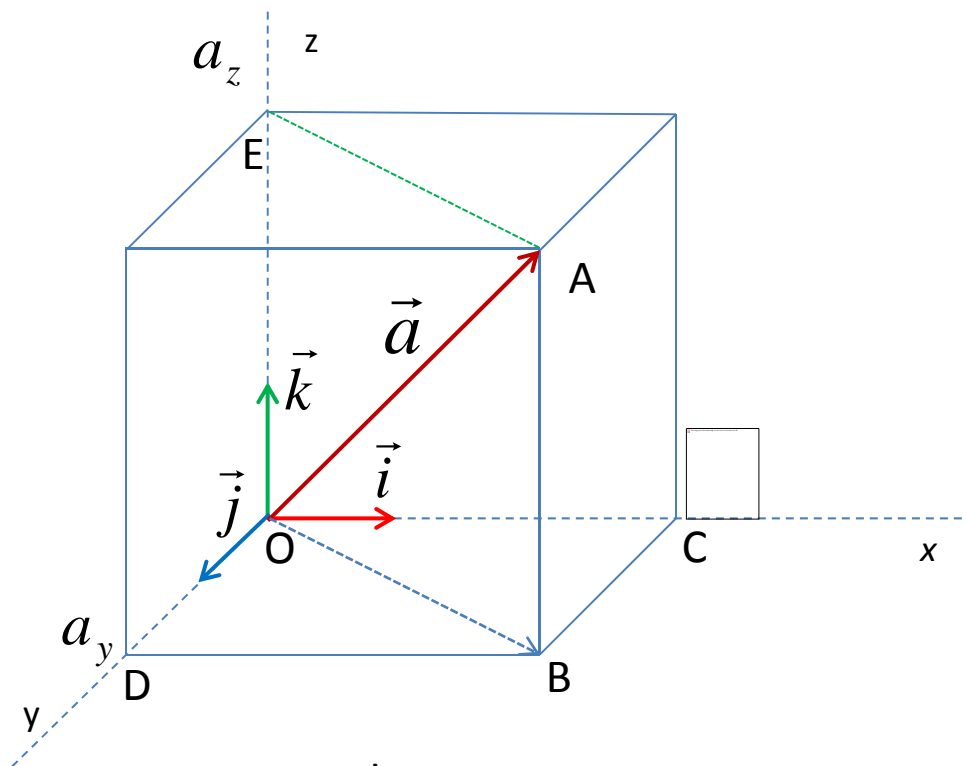


ANALITIČKA GEOMETRIJA U PROSTORU

VEKTOR POLOŽAJA TAČKE

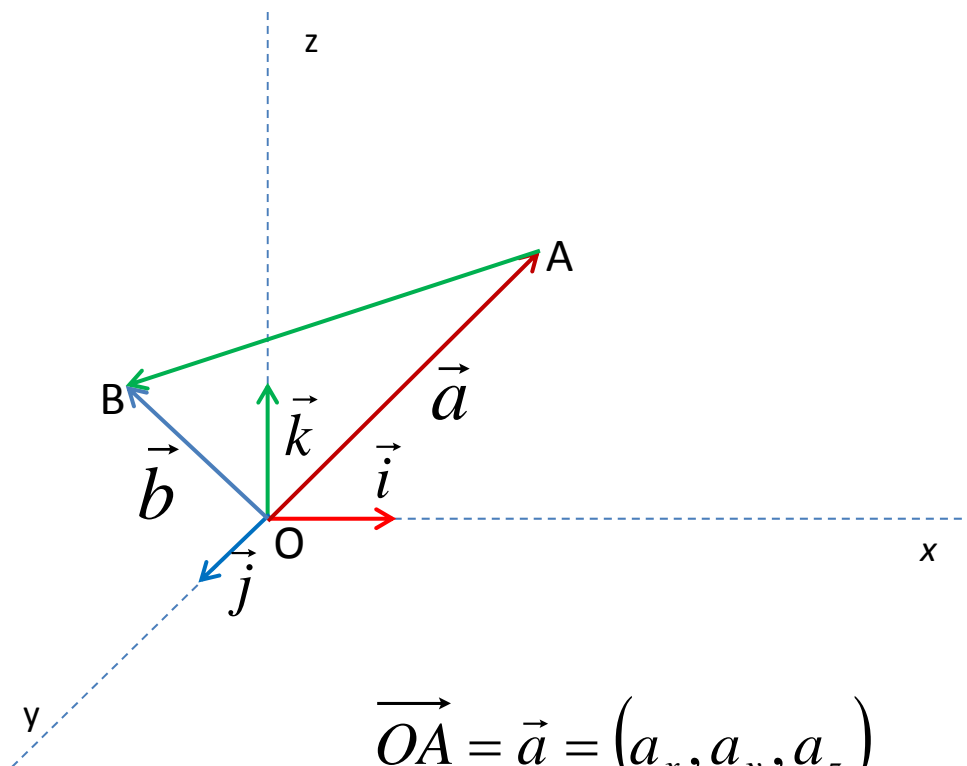


Vektor

$$\vec{a} = \overrightarrow{OA} = (a_x, a_y, a_z) = a_x \vec{i} + a_y \vec{j} + a_z \vec{k}$$

je vektor položaja tačke $A(a_x, a_y, a_z)$.

KOORDINATE VEKTORA ODREDJENOG POČETNOM I ZAVRŠNOM TAČKOM

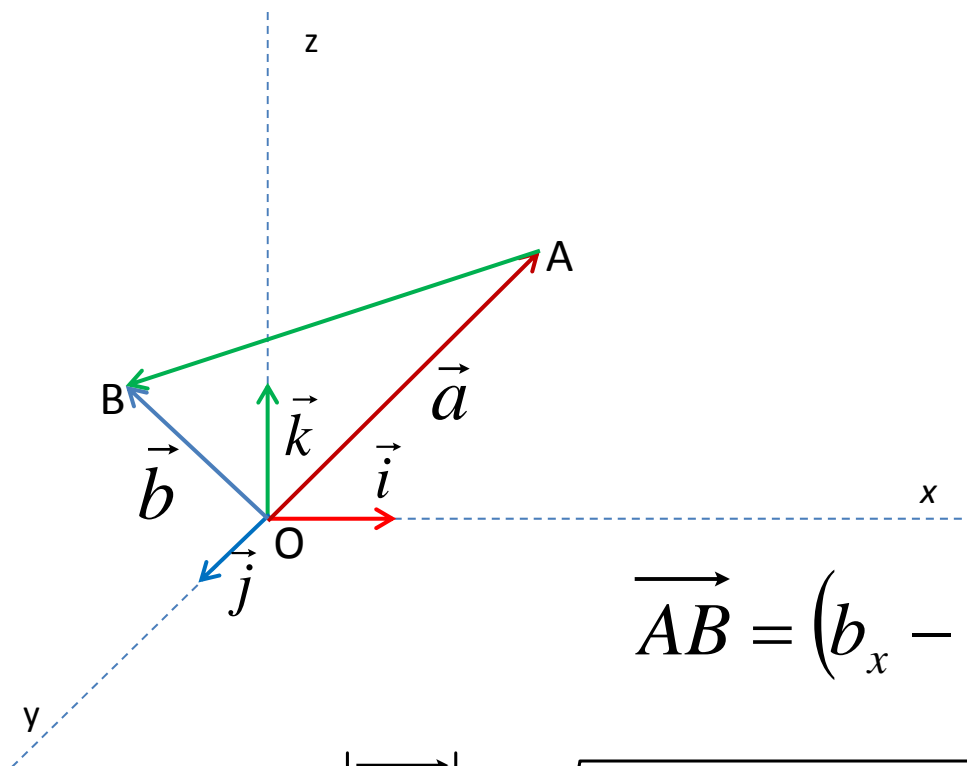


$$\overrightarrow{OA} = \vec{a} = (a_x, a_y, a_z) \quad \overrightarrow{OB} = \vec{b} = (b_x, b_y, b_z)$$

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \vec{b} - \vec{a} = (b_x, b_y, b_z) - (a_x, a_y, a_z) = (b_x - a_x, b_y - a_y, b_z - a_z)$$

$$\overrightarrow{AB} = (b_x - a_x, b_y - a_y, b_z - a_z)$$

RASTOJANJE IZMEDJU DVE TAČKE



$$A(a_x, a_y, a_z)$$

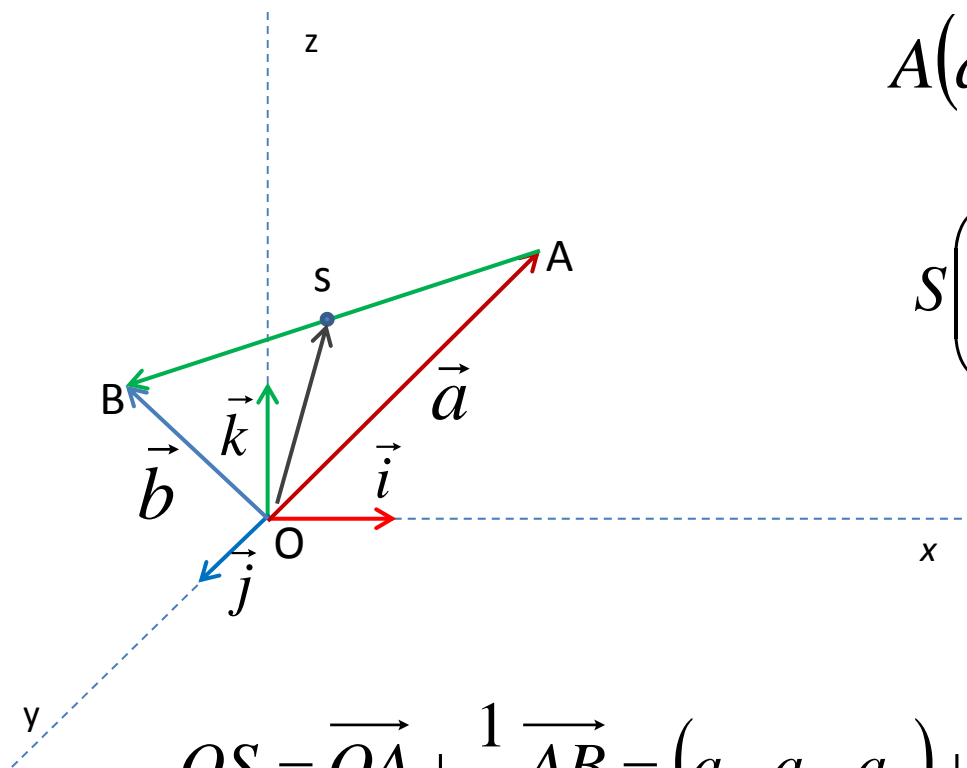
$$B(b_x, b_y, b_z)$$

$$\vec{AB} = (b_x - a_x, b_y - a_y, b_z - a_z)$$

$$d(A, B) = |\vec{AB}| = \sqrt{(b_x - a_x)^2 + (b_y - a_y)^2 + (b_z - a_z)^2}$$

Rastojanje izmedju dve tačke A i B je intezitet vektora \vec{AB} .

KOORDINATE SREDIŠTA DUŽI



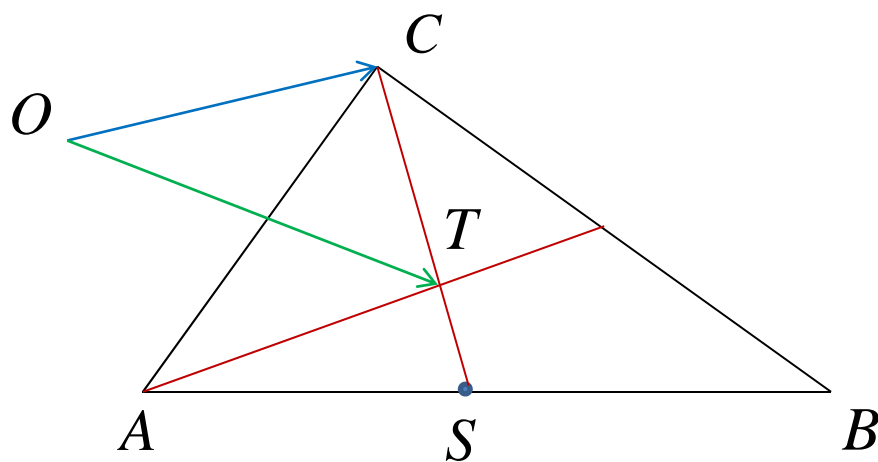
$$A(a_x, a_y, a_z) \quad B(b_x, b_y, b_z)$$

$$S\left(\frac{a_x + b_x}{2}, \frac{a_y + b_y}{2}, \frac{a_z + b_z}{2}\right)$$

$$\overrightarrow{OS} = \overrightarrow{OA} + \frac{1}{2} \overrightarrow{AB} = (a_x, a_y, a_z) + \frac{1}{2} (b_x - a_x, b_y - a_y, b_z - a_z)$$

$$\overrightarrow{OS} = \overrightarrow{OA} + \frac{1}{2} \overrightarrow{AB} = \left(\frac{a_x + b_x}{2}, \frac{a_y + b_y}{2}, \frac{a_z + b_z}{2} \right)$$

TEŽIŠTE TROUGLA



$$A(a_x, a_y, a_z)$$

$$B(b_x, b_y, b_z)$$

$$C(c_x, c_y, c_z)$$

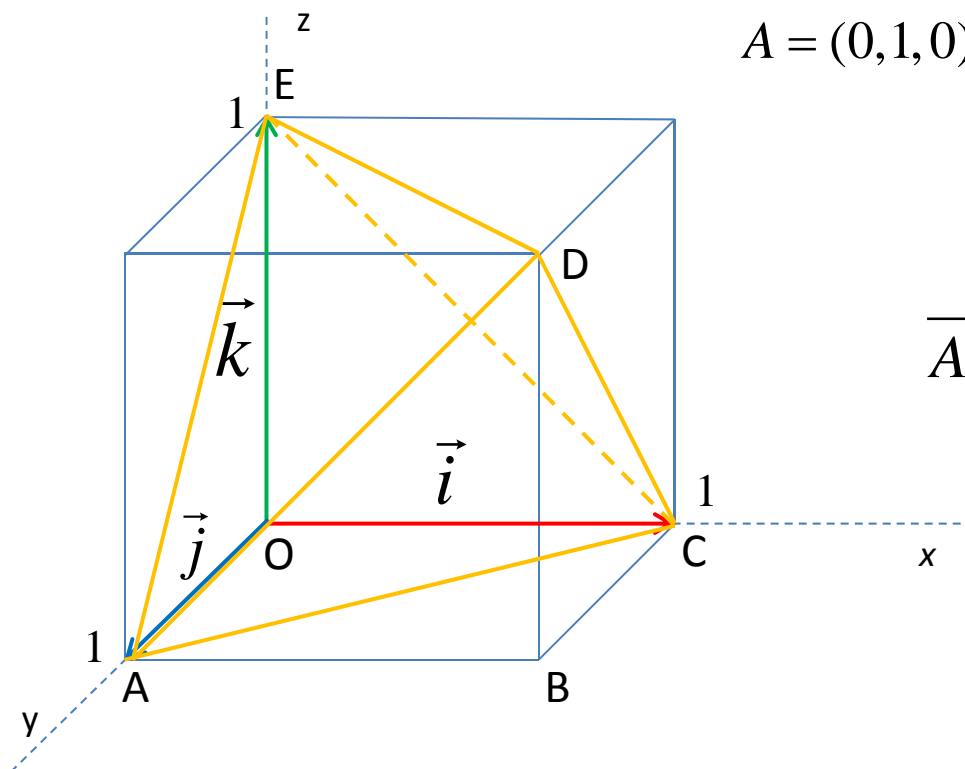
$$S\left(\frac{a_x + b_x}{2}, \frac{a_y + b_y}{2}, \frac{a_z + b_z}{2}\right)$$

$$\overrightarrow{CS} = \left(\frac{a_x + b_x}{2} - c_x, \frac{a_y + b_y}{2} - c_y, \frac{a_z + b_z}{2} - c_z \right)$$

$$\overrightarrow{OT} = \overrightarrow{OC} + \frac{2}{3} \overrightarrow{CS} = \left(\frac{a_x + b_x + c_x}{3}, \frac{a_y + b_y + c_y}{3}, \frac{a_z + b_z + c_z}{3} \right)$$

PRIMER

Data je kocka osnovne ivice $a = 1$ sa jednim temenom u koordinatnom početku O i sa ivicama koje polaze iz tog temena i leze na pozitivnim delovima koordinatnih osa. U tu kocku upisan je tetraedar ACDE prikazan na slici čije su ivice dijagonale strana date kocke. Odrediti ugao između vektora \overrightarrow{AD} i \overrightarrow{AE} .



$$A = (0,1,0) \quad C = (1,0,0) \quad D = (1,1,1) \quad E = (0,0,1)$$

$$\overrightarrow{AE} = (0-0, 0-1, 1-0) = (0, -1, 1)$$

$$\overrightarrow{AD} = (1-0, 1-1, 1-0) = (1, 0, 1)$$

$$\overrightarrow{AD} \cdot \overrightarrow{AE} = 0 \cdot 1 + (-1) \cdot 0 + 1 \cdot 1 = 1$$

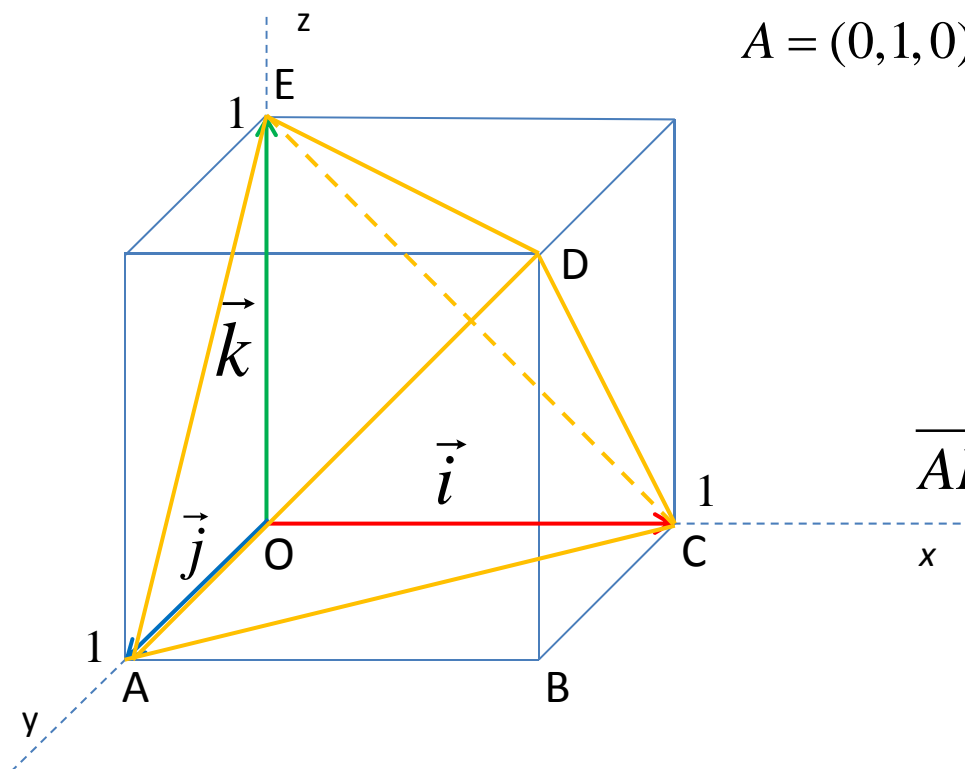
$$|\overrightarrow{AD}| = |\overrightarrow{AE}| = \sqrt{2}$$

$$\cos \varphi = \frac{\overrightarrow{AD} \cdot \overrightarrow{AE}}{|\overrightarrow{AD}| \cdot |\overrightarrow{AE}|} = \frac{1}{\sqrt{2} \cdot \sqrt{2}} = \frac{1}{2}$$

$$\varphi = \frac{\pi}{3} = 60^\circ$$

PRIMER

Data je kocka osnovne ivice $a = 1$ sa jednim temenom u koordinatnom početku O i sa ivicama koje polaze iz tog temena i leze na pozitivnim delovima koordinatnih osa. U tu kocku upisan je tetraedar ACDE prikazan na slici cije su ivice dijagonale strana date kocke. Odrediti $\overrightarrow{AD} \times \overrightarrow{AE}$ i ort tog vektora.



$$A = (0,1,0) \quad C = (1,0,0) \quad D = (1,1,1) \quad E = (0,0,1)$$

$$\overrightarrow{AE} = (0-0, 0-1, 1-0) = (0, -1, 1)$$

$$\overrightarrow{AD} = (1-0, 1-1, 1-0) = (1, 0, 1)$$

$$\overrightarrow{AD} \times \overrightarrow{AE} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 1 \\ 0 & -1 & 1 \end{vmatrix} = \vec{i} - \vec{j} - \vec{k}$$

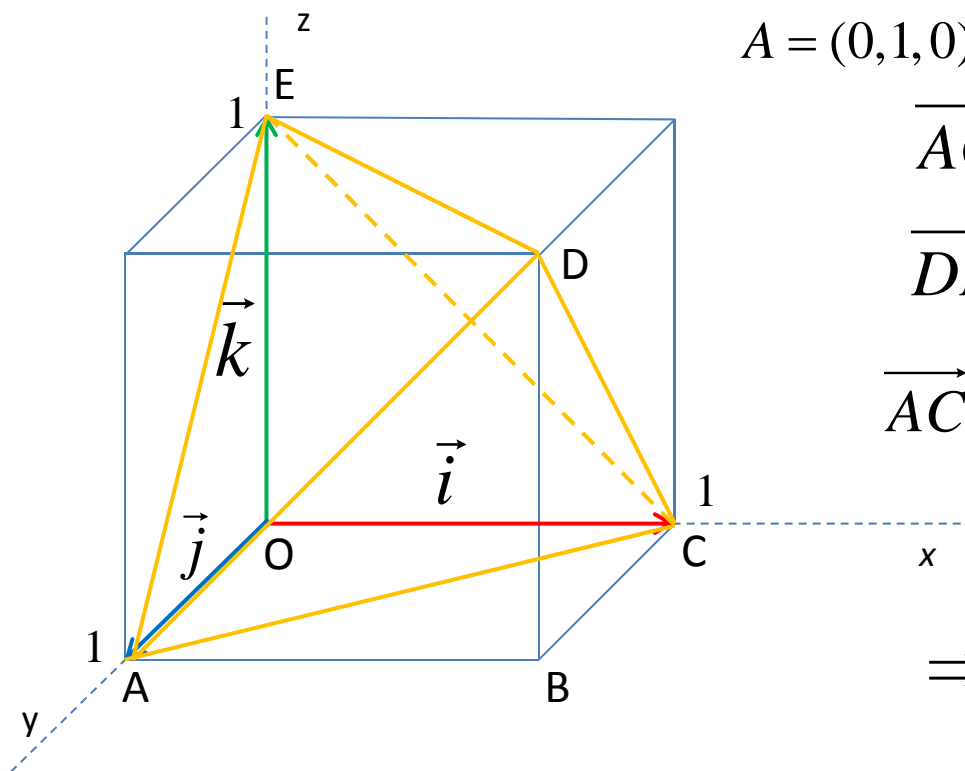
$$\overrightarrow{AD} \times \overrightarrow{AE} = (1, -1, -1)$$

$$\left| \overrightarrow{AD} \times \overrightarrow{AE} \right| = \sqrt{1^2 + (-1)^2 + (-1)^2} = \sqrt{3}$$

$$\text{ort}(\overrightarrow{AD} \times \overrightarrow{AE}) = \left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}} \right)$$

PRIMER

Data je kocka osnovne ivice $a = 1$ sa jednim temenom u koordinatnom početku O i sa ivicama koje polaze iz tog temena i leze na pozitivnim delovima koordinatnih osa. U tu kocku upisan je tetraedar $ACDE$ prikazan na slici čije su ivice dijagonale strana date kocke. Izračunati $\overrightarrow{AC} \cdot \overrightarrow{DE}$



$$A = (0,1,0) \quad C = (1,0,0) \quad D = (1,1,1) \quad E = (0,0,1)$$

$$\overrightarrow{AC} = (1-0, 0-1, 0-0) = (1, -1, 0)$$

$$\overrightarrow{DE} = (0-1, 0-1, 1-1) = (-1, -1, 0)$$

$$\overrightarrow{AC} \cdot \overrightarrow{DE} = 1 \cdot (-1) + (-1) \cdot (-1) + 0 \cdot 0 = 0$$

$$\Rightarrow \overrightarrow{AC} \perp \overrightarrow{DE} \quad \text{Uzajamno ortogonalni}$$

PRIMER

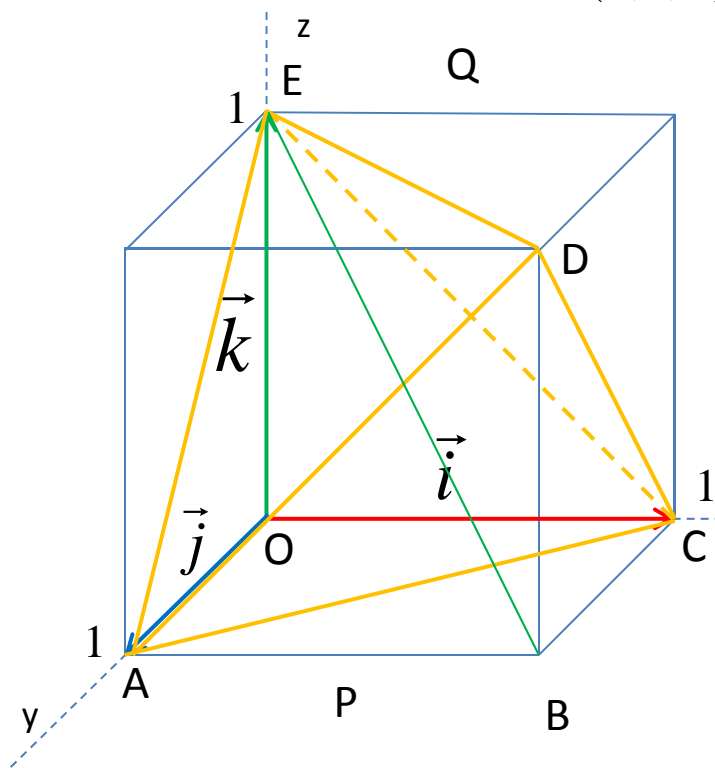
Data je kocka osnovne ivice $a = 1$ sa jednim temenom u koordinatnom početku O i sa ivicama koje polaze iz tog temena koje leze na pozitivnim delovima koordinatnih osa. U tu kocku upisan je tetraedar $ACDE$ prikazan na slici čije su ivice dijagonale strana date kocke. Izračunati $\overrightarrow{AC} \cdot \overrightarrow{BE}$.

$$A = (0, 1, 0) \quad B = (1, 1, 0) \quad C = (1, 0, 0) \quad D = (1, 1, 1) \quad E = (0, 0, 1)$$

$$\overrightarrow{AC} = (1 - 0, 0 - 1, 0 - 0) = (1, -1, 0)$$

$$\overrightarrow{BE} = (0 - 1, 0 - 1, 1 - 0) = (-1, -1, 1)$$

$$\overrightarrow{AC} \cdot \overrightarrow{BE} = 1 \cdot (-1) + (-1) \cdot (-1) + 0 \cdot 1 = 0$$

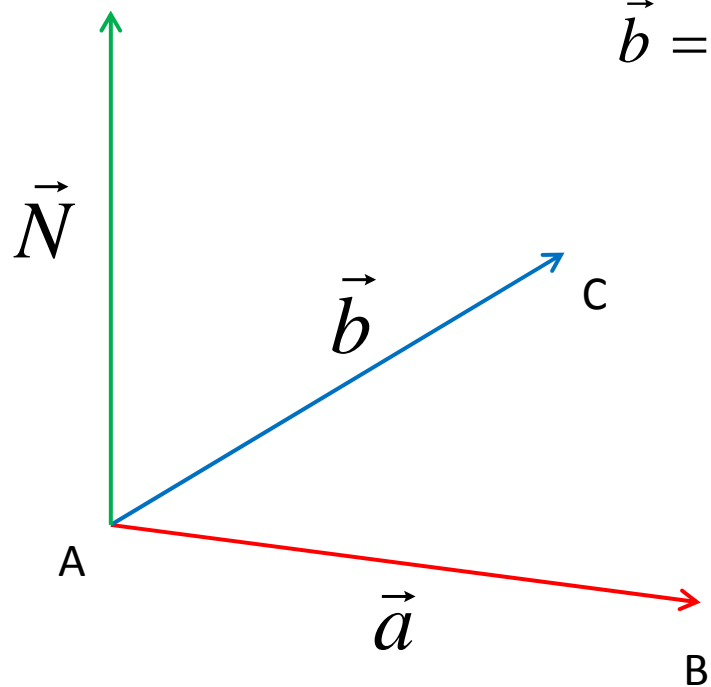


$\Rightarrow \overrightarrow{AC} \perp \overrightarrow{BE}$ Uzajamno ortogonalni

PRIMER

Date su tačke $A = (1, -1, 2)$ $B = (4, 0, -2)$ $C = (-2, 1, -1)$

Odrediti vektor normalan na ravan koju odredjuju te tri tačke.



$$\vec{a} = \overrightarrow{AB} = (4 - 1, 0 - (-1), -2 - 2) = (3, 1, -4)$$

$$\vec{b} = \overrightarrow{AC} = (-2 - 1, 1 - (-1), -1 - 2) = (-3, 2, -3)$$

$$\vec{N} = \vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 1 & -4 \\ -3 & 2 & -3 \end{vmatrix}$$

$$\vec{N} = \vec{i} \begin{vmatrix} 1 & -4 \\ 2 & -3 \end{vmatrix} - \vec{j} \begin{vmatrix} 3 & -4 \\ -3 & -3 \end{vmatrix} + \vec{k} \begin{vmatrix} 3 & 1 \\ -3 & 2 \end{vmatrix}$$

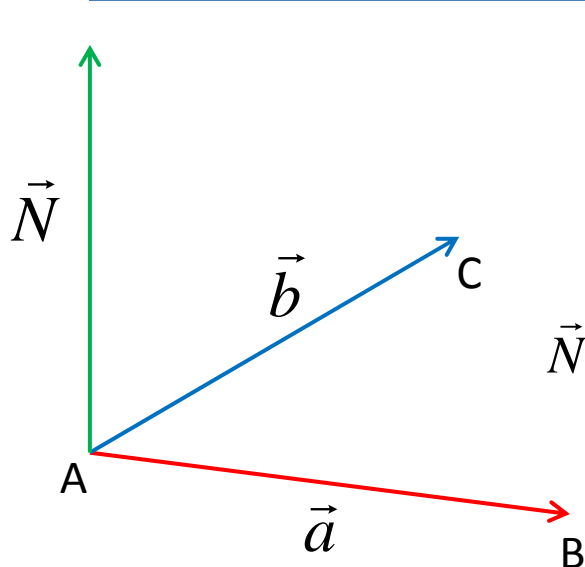
$$\vec{N} = 5\vec{i} + 21\vec{j} + 9\vec{k}$$

$$\vec{N} = (5, 21, 9)$$

PRIMER

Date su tačke $A = (0, 2, 0)$ $B = (2, 0, 0)$ $C = (2, 2, 2)$

Odrediti vektor normalan na ravan koju odredjuju te tri tačke.



$$\vec{a} = \overrightarrow{AB} = (2 - 0, 0 - 2, 0 - 0) = (2, -2, 0)$$

$$\vec{b} = \overrightarrow{AC} = (2 - 0, 2 - 2, 2 - 0) = (2, 0, 2)$$

$$\vec{N} = \vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -2 & 0 \\ 2 & 0 & 2 \end{vmatrix} = \vec{i} \begin{vmatrix} -2 & 0 \\ 0 & 2 \end{vmatrix} - \vec{j} \begin{vmatrix} 2 & 0 \\ 2 & 2 \end{vmatrix} + \vec{k} \begin{vmatrix} 2 & -2 \\ 2 & 0 \end{vmatrix}$$

$$\vec{N} = -4\vec{i} - 4\vec{j} + 4\vec{k} \quad \underline{\underline{\vec{N} = (-4, -4, 4)}}$$

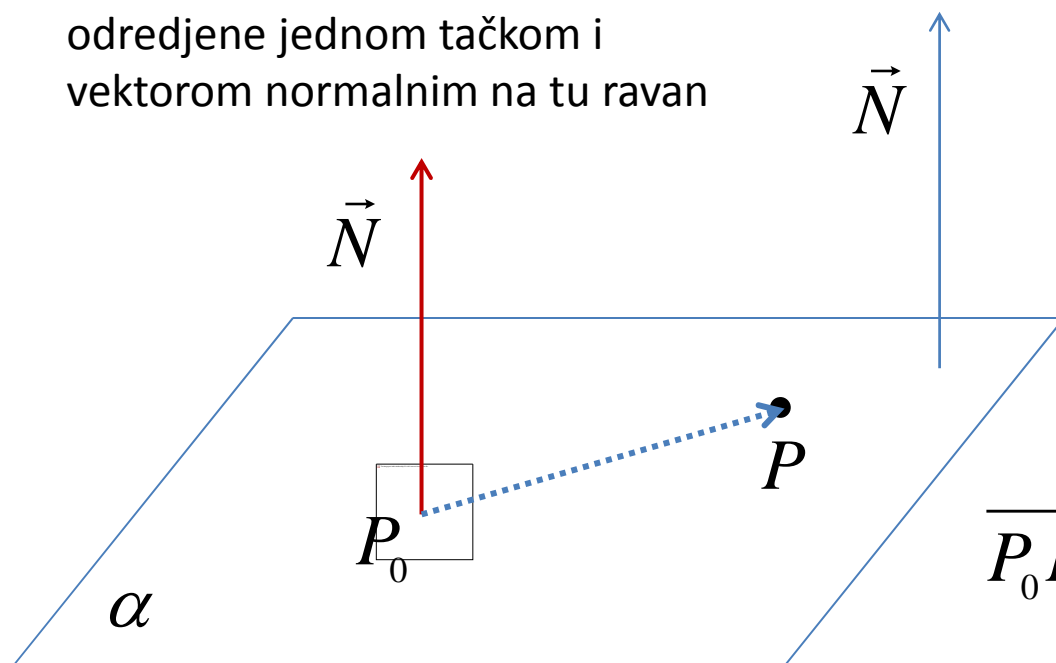
$$\underline{\underline{\lambda \vec{N} = (-4\lambda, -4\lambda, 4\lambda)}} \quad \lambda = \frac{1}{4} \quad \underline{\underline{\frac{1}{4} \vec{N} = \frac{\vec{N}}{4} = (-1, -1, 1)}}$$

$$\underline{\underline{ort \vec{N} = \left(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)}}$$

Vektori normalni na ravan

JEDNAČINA RAVNI

odredjene jednom tačkom i
vektorom normalnim na tu ravan



α :

$$P_0(x_0, y_0, z_0)$$

$$\vec{N} = (A, B, C)$$

$$P(x, y, z) \in \alpha$$

$$\overrightarrow{P_0P} = (x - x_0, y - y_0, z - z_0)$$

Tačka $P(x, y, z)$ pripada ravni α ako i samo su vektori $\overrightarrow{P_0P}$ i \vec{N}

uzajamno ortogonalni. Iz uslova ortogonalnosti $\vec{N} \cdot \overrightarrow{P_0P} = 0$
sledi jednačina ravni:

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$$

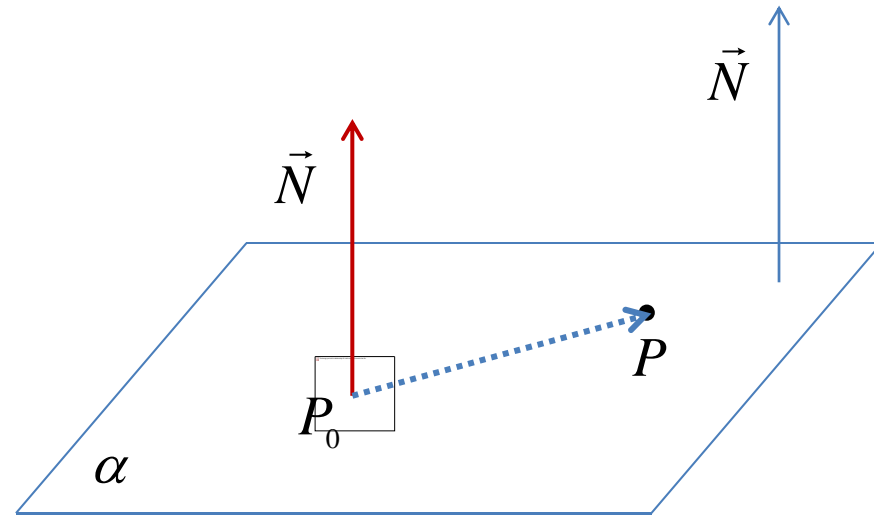
PRIMER

Napisati jednačinu ravni α
koja sadrži tačku

$$P_0(2, -5, -6)$$

i normalna je na vektor

$$\vec{N} = (-1, 2, -3).$$



$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$$

$$-(x - 2) + 2(y + 5) - 3(z + 6) = 0$$

$$-x + 2y - 3z - 6 = 0$$

JEDNAČINA RAVNI

Vektor normalan na ravan

$$\vec{N} = (A, B, C)$$

$$P_0(x_0, y_0, z_0)$$

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$$

$$Ax + By + Cz + D = 0$$

$$D = -Ax_0 - By_0 - Cz_0$$

$$Ax + By + Cz + D = 0 \Rightarrow \vec{N} = (A, B, C)$$

PRIMER

Napisati jednačinu ravni β
koja sadrži tačku

$$P_0(-2, 5, 3)$$

i paralelna je sa ravni

$$\alpha: -x + 2y - 3z - 6 = 0$$

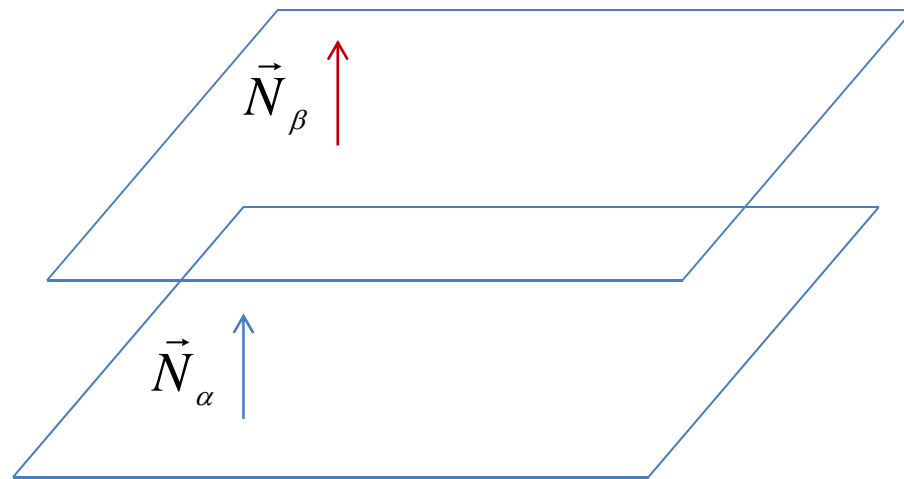
$$\vec{N}_\beta = \vec{N}_\alpha = (-1, 2, -3)$$

$$\beta: P_0(-2, 5, 3) \quad \vec{N}_\beta = (-1, 2, -3)$$

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$$

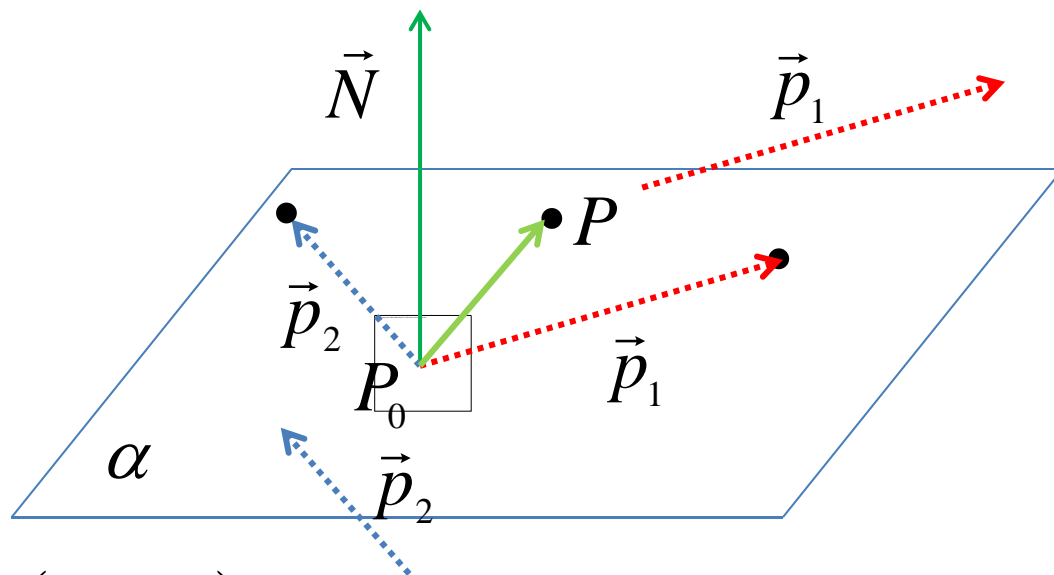
$$-(x + 2) + 2(y - 5) - 3(z - 3) = 0$$

$$-x + 2y - 3z - 3 = 0$$



JEDNAČINA RAVNI

odredjene jednom tačkom i dva
vektora paralelna sa tom ravni



$$P(x, y, z) \in \alpha$$

$$\overrightarrow{P_0P} = (x - x_0, y - y_0, z - z_0)$$

α :

$$P_0(x_0, y_0, z_0)$$

$$\vec{p}_1 = (a_1, b_1, c_1)$$

$$\vec{p}_2 = (a_2, b_2, c_2)$$

$$\vec{N} = \vec{p}_1 \times \vec{p}_2 = (A, B, C)$$

α :

$$P_0(x_0, y_0, z_0)$$

$$\vec{N} = \vec{p}_1 \times \vec{p}_2 = (A, B, C)$$

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$$

PRIMER

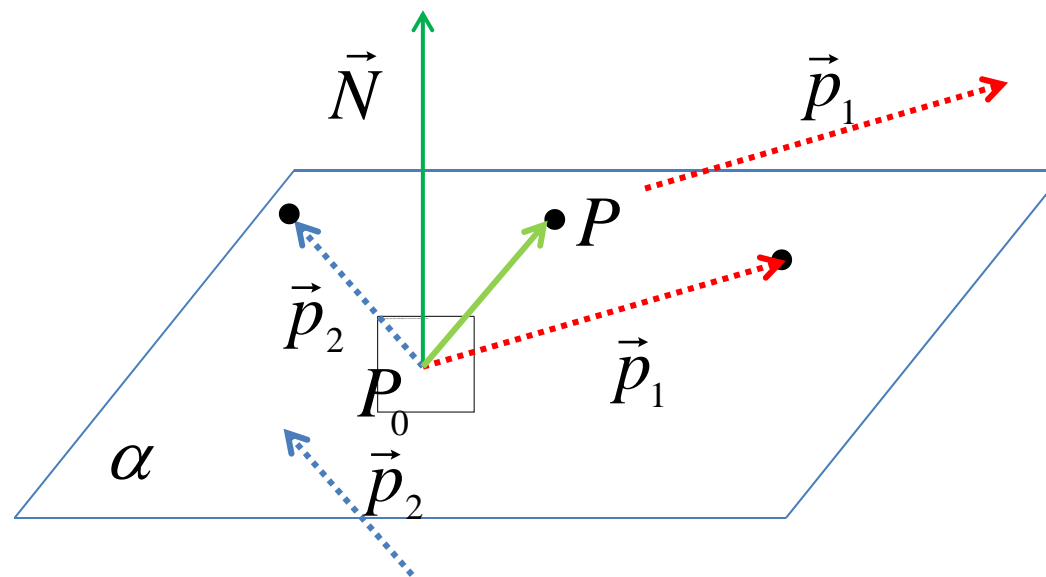
Napisati jednačinu ravni α
koja sadrži tačku

$$P_0(2, -5, -6)$$

i paralelna je vektorima

$$\vec{p}_1 = (1, -3, -2)$$

$$\vec{p}_2 = (1, 0, -1)$$



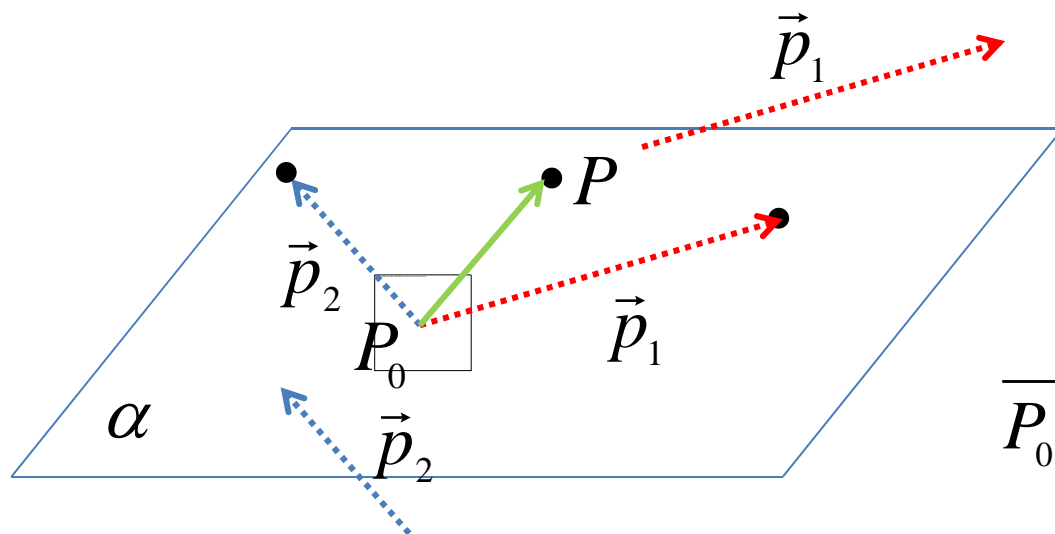
$$\vec{N} = \vec{p}_1 \times \vec{p}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -3 & -2 \\ 1 & 0 & -1 \end{vmatrix} = \vec{i} \begin{vmatrix} -3 & -2 \\ 0 & -1 \end{vmatrix} - \vec{j} \begin{vmatrix} 1 & -2 \\ 1 & -1 \end{vmatrix} + \vec{k} \begin{vmatrix} 1 & -3 \\ 1 & 0 \end{vmatrix} = 3\vec{i} - \vec{j} + 3\vec{k}$$

$$\alpha: P_0(2, -5, -6) \quad \vec{N} = (3, -1, 3)$$

$$3(x-2) - 1(y+5) + 3(z+6) = 0 \quad 3x - y + 3z + 7 = 0$$

JEDNAČINA RAVNI

odredjene jednom tačkom i dva
vektora paralelna sa tom ravni



α :

$$P_0(x_0, y_0, z_0)$$

$$\vec{p}_1 = (a_1, b_1, c_1)$$

$$\vec{p}_2 = (a_2, b_2, c_2)$$

$$P(x, y, z) \in \alpha$$

$$\overrightarrow{P_0P} = (x - x_0, y - y_0, z - z_0)$$

Tačka $P(x, y, z) \in \alpha$ ako i samo ako su vektori $\overrightarrow{P_0P}, \vec{p}_1, \vec{p}_2$ komplanarni.

$$\begin{vmatrix} x - x_0 & y - y_0 & z - z_0 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

PRIMER

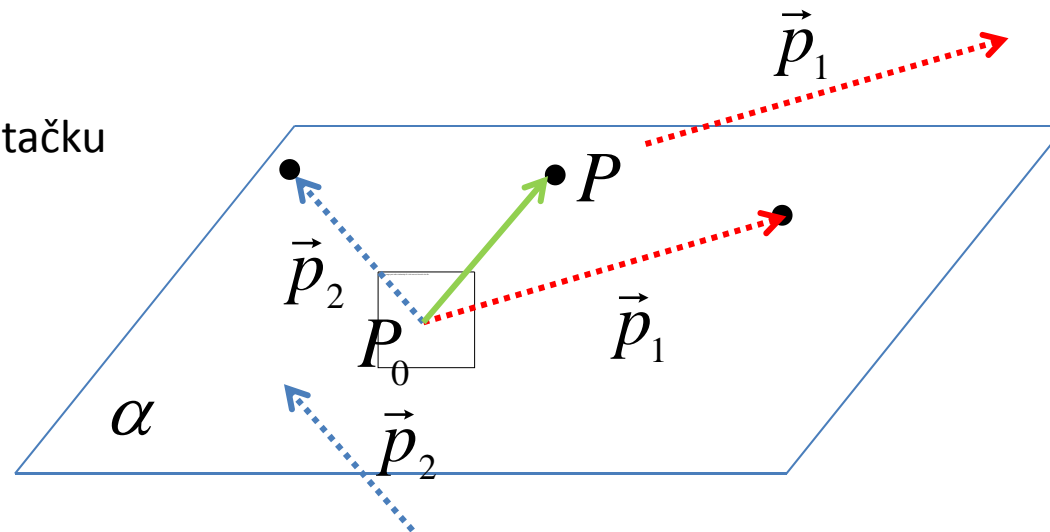
Napisati jednačinu ravni koja sadrži tačku

$$P_0(-2, 1, 3)$$

i paralelna je sa vektorima

$$\vec{p}_1 = (1, -3, -2)$$

$$\vec{p}_2 = (1, 0, -1)$$



$$\alpha : P_0(-2, 1, 3)$$

$$\vec{p}_1 = (1, -3, -2)$$

$$\vec{p}_2 = (1, 0, -1)$$

$$\begin{vmatrix} x+2 & y-1 & z-3 \\ 1 & -3 & -2 \\ 1 & 0 & -1 \end{vmatrix} = 0$$

$$(x+2) \begin{vmatrix} -3 & -2 \\ 0 & -1 \end{vmatrix} - (y-1) \begin{vmatrix} 1 & -2 \\ 1 & -1 \end{vmatrix} + (z-3) \begin{vmatrix} 1 & -3 \\ 1 & 0 \end{vmatrix} = 0$$

$$3(x+2) - (y-1) + 3(z-3) = 0$$

$$3x - y + 3z - 2 = 0$$

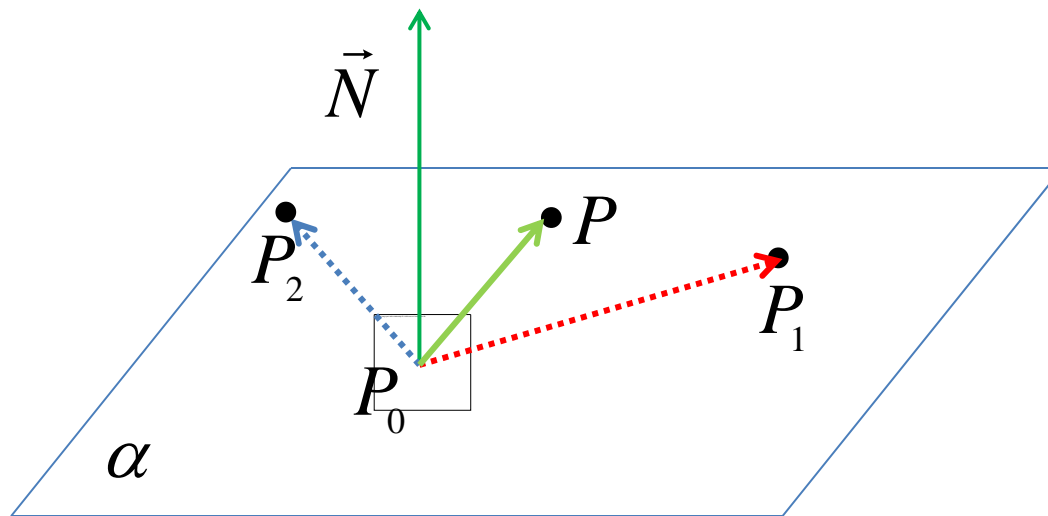
JEDNAČINA RAVNI
odredjene sa tri tačke

α :

$$P_0(x_0, y_0, z_0)$$

$$P_1(x_1, y_1, z_1)$$

$$P_2(x_2, y_2, z_2)$$



$$P(x, y, z) \in \alpha$$

$$\overrightarrow{P_0P_1} = (x_1 - x_0, y_1 - y_0, z_1 - z_0)$$

$$\overrightarrow{P_0P_2} = (x_2 - x_0, y_2 - y_0, z_2 - z_0)$$

$$\overrightarrow{P_0P} = (x - x_0, y - y_0, z - z_0)$$

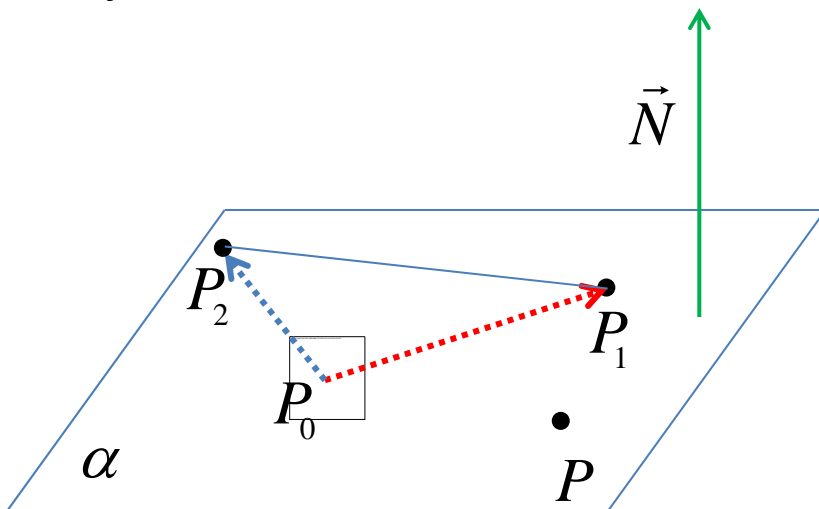
$$\vec{N} = \overrightarrow{P_0P_1} \times \overrightarrow{P_0P_2} = (A, B, C)$$

$$\alpha : P_0(x_0, y_0, z_0)$$

$$\vec{N} = (A, B, C)$$

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$$

JEDNAČINA RAVNI
odredjene sa tri tačke



$$\vec{N} = \overrightarrow{P_0P_1} \times \overrightarrow{P_0P_2}$$

$$\vec{N} = \overrightarrow{P_1P_0} \times \overrightarrow{P_1P_2}$$

$$\vec{N} = \overrightarrow{P_2P_0} \times \overrightarrow{P_2P_1}$$

$$\vec{N} = (A, B, C)$$

α :

$$P_0(x_0, y_0, z_0)$$

$$P_1(x_1, y_1, z_1)$$

$$P_2(x_2, y_2, z_2)$$

$$P(x, y, z) \in \alpha$$

$$\alpha : P_0(x_0, y_0, z_0)$$

$$\vec{N} = (A, B, C)$$

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$$

$$\alpha : P_1(x_1, y_1, z_1)$$

$$\vec{N} = (A, B, C)$$

$$A(x - x_1) + B(y - y_1) + C(z - z_1) = 0$$

$$\alpha : P_2(x_2, y_2, z_2)$$

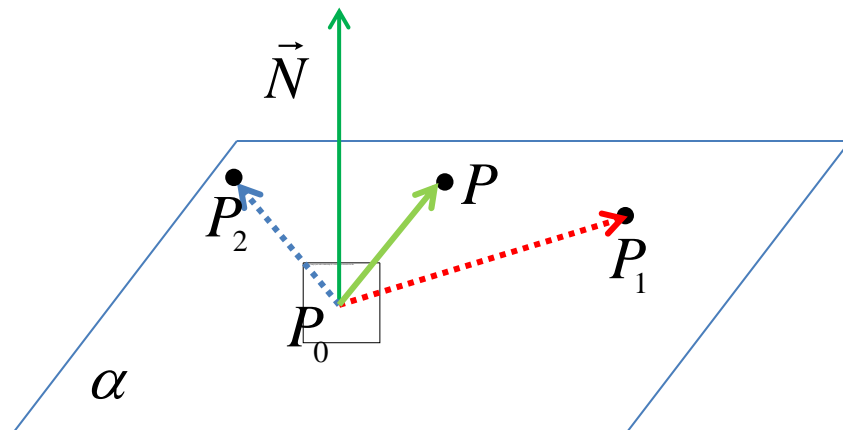
$$\vec{N} = (A, B, C)$$

$$A(x - x_2) + B(y - y_2) + C(z - z_2) = 0$$

PRIMER

Napisati jednačinu ravni
koja sadrži tačke

$$P_0(1, 2, 2) \quad P_1(2, 0, 0) \quad P_2(2, 2, 3)$$



$$\vec{a} = \overrightarrow{P_0P_1} = (2 - 1, 0 - 2, 0 - 2) = (1, -2, -2)$$

$$\vec{b} = \overrightarrow{P_0P_2} = (2 - 1, 2 - 2, 3 - 2) = (1, 0, 1)$$

$$\vec{N} = \vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -2 & -2 \\ 1 & 0 & 1 \end{vmatrix} = \vec{i} \begin{vmatrix} -2 & -2 \\ 0 & 1 \end{vmatrix} - \vec{j} \begin{vmatrix} 1 & -2 \\ 1 & 1 \end{vmatrix} + \vec{k} \begin{vmatrix} 1 & -2 \\ 1 & 0 \end{vmatrix} = -2\vec{i} - 3\vec{j} + 2\vec{k}$$

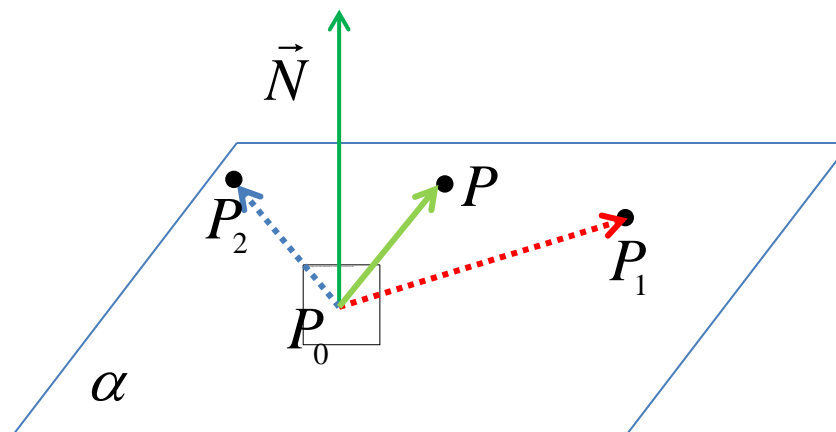
$$\vec{N} = (-2, -3, 2) \quad -2(x - 1) - 3(y - 2) + 2(z - 2) = 0$$

$$-2x - 3y + 2z + 4 = 0$$

PRIMER

Napisati jednačinu ravni
koja sadrži tačke

$$P_0(1, 2, 2) \quad P_1(2, 0, 0) \quad P_2(2, 2, 3)$$



$$\vec{a} = \overrightarrow{P_0P_1} = (2 - 1, 0 - 2, 0 - 2) = (1, -2, -2)$$

$$\vec{b} = \overrightarrow{P_0P_2} = (2 - 1, 2 - 2, 3 - 2) = (1, 0, 1)$$

$$\vec{N} = \vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -2 & -2 \\ 1 & 0 & 1 \end{vmatrix} = \vec{i} \begin{vmatrix} -2 & -2 \\ 0 & 1 \end{vmatrix} - \vec{j} \begin{vmatrix} 1 & -2 \\ 1 & 1 \end{vmatrix} + \vec{k} \begin{vmatrix} 1 & -2 \\ 1 & 0 \end{vmatrix} = -2\vec{i} - 3\vec{j} + 2\vec{k}$$

$$\vec{N} = (-2, -3, 2)$$

$$\begin{aligned} -2(x - 2) - 3(y - 0) + 2(z - 0) &= 0 \\ -2x - 3y + 2z + 4 &= 0 \end{aligned}$$

JEDNAČINA RAVNI

Odredjene sa tri tačke

α :

$$P_0(x_0, y_0, z_0)$$

$$P_1(x_1, y_1, z_1)$$

$$P_2(x_2, y_2, z_2)$$

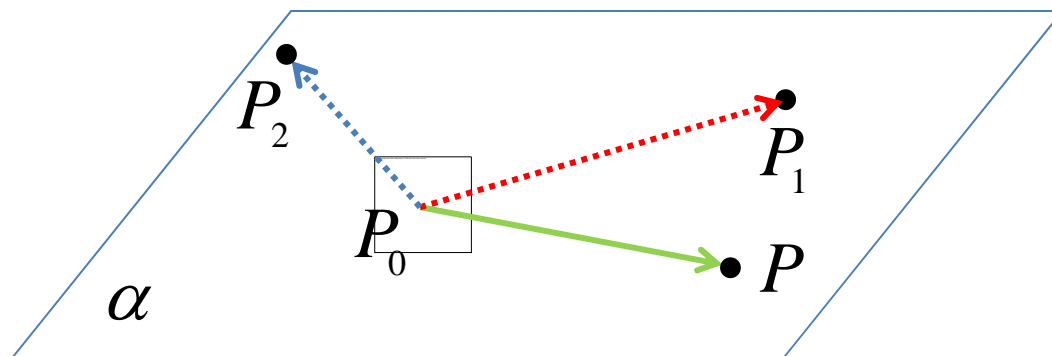
$$P(x, y, z) \in \alpha$$

$$\overrightarrow{P_0P} = (x - x_0, y - y_0, z - z_0)$$

$$\overrightarrow{P_0P_1} = (x_1 - x_0, y_1 - y_0, z_1 - z_0)$$

$$\overrightarrow{P_0P_2} = (x_2 - x_0, y_2 - y_0, z_2 - z_0)$$

Tačka $P(x, y, z)$ pripada ravni α
ako i samo ako su ovi vektori
komplanarni.



$$\left[\overrightarrow{P_0P}, \overrightarrow{P_0P_1}, \overrightarrow{P_0P_2} \right] = \left(\overrightarrow{P_0P} \times \overrightarrow{P_0P_1} \right) \cdot \overrightarrow{P_0P_2} = 0$$

Jednačina ravni:

$$\begin{vmatrix} x - x_0 & y - y_0 & z - z_0 \\ x_1 - x_0 & y_1 - y_0 & z_1 - z_0 \\ x_2 - x_0 & y_2 - y_0 & z_2 - z_0 \end{vmatrix} = 0$$

PRIMER

Napisati jednačinu ravni koja sadrži tačke $P_0(1, 2, 2)$ $P_1(2, 0, 0)$ $P_2(2, 2, 3)$

$$P(x, y, z) \in \alpha$$

$$\overrightarrow{P_0P} = (x-1, y-2, z-2)$$

$$\overrightarrow{P_0P_1} = (2-1, 0-2, 0-2) = (1, -2, -2)$$

$$\overrightarrow{P_0P_2} = (2-1, 2-2, 3-2) = (1, 0, 1)$$

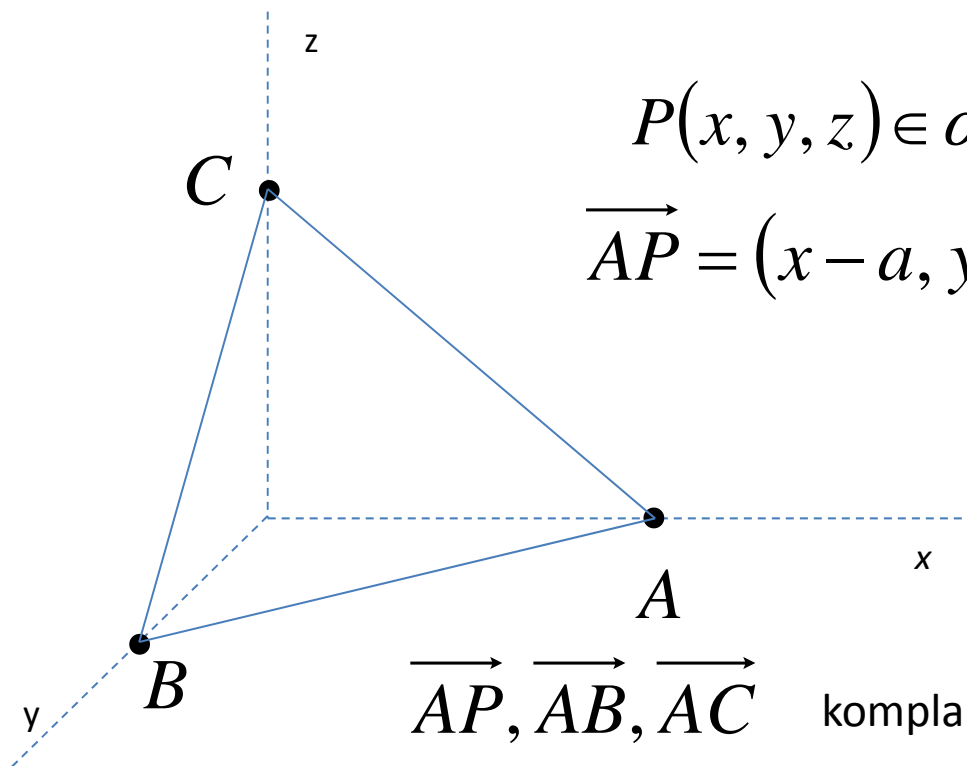
$$\begin{vmatrix} x-1 & y-2 & z-2 \\ 1 & -2 & -2 \\ 1 & 0 & 1 \end{vmatrix} = 0$$

$$(x-1) \begin{vmatrix} -2 & -2 \\ 0 & 1 \end{vmatrix} - (y-2) \begin{vmatrix} 1 & -2 \\ 1 & 1 \end{vmatrix} + (z-2) \begin{vmatrix} 1 & -2 \\ 1 & 0 \end{vmatrix} = 0$$

$$-2(x-1) - 3(y-2) + 2(z-2) = 0$$

$$-2x - 3y + 2z + 4 = 0$$

JEDNAČINA RAVNI – SEGMENTNI OBLIK



$$P(x, y, z) \in \alpha$$

$$\overrightarrow{AP} = (x - a, y, z)$$

α :

$$A(a, 0, 0)$$

$$B(0, b, 0)$$

$$C(0, 0, c)$$

$$\overrightarrow{AB} = (-a, b, 0)$$

$$\overrightarrow{AC} = (-a, 0, c)$$

$$\overrightarrow{AP}, \overrightarrow{AB}, \overrightarrow{AC} \text{ komplanarni} \Leftrightarrow [\overrightarrow{AP}, \overrightarrow{AB}, \overrightarrow{AC}] = 0$$

$$\begin{vmatrix} x - a & y & z \\ -a & b & 0 \\ -a & 0 & c \end{vmatrix} = 0 \quad \Rightarrow \quad \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

PRIMER

Napisati jednačinu ravni koja na osama x,z,y redom odseca

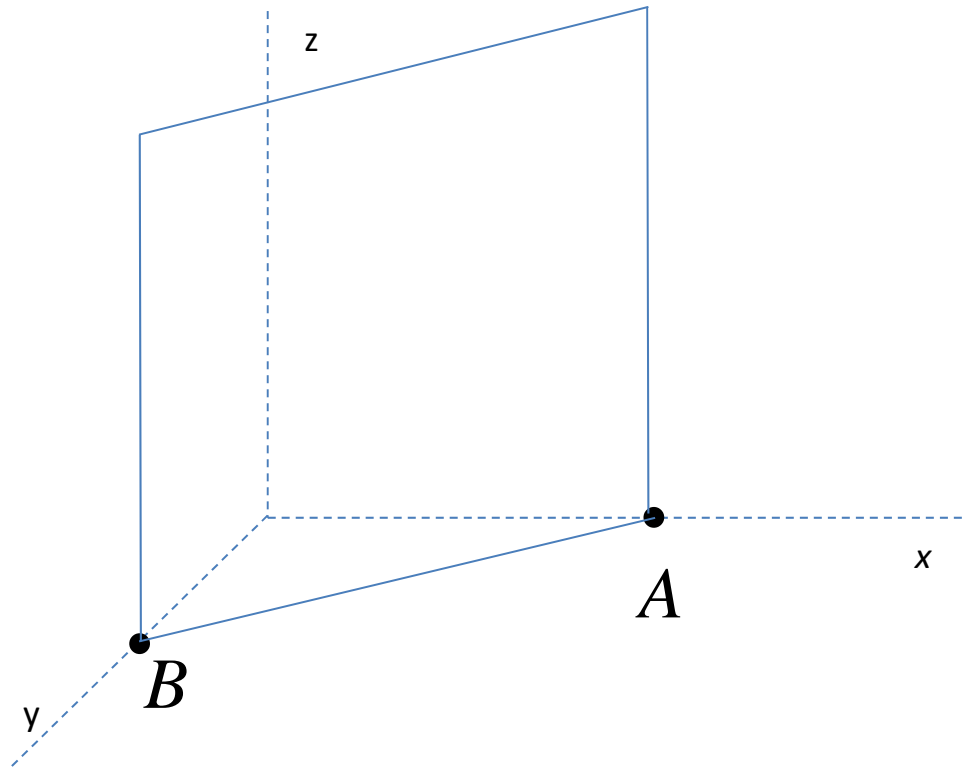
$$a = 2, b = -3, c = 5$$

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

$$\frac{x}{2} - \frac{y}{3} + \frac{z}{5} = 1 \quad | \cdot 30$$

$$15x - 10y + 6z - 30 = 0$$

JEDNAČINA RAVNI – SEGMENTNI OBLIK



α :

$$A(a, 0, 0)$$

$$B(0, b, 0)$$

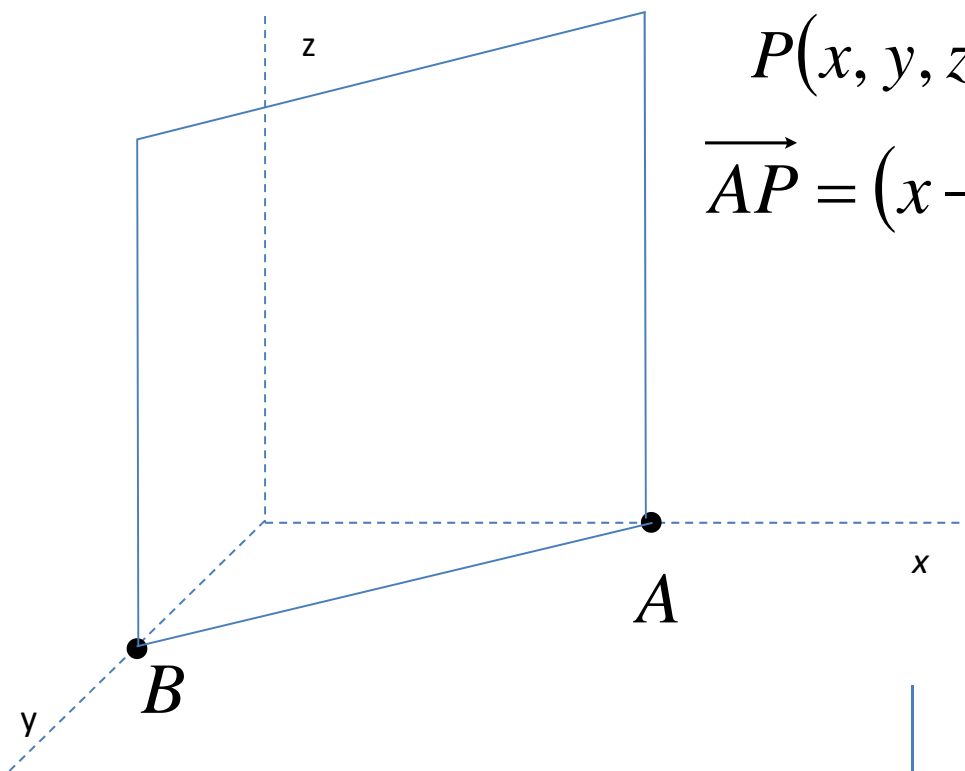
Paralelna z-osi

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

$$c \rightarrow \infty$$

$$\frac{x}{a} + \frac{y}{b} = 1$$

JEDNAČINA RAVNI – SEGMENTNI OBLIK



$$P(x, y, z) \in \alpha$$

$$\overrightarrow{AP} = (x - a, y, z)$$

α :

$$A(a, 0, 0)$$

$$B(0, b, 0)$$

Paralelna z-osi

$$\overrightarrow{AB} = (-a, b, 0)$$

$$\vec{k} = (0, 0, 1)$$

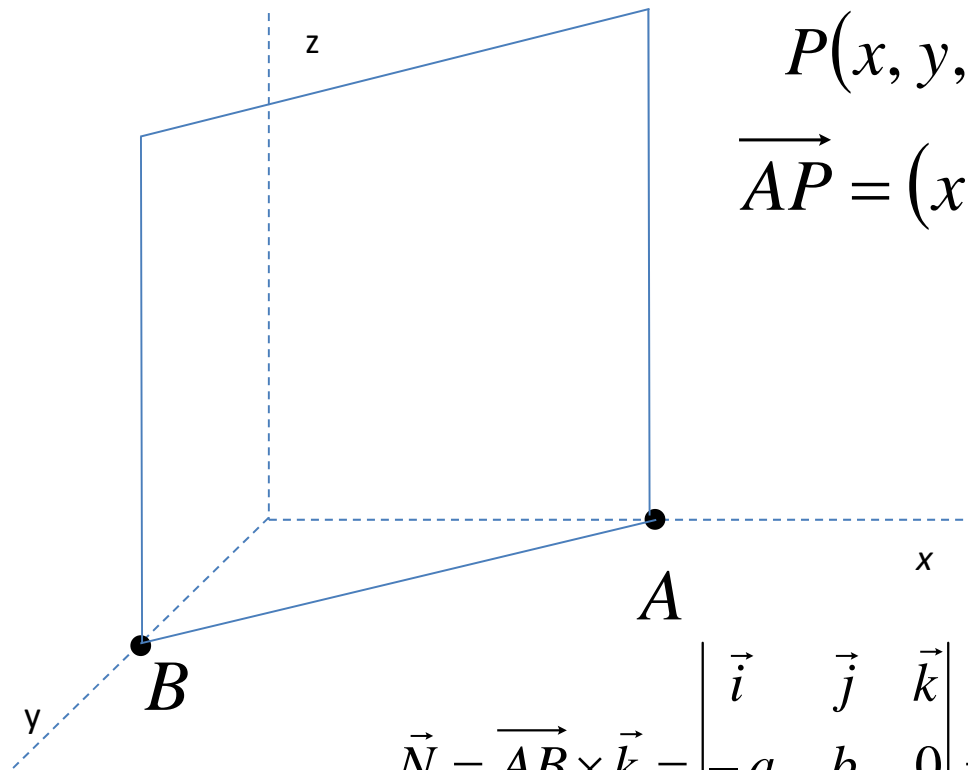
$$\begin{vmatrix} x-a & y & z \\ -a & b & 0 \\ 0 & 0 & 1 \end{vmatrix} = 0 \Rightarrow$$

$$b(x-a) + ay = 0$$

$$bx + ay = ab$$

$$\frac{x}{a} + \frac{y}{b} = 1$$

JEDNAČINA RAVNI – SEGMENTNI OBLIK



$$P(x, y, z) \in \alpha$$

$$\vec{AP} = (x - a, y, z)$$

α :

$$A(a, 0, 0)$$

$$B(0, b, 0)$$

Paralelna z-osi

$$\vec{AB} = (-a, b, 0)$$

$$\vec{k} = (0, 0, 1)$$

$$\vec{N} = \vec{AB} \times \vec{k} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -a & b & 0 \\ 0 & 0 & 1 \end{vmatrix} = \vec{i} \begin{vmatrix} b & 0 \\ 0 & 1 \end{vmatrix} - \vec{j} \begin{vmatrix} -a & 0 \\ 0 & 1 \end{vmatrix} + \vec{k} \begin{vmatrix} -a & b \\ 0 & 0 \end{vmatrix} = b\vec{i} + a\vec{j}$$

$$\vec{N} = (b, a, 0) \quad b(x - a) + a y = 0 \Rightarrow bx + a y = ab \Rightarrow \frac{x}{a} + \frac{y}{b} = 1$$

PRIMER

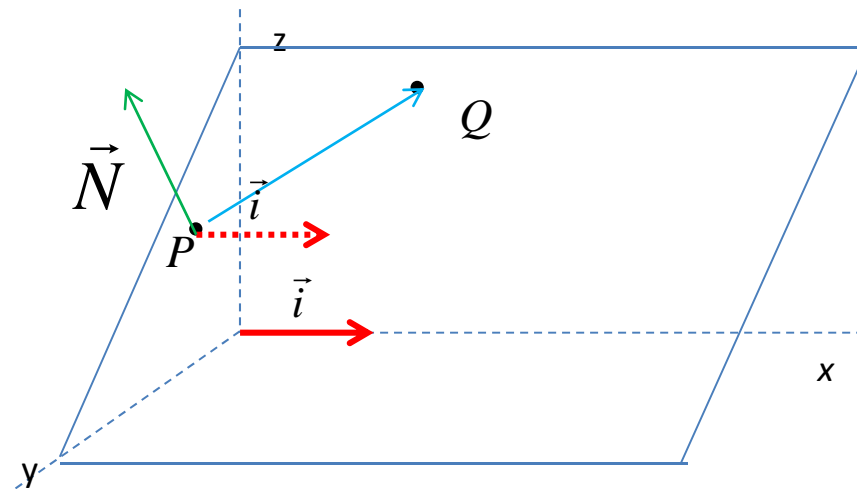
Napisati jednačinu ravni koja sadrži tačke

$$P(1,2,2) \quad Q(3,1,4)$$

i paralelna je x-osi.

$$\overrightarrow{PQ} = (3-1, 1-2, 4-2) = (2, -1, 2)$$

$$\vec{i} = (1, 0, 0)$$



$$\vec{N} = \overrightarrow{PQ} \times \vec{i} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & 2 \\ 1 & 0 & 0 \end{vmatrix} = \vec{i} \begin{vmatrix} -1 & 2 \\ 0 & 0 \end{vmatrix} - \vec{j} \begin{vmatrix} 2 & 2 \\ 1 & 0 \end{vmatrix} + \vec{k} \begin{vmatrix} 2 & -1 \\ 1 & 0 \end{vmatrix} = 2\vec{j} + \vec{k}$$

$$\alpha: \quad \begin{array}{l|l} P(1, 2, 2) & 0 \cdot (x-1) + 2(y-2) + (z-2) = 0 \\ \vec{N} = (0, 2, 1) & 2y + z - 6 = 0 \end{array}$$