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Diffraction of laser light on sharp edge

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Abstract: In the diffraction pattern of a laser light on the sharp edge of a half plane, two regiones can be noticed. In the central region we see deflection of wave into the region of the geometrical shaddow, while in the unscreened region the oscilations of intensity are seen. Very long light traces (plume) were noticed recently on both sides of the edge along the direction normal to the edge of the obstacle. Theoretical explanation is based on the Fresnel-Kirchhof diffraction theory applied on the Gaussian beam propagation behind the half plane.

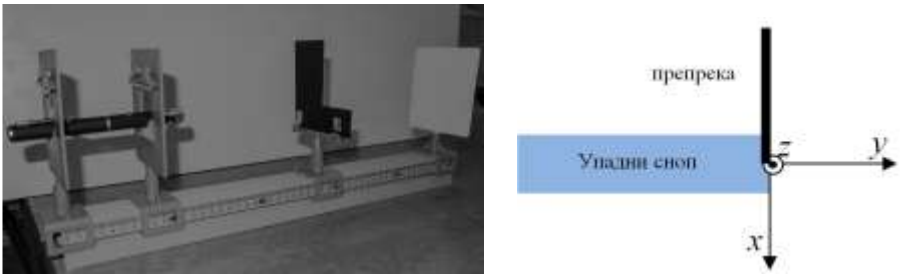
Keywords: diffraction on the sharp edge of a half plane, Gaussian beam

INTRODUCTION

Diffraction is a typical behavior for all kinds of waves. The waves in homogeneous medium propagate in straight lines but when reaching an obstacle they bend entering the space behind the obstacle, as can be clearly seen for water waves. For light this behavior is not so obvious. The scientist in Ancient Greece have considered that the main characteristic of light is namely its straight line propagation. To that conclusion they came by observing sharp edges of shadows that opaque objects led by small light sources, cast on walls. They have concluded that the shadow on the screen is formed in the loci where the straight lines that connect sources with the edges of object would pass through the observation screen. Newton in his “Opticks” [1] comparing the propagation of sound waves with the propagation of light writes: “The waves on the surface of stagnated water, passing by sides of a broad obstacle which stops part of them, bends afterwards and dilate themselves gradually into the quiet water behind the obstacle. The waves, pulses or vibrations of the air, wherein sound consist, bend manifestly, though not so much as the waves of water. For a bell or a cannon may be heard beyond a hill which intercepts the sight of the sounding body, and sounds are propagating readily through crooked pipes as through straight ones**.** But light is never known to follow crooked passages, nor to bend into the shadow”. That led him to the conclusion that the light consists of particles. Therefore for teaching physicist is of great importance to introduce the experiments that demonstrate diffraction of light that is the wave nature of light [2].

A theoretical solution of the plane wave diffraction problem on the sharp edge gave Sommerfeld in 1896, and this solution became the ground for theoretically solving diffraction problems on various two dimensional obstacles [3-5]. The diffraction of the Gaussian beam on the sharp edge has been studied since the late sixties of the last century [6], but the attention was mainly focused on the central part of the diffraction pattern [7,8], which is very intensive and has dimensions comparable to the dimensions of the incident laser beam cross-section. One experimental problem on 40th International Physical Olympiad, that took place in Mexico, was based on diffraction of extended laser beam on the razor edge [8]. The question was how to determine the wavelength of the laser by studding the central part of the diffraction pattern. A competitor from Serbia, winner of a gold medal, was ranked sixth for this problem with 18 out of 20 points [8]. Unlike the central part of the diffraction pattern which was analyzed from 1960 [6], studding of the side pattern that is less intensive, but with length ten and more times longer was analyzed much later [9].

EXPERIMENTAL SETUP AND DIFFRACTION PATTERN

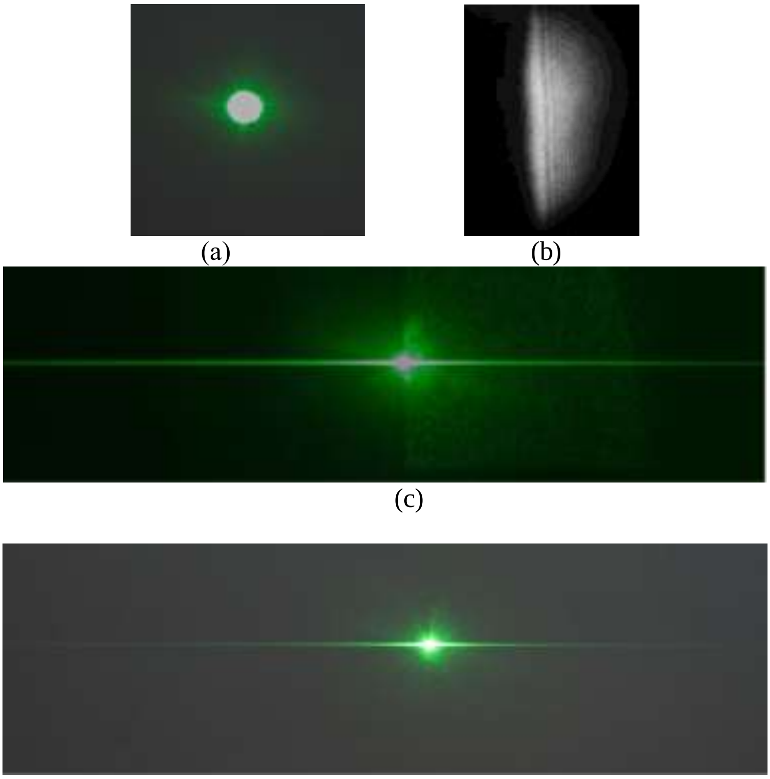


(a) (b)

FIGURE 1. (a) Experimental setup: laser pointer, obstacle with sharp edge and the observation screen on optical bench (b) Drawing of a laser beam which propagates along the y axis and reaches a half plane obstacle with the edge along the z axis.

In Figure 1a the optical bench with the green laser pointer with wavelength $\lambda = 532$ nm, an opaque obstacle to which a razor is taped and a white observation screen are shown. The central part of the diffraction pattern is observed on the observation screen while the side pattern is observed on the white wall when the observation screen is removed. An opaque obstacle is placed so that its edge passes through the vertical center of the beam, so that it covers approximately one half of the laser beam, as shown in Figure 1b.

If the laser beam is allowed to freely propagate on the screen, we observe the light spot of circular shape as shown in Figure 2a. If half of the central part of the beam is shadowed the diffraction pattern is shown in Figure 2b. In the diffraction picture, which is shown in Figures 2c and 2g, one can clearly distinguish the central part of high intensity and long horizontal lines of low-intensity light, which can be clearly seen in a lit room, so the experiment is suitable to be carried out in the classroom. If we want to observe the fluctuations of intensity in the central area (Figure 2a) it would be necessary to use a beam expander [7,8].



(a) (b)

(c)

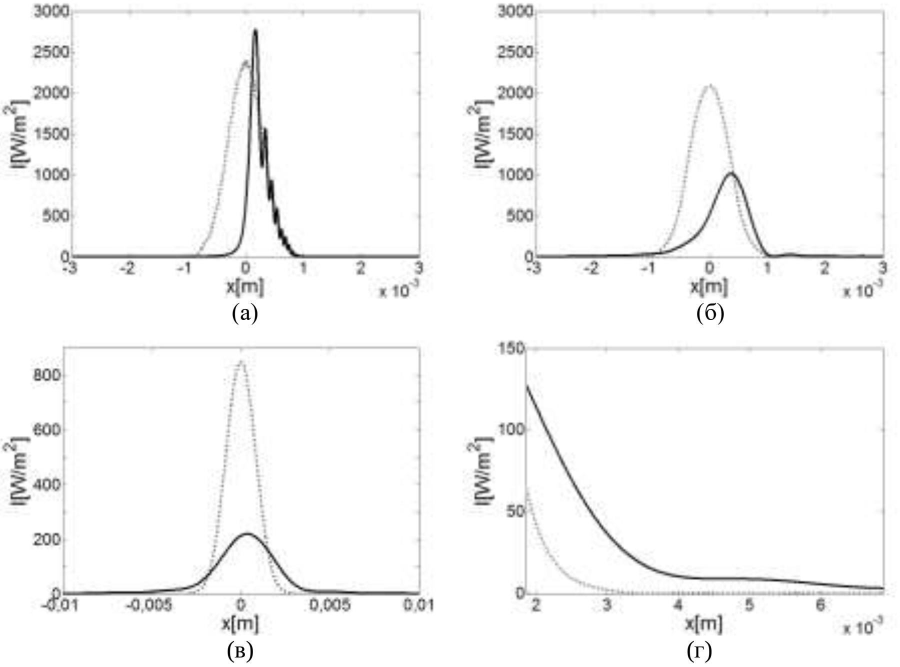
(d)

FIGURE 2. (a) Freely propagating laser beam cross section (b) central part of the diffraction pattern and diffraction patterns at the distances (c) y=1m and (d) y=3m behind the razor blade

THEORETICAL ANALYSYS

Light is the electromagnetic wave, with its electric and magnetic fields satisfying the wave equation in vacuum, which for the electric field reads: $\nabla^2 \vec{E}(\vec{r},t) - \frac{1}{c^2}\frac{\partial^2 \vec{E}(\vec{r},t)}{\partial t^2} = 0$ [5]. Here $\vec{E}$ is the electric field vector, $\vec{r}$ is the position vector, t time, c the speed of light. Let us study a monochromatic wave $\vec{E}(\vec{r},t) = \vec{E}(\vec{r}) \cdot e^{-i\omega t}$, where ω is the circular frequency of the wave. It can be easily seen that the complex amplitude of the electrical field $\vec{E}(\vec{r})$ satisfies the Helmholtz equation $\nabla^2 \vec{E}(\vec{r}) + k^2 \vec{E}(\vec{r}) = 0$, where $k = \omega / c = 2\pi / \lambda$ is the wave number and λ is the wavelength. The problem of diffraction is reduced to finding solutions of the Helmholtz equation behind obstacles that satisfy appropriate boundary conditions at the obstacle.

We will consider a simplified case where electric and magnetic fields do not dependent on the z coordinate, according to the Figure 1b. Then, regardless of the polarization of incident light, electric and magnetic fields can be expressed by a scalar function also satisfying the Helmholtz equation. The solution can be written in the form of a Fresnel-Kirchhoff integral as in [5], that is, since the dimensions of the incident beam is small in relation to the distance between the obstacle and the observation screen the integral is reduced to [10]:



(a) (б)

(в) (г)

FIGURE 3. Dependence of light intensity on the transverse coordinate at a distances (a) y = 5 cm, (b) y = 60 cm, (c) and (g) y = 3 m behind the obstacle. The solid line shows the intensity of the diffracted intensity while the dashed line shows the intensity of the freely propagating Gaussian beam:

$$\Psi(x,y) = \sqrt{\frac{k}{2\pi y}} e^{-i\pi/4} e^{iky} \int \Psi_0(x',0^+) e^{ik(x-x')^2/2y} dx', \qquad (1)$$

where $\Psi_0(x',0^+)$ is proportional to the intensity of the electric field directly behind the obstacle. Since a Gaussian beam is incident on the opaque half plane we have:

$$\Psi_0(x',0^+) = \begin{cases} 0, & x' < 0 \\ Ae^{-\frac{x'^2}{4\sigma^2}}, & x' \geq 0 \end{cases}. \qquad (2)$$

As shown in [11] the light intensity is proportional to the square of the modulus of the function $\Psi(x,y)$. By numerical integration of (1), using MATLAB [12], with the parameters, $\lambda = 532\ \mathrm{nm}$, $\sigma = 300\ \mu\mathrm{m}$ we obtained the shape of the intensity shown in Figure 3, which is in the accordance with the experiment, and agrees with the theoretical calculations presented in [9].

CONCLUSION

Intensity oscillations, that can be seen in Figure 3a, are in the accordance with the diffraction stripes that appear in the half of observation screen that is not sheltered from the incident beam (figure 2b). At larger distances from the obstacle (Figures 3c and 3g) it is shown that the intensity of diffracted light decreases much more slowly than in the case of a free beam. This is consistent with the presence of long bright traces on both sides of the edge, which can be seen in the Figures 2b and 2g.

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