MATRICE

Preslikavanje

$$A: \{1, 2, \ldots, m\} \times \{1, 2, \ldots, n\} \rightarrow R$$

je matrica tipa $m \times n$.

$$A = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{m1} & a_{m2} & \ldots & a_{mn} \end{bmatrix}$$

MATRICE

Oznake

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \qquad A = \begin{Vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{Vmatrix} \qquad A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$$

$$A = \left[a_{ij}\right]_{m,n} = \left\|a_{ij}\right\|_{m,n} = \left(a_{ij}\right)_{m,n}$$

MATRICE

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$$

Uređene *n*-torke

$$A_i = \left(a_{i1}, a_{i2}, \dots, a_{in}\right) \quad i = 1, 2, \dots, m$$

su ***vrste*** matrice.

Uređene *m*-torke

$$A^j = \begin{pmatrix} a_{1j} \\ a_{2j} \\ \dots \\ a_{mj} \end{pmatrix} \quad j = 1, 2, \dots, n$$

su ***kolone*** matrice.

MATRICE

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$$

$$A = \left[a_{ij}\right]_{m,n}$$

$m-$ broj vrsta

$n-$ broj kolona

$m \times n-$ Tip matrice

MATRICE

Matrice su istog tipa ako imaju isti broj vrsta i isti broj kolona.

$$A = \begin{bmatrix} 1 & -3 & 2 \\ 0 & 4 & -5 \end{bmatrix} \qquad B = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix} \qquad C = \begin{bmatrix} 2 & 3 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

Matrice *A* tipa 2×3

Matrice *C* tipa 2×3

Matrica *B* tipa 3×3

Matrice *A* i *B* nisu istog tipa.

Matrice *A* i *C* su istog tipa.

MATRICE

Matrice su istog tipa ako imaju isti broj vrsta i isti broj kolona.

$$A = \begin{bmatrix} 1 & -3 & 2 \\ 0 & 4 & -5 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & -2 & 1 \\ 1 & 3 & -4 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

Matrice *A* i *B* imaju isti broj vrsta i kolona i one su istog tipa 2×3

. Matrica *C* tipa 3×2

i nije istog tipa ni sa jednom od datih matrica *A* i *B* koje su tipa 2×3 .

Matrica *A*, koja ima samo jednu vrstu

$$A = \begin{bmatrix} a_1 & a_2 & \dots & a_n \end{bmatrix}$$

zove se ***matrica - vrsta***.

Matrica *B*, koja ima samo jednu kolonu

$$B = \begin{bmatrix} b_1 \\ b_2 \\ \dots \\ b_m \end{bmatrix}$$

zove se ***matrica - kolona.***

MATRICE

Matrica, čiji su svi elementi nule

$$0 = \begin{bmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 \end{bmatrix}$$

zove se ***nula - matrica***, pri čemu je tip matrice proizvoljan. Nula matrica se često označava sa *0*, ukoliko je jasno da se radi o nula-matrici a ne o reanom broju *0*.

MATRICE

Matrice $A = [a_{ij}]_{m,n}$ i $B = [b_{ij}]_{p,q}$

su ***jednake*** ako su istog tipa : $m = p \wedge n = q$

i ako su im svi odgovarajući elementi jednaki, tj. ako je

$$a_{ij} = b_{ij} , \qquad i = 1,2,...,m \qquad j = 1, 2, \ldots , n$$

SABIRANJE MATRICA

Sabiranje matrica je definisano za matrice istog tipa.

$$A = \left[a_{ij}\right]_{m,n} \qquad B = \left[b_{ij}\right]_{m,n}$$

$$A + B = \left[a_{ij} + b_{ij}\right]_{m,n}$$

$$A + B = \begin{bmatrix} a_{11} + b_{11} & \dots & \dots & a_{1n} + b_{1n} \\ a_{21} + b_{21} & \dots & \dots & a_{2n} + b_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} + b_{m1} & \dots & \dots & a_{mn} + b_{mn} \end{bmatrix}$$

SABIRANJE MATRICA

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \qquad B = \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ \dots & \dots & \dots & \dots \\ b_{m1} & b_{m2} & \dots & b_{mn} \end{bmatrix}$$

$$A + B = \begin{bmatrix} a_{11} + b_{11} & \dots & \dots & a_{1n} + b_{1n} \\ a_{21} + b_{21} & \dots & \dots & a_{2n} + b_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} + b_{m1} & \dots & \dots & a_{mn} + b_{mn} \end{bmatrix}$$

MNOŽENJE MATRICA SKALAROM

$$A = \left[a_{ij} \right]_{m,n}$$

$$k\,A = \left[k\,a_{ij} \right]_{m,n}$$

$$k\,A = \begin{bmatrix} k\,a_{11} & k\,a_{12} & \dots & k\,a_{1n} \\ k\,a_{21} & k\,a_{22} & \dots & k\,a_{2n} \\ \dots & \dots & \dots & \dots \\ k\,a_{m1} & k\,a_{m2} & \dots & k\,a_{mn} \end{bmatrix}$$

MATRICE

$$-A = (-1) \cdot A$$ suprotna matrica

$$A - B = A + (-B)$$ razlika dve matrice

MATRICE - PRIMER

Date su matrice $A = \begin{bmatrix} 1 & -2 & 3 \\ 4 & 5 & -6 \end{bmatrix}$ I $B = \begin{bmatrix} 3 & 0 & 2 \\ -7 & 1 & 8 \end{bmatrix}$

$$A + B = \begin{bmatrix} 1+3 & -2+0 & 3+2 \\ 4-7 & 5+1 & -6+8 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 4 & -2 & 5 \\ -3 & 6 & 2 \end{bmatrix}$$

$$3 \cdot A = \begin{bmatrix} 3 \cdot 1 & 3 \cdot (-2) & 3 \cdot 3 \\ 3 \cdot 4 & 3 \cdot 5 & 3 \cdot (-6) \end{bmatrix}$$

$$3A = \begin{bmatrix} 3 & -6 & 9 \\ 12 & 15 & -18 \end{bmatrix}$$

MATRICE - primer

Date su matrice $A = \begin{bmatrix} 1 & -2 & 3 \\ 4 & 5 & -6 \end{bmatrix}$ i $B = \begin{bmatrix} 3 & 0 & 2 \\ -7 & 1 & 8 \end{bmatrix}$

$$2A - 3B = 2 \cdot \begin{bmatrix} 1 & -2 & 3 \\ 4 & 5 & -6 \end{bmatrix} - 3 \cdot \begin{bmatrix} 3 & 0 & 2 \\ -7 & 1 & 8 \end{bmatrix}$$

$$2A - 3B = \begin{bmatrix} 2 & -4 & 6 \\ 8 & 10 & -12 \end{bmatrix} - \begin{bmatrix} 9 & 0 & 6 \\ -21 & 3 & 24 \end{bmatrix}$$

$$2A - 3B = \begin{bmatrix} 2-9 & -4-0 & 6-6 \\ 8-(-21) & 10-3 & -12-24 \end{bmatrix}$$

$$2A - 3B = \begin{bmatrix} -7 & -4 & 0 \\ 29 & 7 & -36 \end{bmatrix}$$

MNOŽENJE MATRICA

Matrice *A* i *B* su ***saglasne***, u tom poretku, ako je broj kolona prve matrice jednak broju vrsta druge matrice.

Množenje je definisano samo za saglasne matrice.

MNOŽENJE MATRICA

$$A = \begin{bmatrix} a_1 & a_2 & \dots & a_n \end{bmatrix} \qquad B = \begin{bmatrix} b_1 \\ b_2 \\ \dots \\ b_n \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} \sum_{k=1}^{n} a_k \, b_k \end{bmatrix}$$ tipa 1×1

koja se, po dogovoru, svodi na broj - skalar iz *R*

pa se može pisati da je $A \cdot B = \sum_{k=1}^{n} a_k \, b_k$

MNOŽENJE MATRICA - primer

$$A = \begin{bmatrix} 2 & -1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 4 \\ 3 \\ -5 \end{bmatrix}$$

$$A \cdot B = 2 \cdot 4 + (-1) \cdot 3 + 0 \cdot (-5) = 5$$

MNOŽENJE MATRICA

Proizvod *dve proizvoljne* ***saglasne*** *matrice*

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1p} \\ a_{21} & a_{22} & \dots & a_{2p} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mp} \end{bmatrix} \quad \text{i} \quad B = \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ \dots & \dots & \dots & \dots \\ b_{p1} & b_{p2} & \dots & b_{pn} \end{bmatrix}$$

tipa $m \times p$ i $p \times n$ je matrica tipa $m \times n$

$$A \cdot B = \begin{bmatrix} A_1 \cdot B^1 & A_1 \cdot B^2 & \dots & A_1 \cdot B^n \\ A_2 \cdot B^1 & A_2 \cdot B^2 & \dots & A_2 \cdot B^n \\ \dots & \dots & \dots & \dots \\ A_m \cdot B^1 & A_m \cdot B^2 & \dots & A_m \cdot B^n \end{bmatrix}$$

gde je A_i *i*-ta vrsta matrice *A*

$i = 1, 2, \dots, m$

B^j *j*-ta kolona matrice *B*

$j = 1, 2, \dots, n$

MNOŽENJE MATRICA

$$A = \left[a_{ik}\right]_{m,p} \qquad B = \left[b_{kj}\right]_{p,n}$$

$$C = A \cdot B = \left[c_{ij}\right]_{m,n}$$

$$c_{ij} = \sum_{k=1}^{p} a_{ik} b_{kj} = a_{i1} b_{1j} + a_{i2} b_{2j} + \ldots + a_{ip} b_{pj}$$

MNOŽENJE MATRICA - primer

$$A = \begin{bmatrix} 2 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & -2 & 0 \\ 4 & 5 & -3 \end{bmatrix}$$

$$AB = \begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 & 0 \\ 4 & 5 & -3 \end{bmatrix} = \begin{bmatrix} 2\cdot 1 + 1\cdot 4 & 2\cdot(-2) + 1\cdot 5 & 2\cdot 0 + 1\cdot(-3) \end{bmatrix}$$

$$AB = \begin{bmatrix} 6 & 1 & -3 \end{bmatrix}$$ tipa 1×3

MNOŽENJE MATRICA - primer

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & -2 & -5 \\ 3 & 4 & 0 \end{bmatrix}$$

$$A\,B = \begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} 1 & -2 & -5 \\ 3 & 4 & 0 \end{bmatrix} = \begin{bmatrix} 2-3 & -4-4 & -10-0 \\ 1+0 & -2+0 & -5+0 \\ -3+12 & 6+16 & 15+0 \end{bmatrix}$$

$$A\,B = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$$ tipa 3×3

MNOŽENJE MATRICA - primer

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & -2 & -5 \\ 3 & 4 & 0 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & -2 & -5 \\ 3 & 4 & 0 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} 2-2+15 & -1+0-20 \\ 6+4-0 & -3+0+0 \end{bmatrix} = \begin{bmatrix} 15 & -21 \\ 10 & -3 \end{bmatrix}$$

$$BA = \begin{bmatrix} 15 & -21 \\ 10 & -3 \end{bmatrix}$$ tipa 2×2

MNOŽENJE MATRICA

Množenje matrica nije komutativno. U opštem slučaju

$$A \cdot B \neq B \cdot A$$

Ukoliko je

$$A \cdot B = B \cdot A$$

Za matrice *A* i *B* se kaže da su komutativne.

MNOŽENJE MATRICA

$$\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1\cdot(-1)+0\cdot 2 \\ (-2)\cdot(-1)+1\cdot 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & 0 \\ 1 & -2 & -1 \\ 2 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} (-1)\cdot 0+1\cdot 1+0\cdot 2 \\ 1\cdot 0+(-2)\cdot 1+(-1)\cdot 2 \\ 2\cdot 0+0\cdot 1+1\cdot 2 \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \\ 2 \end{bmatrix}$$

MNOŽENJE MATRICA - primer

$$\begin{bmatrix} 1 & 0 & h \\ 0 & 1 & k \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos a & -\sin a & 0 \\ \sin a & \cos a & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos a & -\sin a & h \\ \sin a & \cos a & k \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos a & -\sin a & 0 \\ \sin a & \cos a & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & h \\ 0 & 1 & k \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos a & -\sin a & h\cos a - k\sin a \\ \sin a & \cos a & h\sin a + k\cos a \\ 0 & 0 & 1 \end{bmatrix}$$

MNOŽENJE MATRICA - osobine

$$(AB)C = A(BC)$$

zakon asocijativnosti

$$A(B+C) = AB + AC$$

množenje matrica sa leve strane zakon distributivnosti u odnosu na sabiranje matrica

$$(B+C)A = BA + CA$$

množenje matrica sa desne strane zakon distributivnosti u odnosu na sabiranje matrica

$$k(AB) = (kA)B = A(kB) \qquad k \in R$$

$$0 \cdot A = 0 \qquad B \cdot 0 = 0$$

KVADRATNE MATRICE

Matrice koje imaju isti broj vrsta i kolona nazivaju se ***kvadratne matrice***.

Za kvadratnu matricu koja ima *n* vrsta i *n* kolona (tipa $n \times n$)

kaže se da je ***reda*** *n* ili da je *n*-tog reda.

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}$$

Glavna dijagonala

Elementi glavne dijagonale

$$a_{11}, a_{22}, \dots, a_{nn}$$

Sporedna dijagonala

KVADRATNE MATRICE

Matrica

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

je kvadratna matrica trećeg reda.

Elementi glavne dijagonale su *1, 5* i *9*.

KVADRATNE MATRICE

Gornja trougaona matrica je kvadratna matrica čiji su elementi ispod glavne dijagonale jednaki nuli:

$a_{ij} = 0$ za $i > j$

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ 0 & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & a_{nn} \end{bmatrix} \qquad \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ & a_{22} & \dots & a_{2n} \\ & & \dots & \dots \\ & & & a_{nn} \end{bmatrix}$$

Donja trougaona matrica je kvadratna matrica čiji su elementi iznad glavne dijagonale jednaki nuli:

$a_{ij} = 0$ za $i < j$

$$\begin{bmatrix} a_{11} & 0 & \dots & 0 \\ a_{21} & a_{22} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix} \qquad \begin{bmatrix} a_{11} & & & \\ a_{21} & a_{22} & & \\ \dots & \dots & \dots & \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}$$

Dijagonalna matrica je kvadratna matrica sa elementima jednakim nuli van glavne dijagonale:

$$\begin{bmatrix} a_{11} & 0 & \dots & 0 \\ 0 & a_{22} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & a_{nn} \end{bmatrix} \qquad \begin{bmatrix} a_{11} & & & \\ & a_{22} & & \\ & & \dots & \\ & & & a_{nn} \end{bmatrix}$$

KVADRATNE MATRICE

Dijagonalna kvadratna matrica reda *n* sa elementima jednakim jedinici na glavnoj dijagonali je ***jedinična matrica*** reda *n*.

$$\begin{bmatrix} 1 & 0 & ... & 0 \\ 0 & 1 & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & ... & \\ & & & 1 \end{bmatrix}$$

Označava se sa I_n ili jednostavno sa I.

KVADRATNE MATRICE

Jedinična matrica reda *n* je neutralni element za množenje matrica reda *n*:

$$A\,I = I\,A = A$$

ALGEBRA KVADRATNIH MATRICA

Proizvoljna kvadratna matrica *A* reda *n* je saglasna sama sa sobom i proizvod

$$A \cdot A = A^2$$

je ***kvadrat*** matrice *A*.

ALGEBRA KVADRATNIH MATRICA

Za svaku kvadratnu matricu *A* reda *n*, može se definisati proizvoljan ***stepen*** (matrice *A*) pomoću rekurentnog niza :

$$A^0 = I \qquad A^{n+1} = A^n A \qquad (n = 0,1,2,\dots)$$

$$A^0 = I \quad , \quad A^1 = A \quad , \quad A^2 = A \cdot A \quad , \quad A^3 = A^2 A \quad , \dots$$

ALGEBRA KVADRATNIH MATRICA

Za neki polinom

$$P(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n$$

gde su a_i , $i = 0, 1, 2, \ldots, n$ skalari,

definiše se polinom matrice kao matrica

$$P(A) = a_0 I + a_1 A + a_2 A^2 + \ldots + a_n A^n$$

U slučaju kada je $P(A)$ nula-matrica, za matricu *A* se kaže da

je ***nula polinoma*** $P(x)$ **.**

ALGEBRA KVADRATNIH MATRICA - primer

$$A = \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix} \qquad A^2 = \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix} = \begin{bmatrix} 7 & -6 \\ -9 & 22 \end{bmatrix}$$

$$P(x) = 2x^2 - 3x + 5$$

$$P(A) = 2\begin{bmatrix} 7 & -6 \\ -9 & 22 \end{bmatrix} - 3\begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix} + 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$P(A) = \begin{bmatrix} 16 & -18 \\ -27 & 61 \end{bmatrix}$$

ALGEBRA KVADRATNIH MATRICA - primer

$$A = \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix} \qquad A^2 = \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix} = \begin{bmatrix} 7 & -6 \\ -9 & 22 \end{bmatrix}$$

$$Q(x) = x^2 + 3x - 10$$

$$Q(A) = \begin{bmatrix} 7 & -6 \\ -9 & 22 \end{bmatrix} + 3\begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix} - 10\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$Q(A) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Matrica *A* nula polinoma $Q(x)$.

TRANSPONOVANA MATRICA

Transponovana matrica matrice $A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$

je matrica $A^T = \begin{bmatrix} a_{11} & a_{21} & \dots & a_{m1} \\ a_{12} & a_{22} & \dots & a_{m2} \\ \dots & \dots & \dots & \dots \\ a_{1n} & a_{2n} & \dots & a_{mn} \end{bmatrix}$

koja se dobija zamenom mesta vrsta i kolona date matrice *A*.

TRANSPONOVANA MATRICA - primer

Odrediti transponovane matrice matrica

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -5 & -6 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 4 \\ 2 & -5 \\ 3 & -6 \end{bmatrix} \qquad B^T = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad C^T = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

TRANSPONOVANA MATRICA

Osobine transponovane matrice:

$$(A + B)^T = A^T + B^T$$

$$\left(A^T\right)^T = A$$

$$(kA)^T = k\,A^T \qquad k \in R$$

$$(AB)^T = B^T A^T$$

INVERZNA MATRICA

Inverzna matrica kvadratne matrice A je matrica A^{-1}

takva da je

$$A\,A^{-1} = A^{-1}A = I$$

gde je I jedinična matrica.

Ukoliko postoji inverzna matrica, ona je jedinstvena.

INVERZNA MATRICA

Matrice

$$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} \quad \text{i} \quad \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$$

su uzajamno inverzne zato što je:

$$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

INVERZNA MATRICA

Osobine inverzne matrice:

$$\left(A^{-1}\right)^{-1} = A$$

$$\left(A^{T}\right)^{-1} = \left(A^{-1}\right)^{T}$$

$$\left(A\,B\right)^{-1} = B^{-1}A^{-1}$$

MATRICE

Determinanta matrice *A*

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}$$

$$\det A = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}$$

MATRICE - DETERMINANTE

minori M_{ij}

$(-1)^{i+j}$

Parna i neparna mesta

$$\begin{vmatrix} a_{11} & a_{12} & \dots & a_{1j} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2j} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{i1} & a_{i2} & \dots & a_{ij} & \dots & a_{in} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nj} & \dots & a_{nn} \end{vmatrix}$$

$$\begin{bmatrix} + & - & + & - & \dots \\ - & + & - & + & \dots \\ + & - & + & - & \\ \dots & \dots & \dots & \dots & \dots \end{bmatrix}$$

kofaktori $A_{ij} = (-1)^{i+j} M_{ij}$

MATRICE - DETERMINANTE

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix} \qquad \det A = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}$$

Determinanta matrice *A* je jednaka zbiru proizvoda svih elemenata neke vrste ili kolone i odgovarajućih kofaktora:

$$det(A) = \sum_{j=1}^{n} a_{ij}A_{ij} = a_{i1}A_{i1} + a_{i2}A_{i2} + \dots + a_{in}A_{in}$$

$$det(A) = \sum_{i=1}^{n} a_{ij}A_{ij} = a_{1j}A_{1j} + a_{2j}A_{2j} + \dots + a_{nj}A_{nj}$$

MATRICE – DETERMINANTE - primer

$$D = \begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix} = 2 \cdot 4 - 3 \cdot 1 = 5$$

$$D = \begin{vmatrix} -2 & 5 \\ 1 & 2 \end{vmatrix} = (-2) \cdot 2 - 5 \cdot 1 = -9$$

MATRICE – DETERMINANTE - primer

$$D=\begin{vmatrix}1 & -2 & 5\\ 1 & 2 & 3\\ 3 & 1 & 2\end{vmatrix}=1\cdot\begin{vmatrix}2 & 3\\ 1 & 2\end{vmatrix}-(-2)\cdot\begin{vmatrix}1 & 3\\ 3 & 2\end{vmatrix}+5\cdot\begin{vmatrix}1 & 2\\ 3 & 1\end{vmatrix}$$

$$D=\left(2\cdot 2-3\cdot 1\right)+2\left(1\cdot 2-3\cdot 3\right)+5\left(1\cdot 1-2\cdot 3\right)=-38$$

MATRICE – ADJUNGOVANA MATRICA

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}$$

Transponovana matrica matrice odgovarajućih kofaktora

$$adj\,A = \begin{bmatrix} A_{11} & A_{21} & \dots & A_{n1} \\ A_{12} & A_{22} & \dots & A_{n2} \\ \dots & \dots & \dots & \dots \\ A_{1n} & A_{2n} & \dots & A_{nn} \end{bmatrix}$$

zove se ***adjungovana matrica*** matrice *A*.

MATRICE – ADJUNGOVANA MATRICA

Matrica $adj\, A$ se može formirati na dva načina:

1. Elementi matrice *A* se menjaju odgovarajućim kofaktorima i zatim se dobijena matrica transponuje.

2. U transponovanoj matrici A^T matrice *A* elementi se menjaju odgovarajućim kofaktorima.

MATRICE – ADJUNGOVANA MATRICA - primer

$$A = \begin{bmatrix} 2 & 3 & -4 \\ 0 & -4 & 2 \\ 1 & -1 & 5 \end{bmatrix}$$

$$A_{11} = +\begin{vmatrix} -4 & 2 \\ -1 & 5 \end{vmatrix} = -18 \qquad A_{12} = -\begin{vmatrix} 0 & 2 \\ 1 & 5 \end{vmatrix} = 2 \qquad A_{13} = +\begin{vmatrix} 0 & -4 \\ 1 & -1 \end{vmatrix} = 4$$

$$A_{21} = -\begin{vmatrix} 3 & -4 \\ -1 & 5 \end{vmatrix} = -11 \qquad A_{22} = +\begin{vmatrix} 2 & -4 \\ 1 & 5 \end{vmatrix} = 14 \qquad A_{23} = -\begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix} = 5$$

$$A_{31} = +\begin{vmatrix} 3 & -4 \\ -4 & 2 \end{vmatrix} = -10 \qquad A_{32} = -\begin{vmatrix} 2 & -4 \\ 0 & 2 \end{vmatrix} = -4 \qquad A_{33} = +\begin{vmatrix} 2 & 3 \\ 0 & -4 \end{vmatrix} = -8$$

$$adj\,A = \begin{bmatrix} -18 & -11 & -10 \\ 2 & 14 & -4 \\ 4 & 5 & -8 \end{bmatrix}$$

MATRICE – ADJUNGOVANA MATRICA - primer

$$A = \begin{bmatrix} 2 & 3 & -4 \\ 0 & -4 & 2 \\ 1 & -1 & 5 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 2 & 0 & 1 \\ 3 & -4 & -1 \\ -4 & 2 & 5 \end{bmatrix}$$

$$A_{11}^T = +\begin{vmatrix} -4 & 2 \\ -1 & 5 \end{vmatrix} = -18 \qquad A_{12}^T = -\begin{vmatrix} 3 & -4 \\ -1 & 5 \end{vmatrix} = -11 \qquad A_{13}^T = +\begin{vmatrix} 3 & -4 \\ -4 & 2 \end{vmatrix} = -10$$

$$A_{21}^T = -\begin{vmatrix} 0 & 2 \\ 1 & 5 \end{vmatrix} = 2 \qquad A_{22}^T = +\begin{vmatrix} 2 & -4 \\ 1 & 5 \end{vmatrix} = 14 \qquad A_{23}^T = -\begin{vmatrix} 2 & -4 \\ 0 & 2 \end{vmatrix} = -4$$

$$A_{31}^T = +\begin{vmatrix} 0 & -4 \\ 1 & -1 \end{vmatrix} = 4 \qquad A_{32}^T = -\begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix} = 5 \qquad A_{33}^T = +\begin{vmatrix} 2 & 3 \\ 0 & -4 \end{vmatrix} = -8$$

$$adj\,A = \begin{bmatrix} -18 & -11 & -10 \\ 2 & 14 & -4 \\ 4 & 5 & -8 \end{bmatrix}$$

MATRICE – ADJUNGOVANA MATRICA - primer

$$A = \begin{bmatrix} 2 & 3 \\ 0 & -4 \end{bmatrix}$$

$A_{11} = -4$ $A_{12} = 0$

$A_{21} = -3$ $A_{22} = 2$

$$adj\,A = \begin{bmatrix} -4 & -3 \\ 0 & 2 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 2 & 0 \\ 3 & -4 \end{bmatrix}$$

$A_{11}^T = -4$ $A_{12}^T = -3$

$A_{21}^T = 0$ $A_{22}^T = 2$

$$adj\,A = \begin{bmatrix} -4 & -3 \\ 0 & 2 \end{bmatrix}$$

INVERZNA MATRICA

Za proizvoljnu kvadratnu matricu A važi matrična jednakost

$$A \cdot (adj\, A) = (adj\, A) \cdot A = det(A) \cdot I$$

gde je I jedinična matrica.

Za datu kvadratnu matricu *A* postoji inverzna matrica A^{-1}

ako i samo ako je $det(A) \neq 0$ (regularna matrica).

Tada je

$$A^{-1} = \frac{1}{\det(A)}(adj\, A)$$

INVERZNA MATRICA - primer

$$A = \begin{bmatrix} 2 & 3 & -4 \\ 0 & -4 & 2 \\ 1 & -1 & 5 \end{bmatrix} \qquad adj\,A = \begin{bmatrix} -18 & -11 & -10 \\ 2 & 14 & -4 \\ 4 & 5 & -8 \end{bmatrix}$$

$$\det(A) = \begin{vmatrix} 2 & 3 & -4 \\ 0 & -4 & 2 \\ 1 & -1 & 5 \end{vmatrix} = 2\cdot(-18) + 0\cdot(-11) + 1\cdot(-10) = -46$$

$$A^{-1} = \frac{1}{det(A)}\,adj\,A = \frac{1}{-46}\begin{bmatrix} -18 & -11 & -10 \\ 2 & 14 & -4 \\ 4 & 5 & -8 \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} \frac{9}{23} & \frac{11}{46} & \frac{5}{23} \\ -\frac{1}{23} & -\frac{7}{23} & \frac{2}{23} \\ -\frac{2}{23} & -\frac{5}{46} & \frac{4}{23} \end{bmatrix}$$

INVERZNA MATRICA - primer

$$A = \begin{bmatrix} 2 & 3 \\ 0 & -4 \end{bmatrix}$$

$$A_{11} = -4 \qquad A_{12} = 0$$

$$A_{21} = -3 \qquad A_{22} = 2$$

$$adj\,A = \begin{bmatrix} -4 & -3 \\ 0 & 2 \end{bmatrix} \quad \det A = \begin{vmatrix} 2 & 3 \\ 0 & -4 \end{vmatrix} = -8 \neq 0$$

$$A^{-1} = \frac{1}{-8}\,adj\,A = -\frac{1}{8}\begin{bmatrix} -4 & -3 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{3}{8} \\ 0 & -\frac{1}{4} \end{bmatrix}$$