = BH$$



$$V_{\text{piramide}} = \frac{1}{3} BH = \frac{1}{3} V_{\text{paraleloipeda}} = \frac{1}{3} |[\vec{a}, \vec{b}, \vec{c}]|$$

$$V_{\text{tetraedra}} = \frac{1}{3} B_1 H = \frac{1}{3} \frac{1}{2} BH = \frac{1}{6} V_{\text{paraleloipeda}} = \frac{1}{6} |[\vec{a}, \vec{b}, \vec{c}]|$$