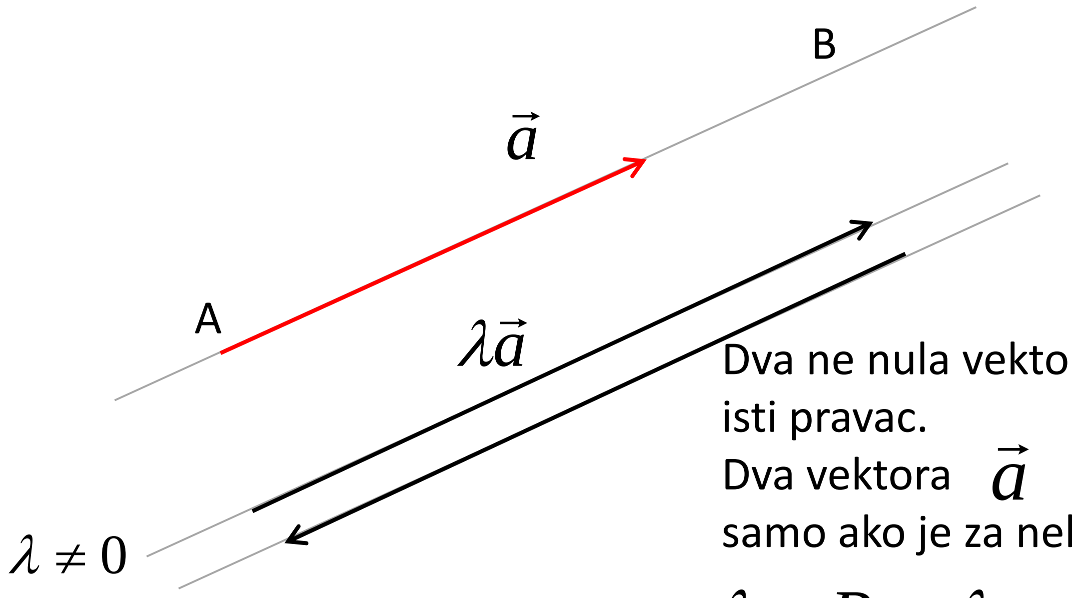
KOLINEARNOST VEKTORA ZADATIH POMOĆU KOORDINATA



Dva ne nula vektora su kolinearni ako imaju isti pravac.
Dva vektora $\vec{a}$ i $\vec{b}$ su kolinearni ako i samo ako je za neko

$$\lambda \in R, \quad \lambda \neq 0, \qquad \vec{b} = \lambda\vec{a}.$$

$$\vec{a} = (a_x, a_y, a_z)$$

$$\vec{b} = (b_x, b_y, b_z) \qquad \vec{b} = \lambda\vec{a} \qquad (b_x, b_y, b_z) = (\lambda a_x, \lambda a_y, \lambda a_z)$$

$$\frac{b_x}{a_x} = \frac{b_y}{a_y} = \frac{b_z}{a_z}$$

Uslov kolinearnosti dva vektora

PRIMER

Vektori

$\vec{a} = (-1, 2, -3) \qquad \vec{b} = (2, -4, 6)$

Su kolinearni zato što je

$$\frac{2}{-1} = \frac{-4}{2} = \frac{6}{-3} \quad = -2$$

Vektori

$\vec{a} = (-1, 2, -3) \qquad \vec{b} = (2, -4, 3)$

nisu kolinearni zato što je

$$\frac{2}{-1} = \frac{-4}{2} \neq \frac{3}{-3}$$

PRIMER

Odrediti vrednosti parametre λ i μ tako da su vektori

$\vec{a} = (\lambda, 3, 2)$ i $\vec{b} = (4, 6, \mu)$

kolinearni.

$$\frac{4}{\lambda} = \frac{6}{3} = \frac{\mu}{2} \Rightarrow \lambda = 2, \mu = 4$$

ZADATAK

Dati su vektori

$\vec{a} = (1,1,1)$ $\vec{b} = (0,2,0)$

Odrediti vrednost parametra λ tako da su vektori

$\vec{p} = \lambda\vec{a} + 5\vec{b}$ i $\vec{q} = 3\vec{a} - \vec{b}$ kolinearni.

$\vec{p} = (\lambda, \lambda + 10, \lambda)$ $\vec{q} = (3,1,3)$

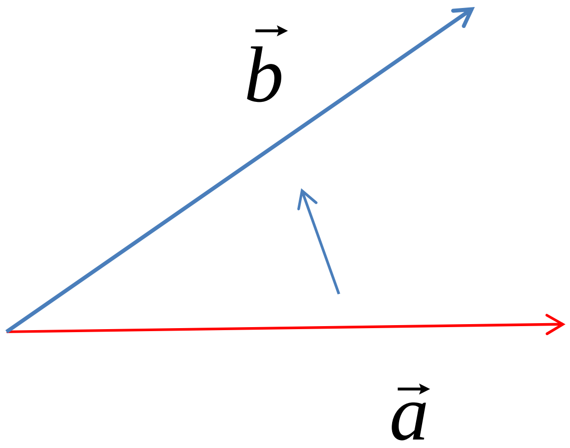
$$\frac{\lambda}{3} = \frac{\lambda + 10}{1} = \frac{\lambda}{3} \implies \frac{\lambda}{3} = \frac{\lambda + 10}{1} \implies \lambda = 3(\lambda + 10) \implies \lambda = 3(\lambda + 10) \implies$$

$$\implies -2\lambda = 30 \implies \lambda = -15$$

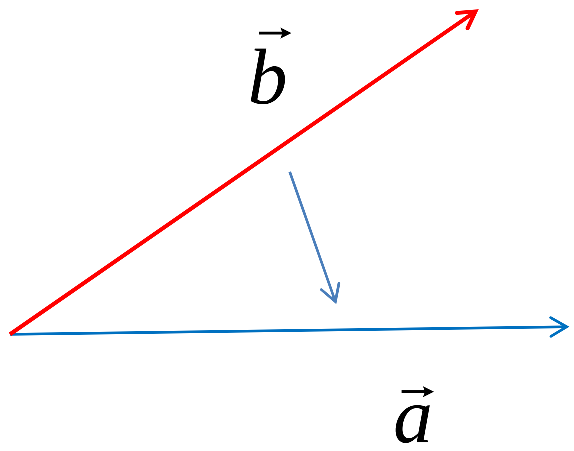
UGAO IZMEDJU DVA VEKTORA

Ugao izmedju dva vektora je ugao za koji treba zarotirati jedan od njih da bi se poklopio po pravcu I smeru sa drugim vektorom.

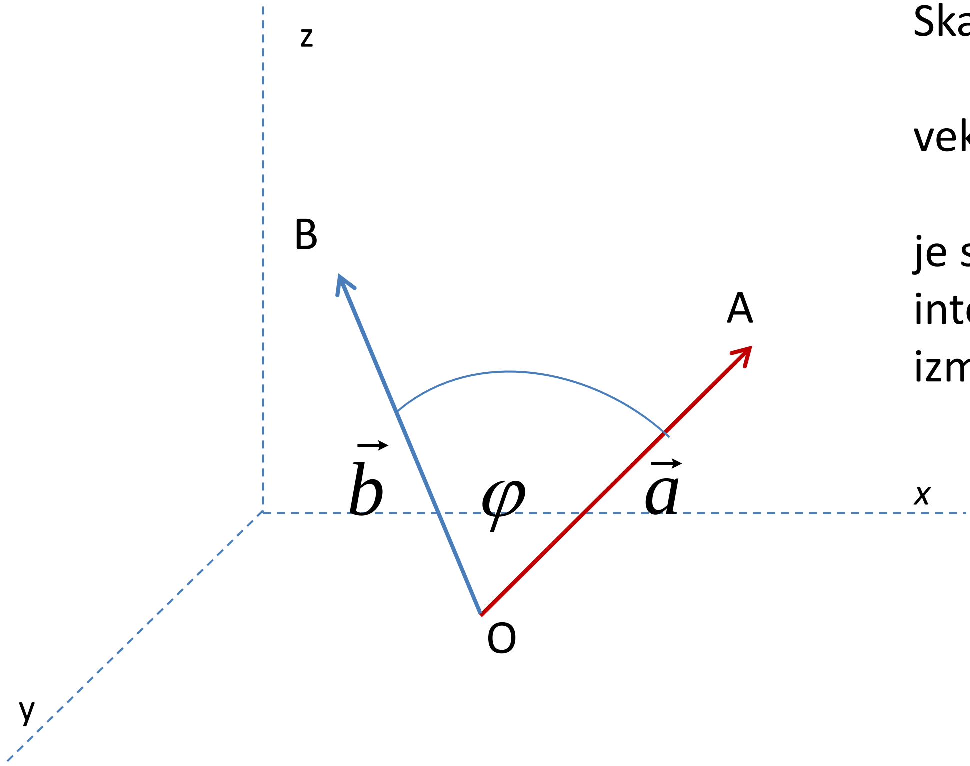
$\angle(\vec{a},\vec{b})$



$\angle(\vec{b},\vec{a})$



SKALARNI PROIZVOD VEKTORA



Skalarni proizvod $\vec{a} \cdot \vec{b}$ dva

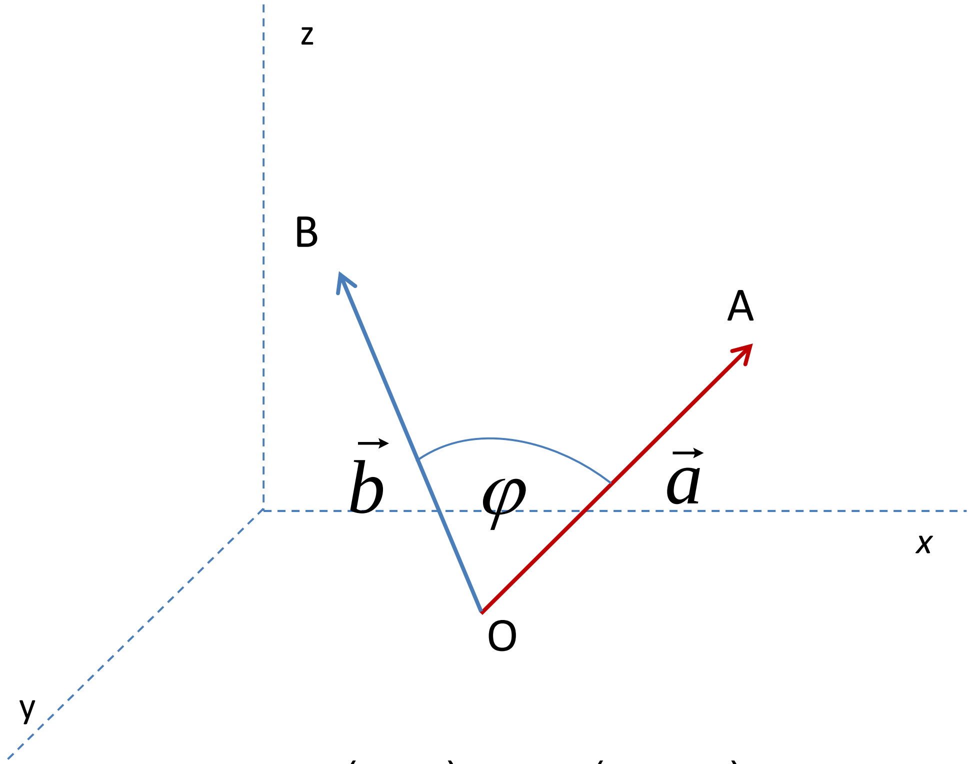
vektora $\vec{a}$ i $\vec{b}$

je skalar, koji je jedanak proizvodu intenziteta vektora i kosinusa ugla izmedju njih.

$\vec{a}$ $\vec{b}$

$$\vec{a} \cdot \vec{b} = \left|\vec{a}\right| \cdot \left|\vec{b}\right| \cdot \cos\varphi$$

SKALARNI PROIZVOD VEKTORA



$$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos\varphi$$

Osobine skalarnog proizvoda

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$$

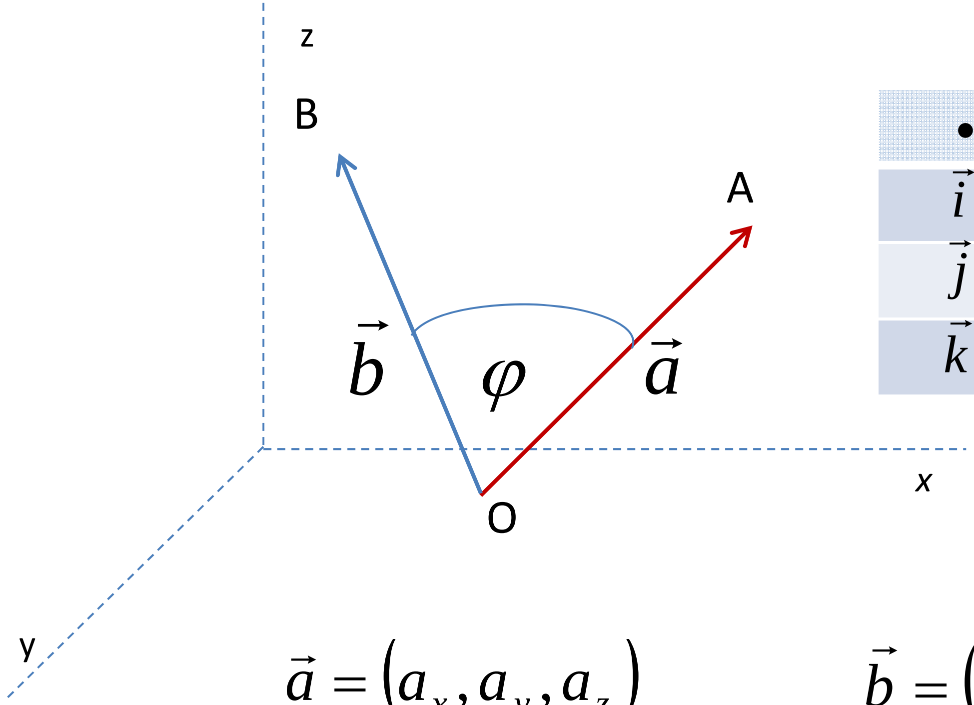
$$(\lambda \vec{a}) \cdot \vec{b} = \vec{a} \cdot (\lambda \vec{b}) = \lambda (\vec{a} \cdot \vec{b})$$

$$\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$$

$$\vec{a} \cdot \vec{a} = |\vec{a}|^2 = |\vec{a}|^2 \geq 0$$

$$\vec{a} \cdot \vec{a} = 0 \Leftrightarrow \vec{a} = 0$$

SKALARNI PROIZVOD VEKTORA ZADATIH POMOĆU KOORDINATA



$\bullet$	$\vec{i}$	$\vec{j}$	$\vec{k}$
$\vec{i}$	1	0	0
$\vec{j}$	0	1	0
$\vec{k}$	0	0	1

$$\vec{a} = \left(a_x, a_y, a_z\right) \qquad \vec{b} = \left(b_x, b_y, b_z\right)$$

$$\vec{a} = a_x\vec{i} + a_y\vec{j} + a_z\vec{k} \qquad \vec{b} = b_x\vec{i} + b_y\vec{j} + b_z\vec{k}$$

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$$

PRIMER

Izračunati skalarni proizvod datih vektora:

$\vec{a} = (1,1,-2) \qquad \vec{b} = (1,-1,4)$

$\vec{a} \cdot \vec{b} = 1 \cdot 1 + 1 \cdot (-1) + (-2) \cdot 4 = -8$

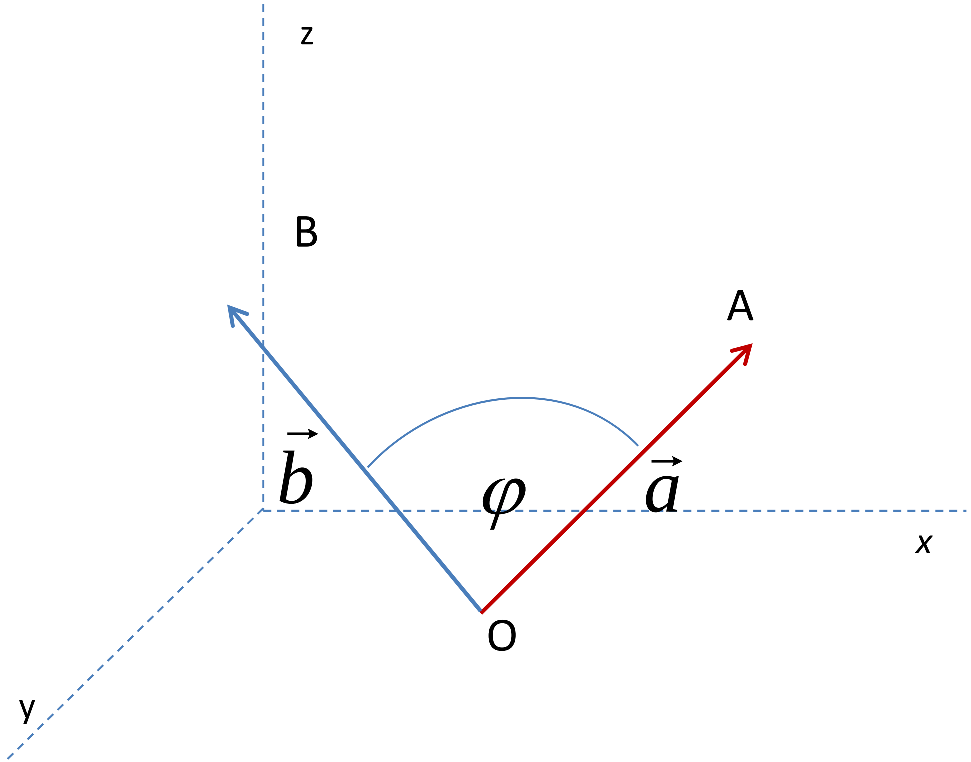
$\vec{a} = (1,5,-2) \qquad \vec{b} = (1,-1,4)$

$\vec{a} \cdot \vec{b} = 1 \cdot 1 + 5 \cdot (-1) + (-2) \cdot 4 = -12$

$\vec{a} = (1,-2,-2) \qquad \vec{b} = (1,-1,3)$

$\vec{a} \cdot \vec{b} = 1 \cdot 1 + (-2) \cdot (-1) + (-2) \cdot 3 = -3$

SKALARNI PROIZVOD VEKTORA - USLOV ORTOGONALNOSTI DVA VEKTORA



$$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos\varphi$$

Dva nenula vektora su ortogonalni ako I samo ako je njihov skalarni proizvod jednak nuli.

$$\vec{a} \cdot \vec{b} = 0 \Leftrightarrow \varphi = 90^0$$

Uslov ortogonalnosti vektora zadatih pomoću koordinata

$$\vec{a} = (a_x, a_y, a_z) \qquad \vec{b} = (b_x, b_y, b_z)$$

$$\vec{a} \cdot \vec{b} = 0 \Leftrightarrow a_x b_x + a_y b_y + a_z b_z = 0$$

PRIMER

Vektori

$\vec{a} = (-3, 3, 2)$ $\vec{b} = (4, 6, -3)$

Su ortogonalni zato sto je

$$\vec{a} \cdot \vec{b} = (-3) \cdot 4 + 3 \cdot 6 + 2 \cdot (-3) = -12 + 18 - 6 = 0$$

Vektori

$\vec{a} = (-3, 3, 2)$ $\vec{b} = (4, 6, -2)$

nisu ortogonalni zato sto je

$$\vec{a} \cdot \vec{b} = (-3) \cdot 4 + 3 \cdot 6 + 2 \cdot (-2) = -12 + 18 - 4 = 2 \neq 0$$

PRIMER

Odrediti vrednost parametra λ tako da su vektori $\vec{a} = (\lambda, 3, 2)$ i $\vec{b} = (4, 6, \lambda)$ ortogonalni.

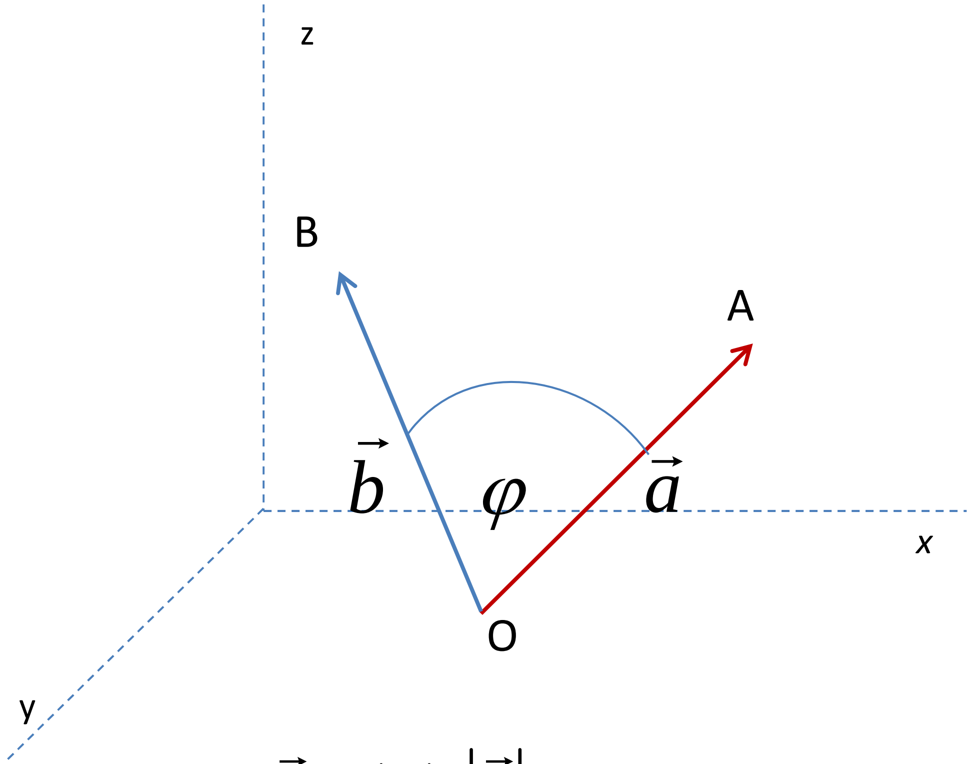
$$\vec{a} \perp \vec{b} \iff \vec{a} \cdot \vec{b} = 0$$

$$\vec{a} \cdot \vec{b} = \lambda \cdot 4 + 3 \cdot 6 + 2 \cdot \lambda = 0$$

$$6\lambda + 18 = 0$$

$$\lambda = -3$$

SKALARNI PROIZVOD VEKTORA - UGAO IZMEDJU DVA VEKTORA



$$\vec{a} = (a_x, a_y, a_z)$$

$$\vec{b} = (b_x, b_y, b_z)$$

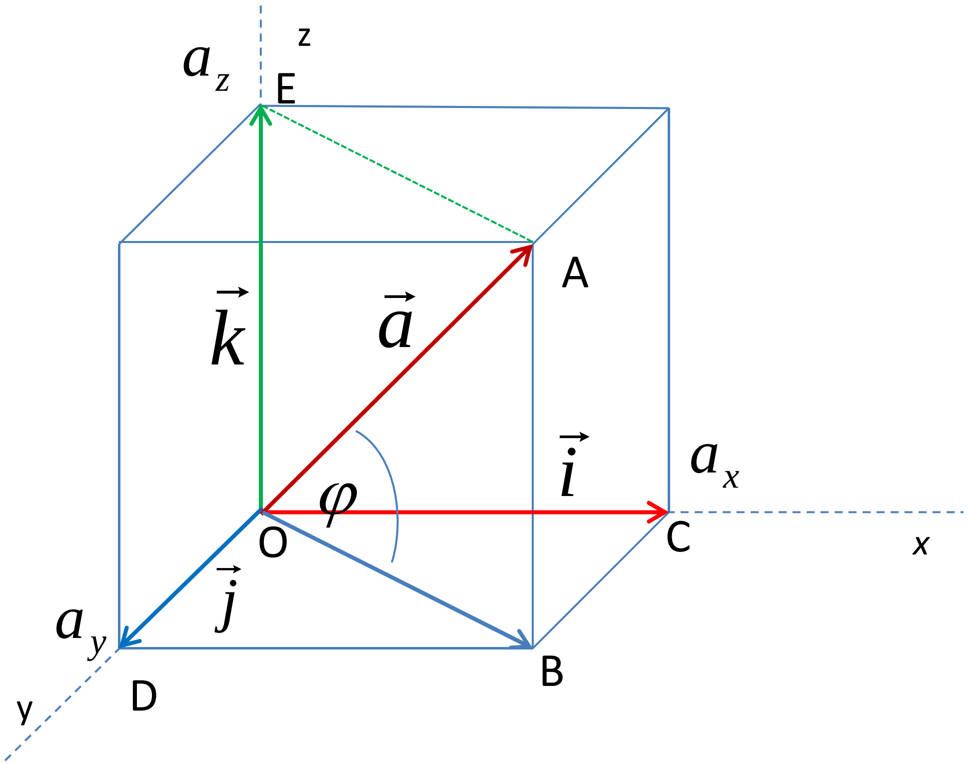
$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$$

$$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos\varphi \qquad \cos\varphi = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|}$$

$$\cos\varphi = \frac{a_x b_x + a_y b_y + a_z b_z}{\sqrt{a_x^2 + a_y^2 + a_z^2}\sqrt{b_x^2 + b_y^2 + b_z^2}}$$

PRIMER

Data je kocka osnovne ivice $a = 1$ sa jednim temenom u koordinatnom početku O i sa ivicama koje polaze iz tog temena koje leze na pozitivnim delovima koordinatnih osa. OA je dijagonala te kocke. OB je dijagonala donje osnove te kocke.
Odrediti kosinus ugla izmedju vektora $\overrightarrow{OA}$ i $\overrightarrow{OB}$



$$\vec{a} = \overrightarrow{OA} = (1,1,1)$$

$$\vec{b} = \overrightarrow{OB} = (1,1,0)$$

$$|\vec{a}| = \sqrt{3} \qquad |\vec{b}| = \sqrt{2}$$

$$\cos\varphi = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|}$$

$$\cos\varphi = \frac{1\cdot1+1\cdot1+1\cdot0}{\sqrt{2}\cdot\sqrt{3}} = \frac{\sqrt{6}}{3}$$

PRIMER

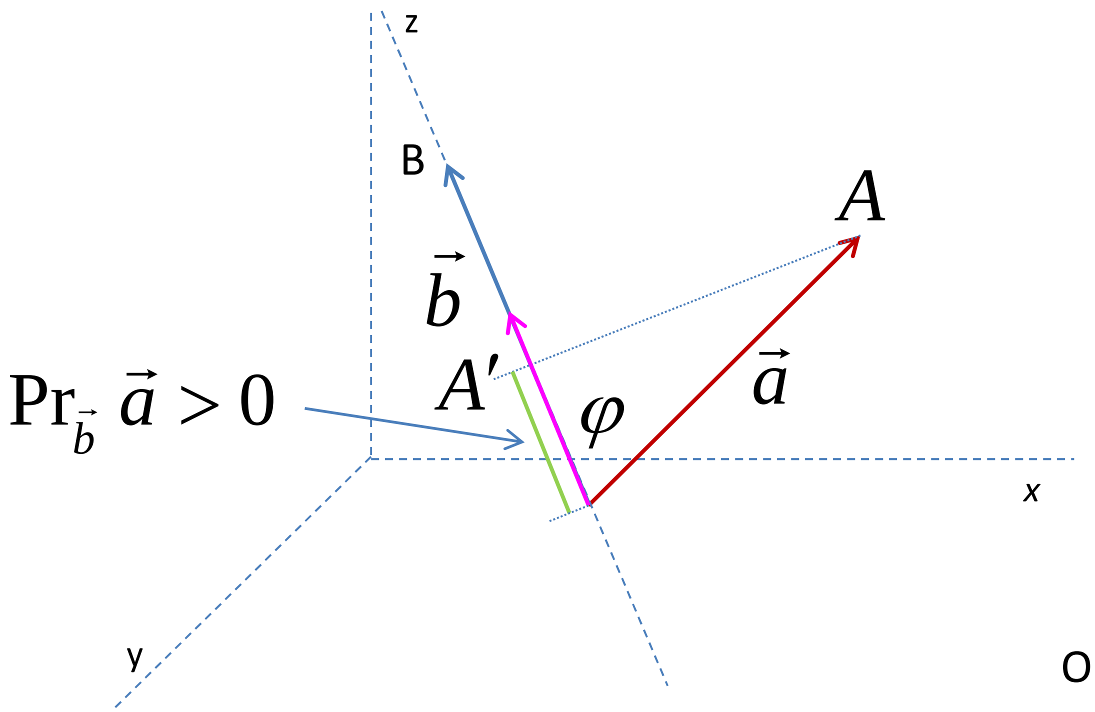
Odrediti ugao izmedju vektora $\vec{a} = (1,0,1)$ i $\vec{b} = (0,-1,1)$.

$$\cos\varphi = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|}$$

$$\cos\varphi = \frac{1\cdot 0 + 0\cdot(-1) + 1\cdot 1}{\sqrt{1^2+0^2+1^2}\cdot\sqrt{0^2+(-1)^2+1^2}} = \frac{1}{\sqrt{2}\cdot\sqrt{2}} = \frac{1}{2}$$

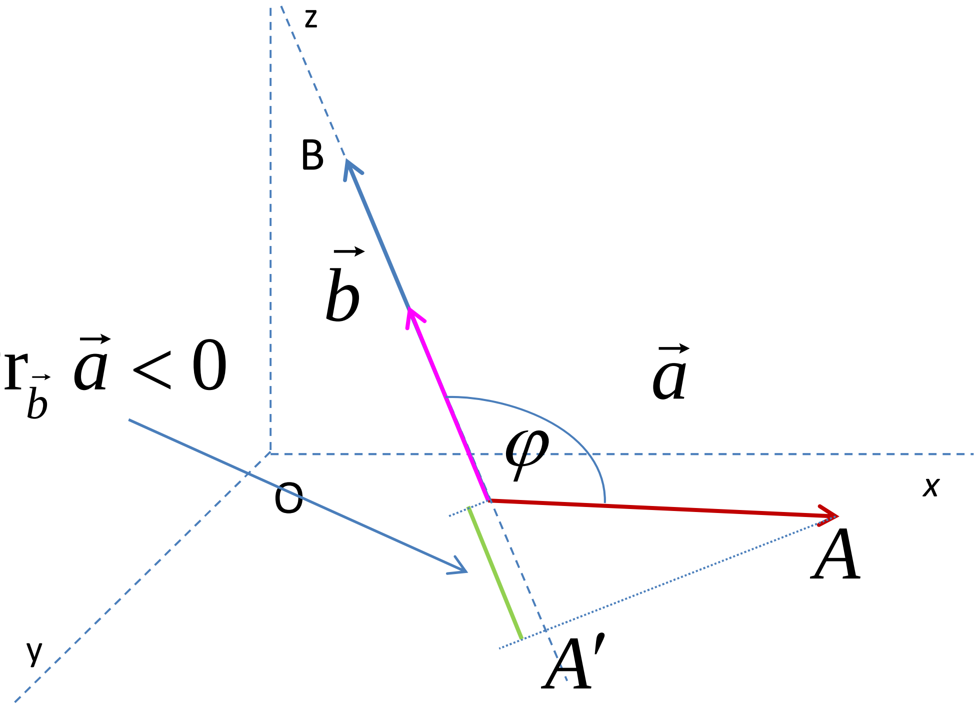
$$\varphi = \frac{\pi}{3} = 60^0$$

ORTOGONALNA ALGEBARSKA PROJEKCIJA VEKTORA



$$ort\,\vec{b} = \frac{\vec{b}}{\left|\vec{b}\right|} \qquad \mathrm{Pr}_{\vec{b}}\ \vec{a} = \left|\vec{a}\right| \cdot \cos\varphi = \vec{a} \cdot ort\,\vec{b}$$

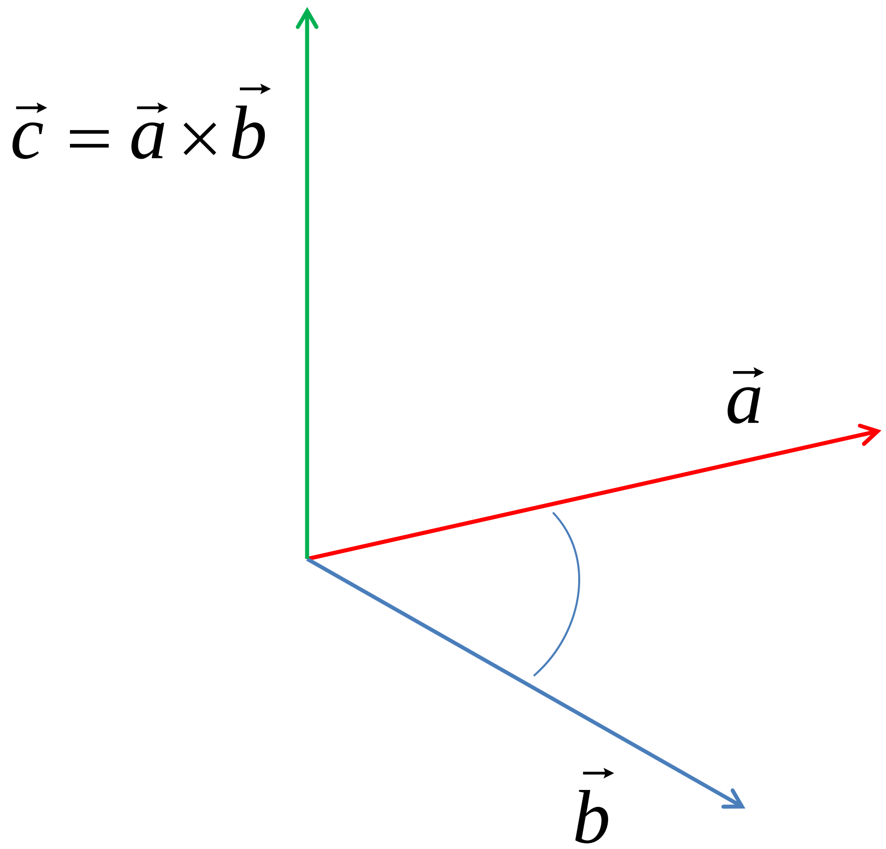
ORTOGONALNA ALGEBARSKA PROJEKCIJA VEKTORA



$$\mathrm{Pr}_{\vec{b}}\,\vec{a} < 0$$

$$ort\,\vec{b} = \frac{\vec{b}}{\left|\vec{b}\right|} \qquad \mathrm{Pr}_{\vec{b}}\,\vec{a} = \left|\vec{a}\right| \cdot \cos\varphi = \vec{a} \cdot ort\,\vec{b}$$

VEKTORSKI PROIZVOD



Vektorski proizvod vektora $\vec{a}$ i $\vec{b}$
je vektor

$$\vec{c} = \vec{a} \times \vec{b}$$

čiji je intezitet jednak površini paralelograma konstruisanog nad

vektorima $\vec{a}$ i $\vec{b}$,

pravac normalan na ravan odredjenu tim vektorima,

a smer je takav da vektori

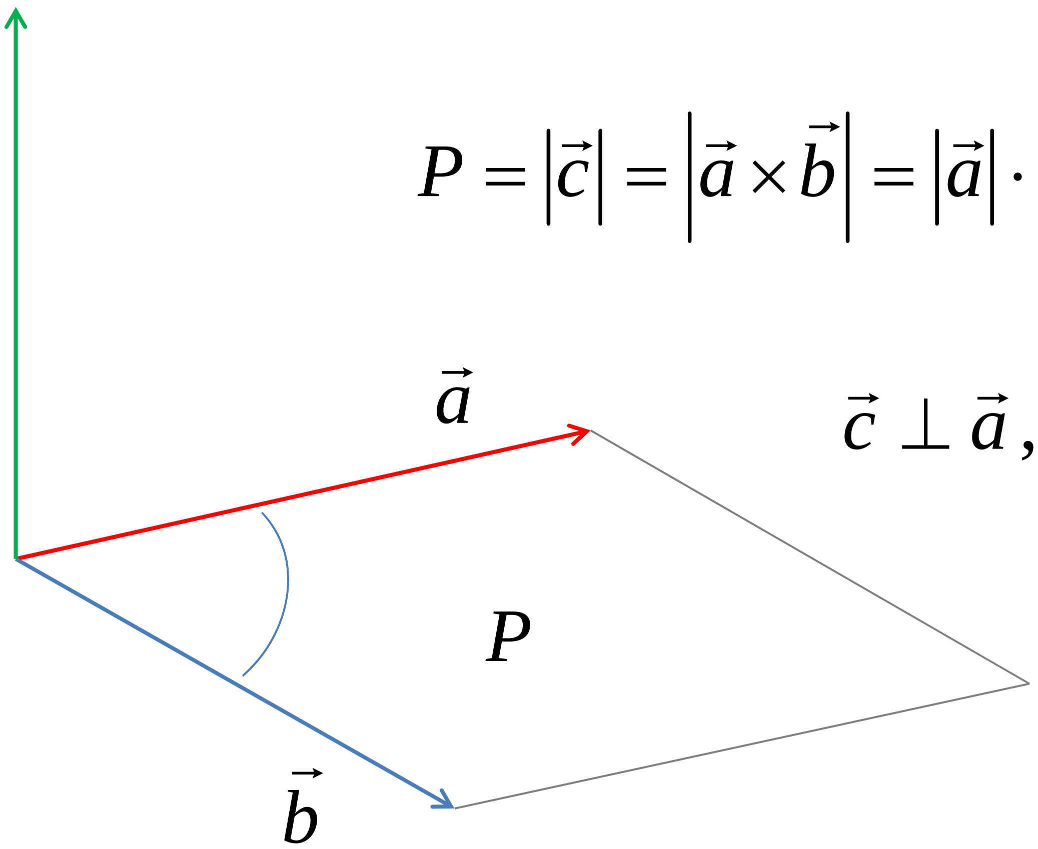
$$\vec{a},\ \vec{b},\ \vec{c}$$

čine trijedar iste orjentacije kao i koordinatni sistem.

VEKTORSKI PROIZVOD

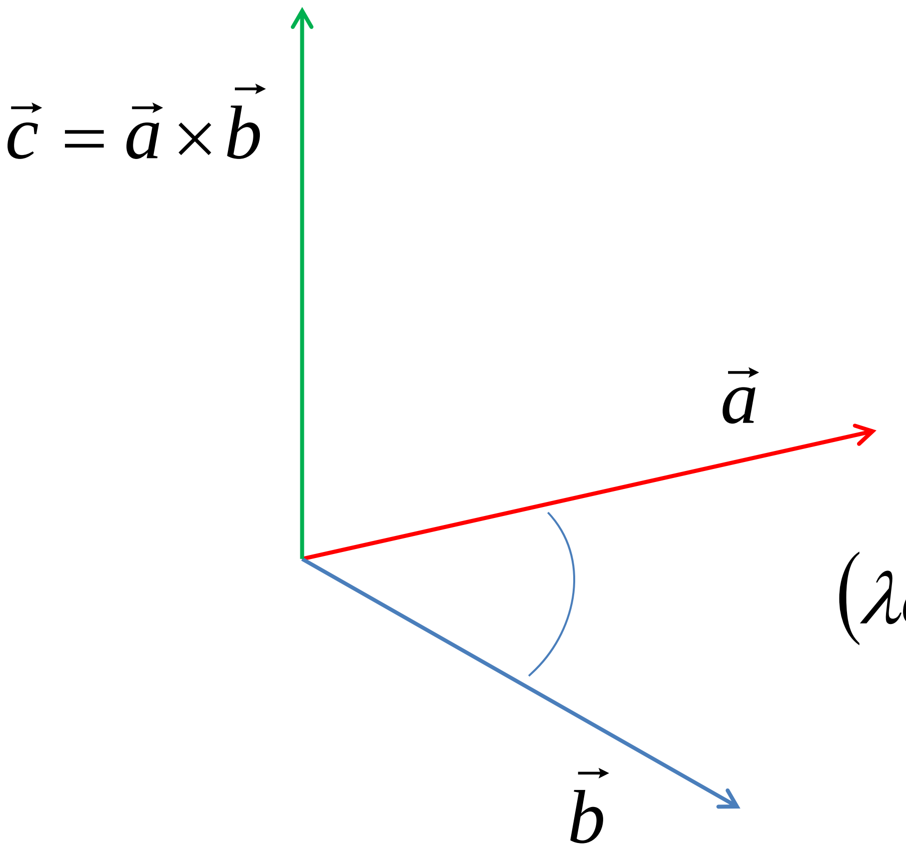
$$\vec{c} = \vec{a} \times \vec{b}$$

$$P = |\vec{c}| = \left|\vec{a} \times \vec{b}\right| = |\vec{a}| \cdot \left|\vec{b}\right| \cdot \sin \angle\left(\vec{a}, \vec{b}\right)$$



$$\vec{c} \perp \vec{a}, \ \vec{c} \perp \vec{b}$$

VEKTORSKI PROIZVOD - OSOBINE

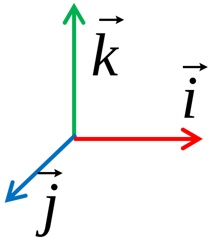


$$\vec{b} \times \vec{a} = -\vec{a} \times \vec{b}$$

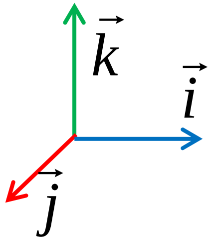
$$(\lambda \vec{a}) \times \vec{b} = \vec{a} \times (\lambda \vec{b}) = \lambda (\vec{a} \times \vec{b})$$

$$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$$

VEKTORSKI PROIZVOD - IZRAČUNAVANJE



$\times$	$\vec{i}$	$\vec{j}$	$\vec{k}$
$\vec{i}$	0	$\vec{k}$	$-\vec{j}$
$\vec{j}$	$-\vec{k}$	0	$\vec{i}$
$\vec{k}$	$\vec{j}$	$-\vec{i}$	0



$$\vec{a} = (a_x, a_y, a_z) \qquad \vec{a} = a_x\vec{i} + a_y\vec{j} + a_z\vec{k}$$

$$\vec{b} = (b_x, b_y, b_z) \qquad \vec{b} = b_x\vec{i} + b_y\vec{j} + b_z\vec{k}$$

$$\vec{a} \times \vec{b} = \left(a_x\vec{i} + a_y\vec{j} + a_z\vec{k}\right) \times \left(b_x\vec{i} + b_y\vec{j} + b_z\vec{k}\right)$$

$$\vec{a} \times \vec{b} = \vec{i}\left(a_y b_z - a_z b_y\right) - \vec{j}\left(a_x b_z - a_z b_x\right) + \vec{k}\left(a_x b_y - a_y b_x\right)$$

VEKTORSKI PROIZVOD - IZRAČUNAVANJE

$$\vec{a}\times\vec{b}=\vec{i}\begin{vmatrix}a_y & a_z\\ b_y & b_z\end{vmatrix}-\vec{j}\begin{vmatrix}a_x & a_z\\ b_x & b_z\end{vmatrix}+\vec{k}\begin{vmatrix}a_x & a_y\\ b_x & b_y\end{vmatrix}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ a_x & a_y & a_z\\ b_x & b_y & b_z\end{vmatrix}$$

$$\vec{a}\times\vec{b}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ a_x & a_y & a_z\\ b_x & b_y & b_z\end{vmatrix}=\vec{i}\begin{vmatrix}a_y & a_z\\ b_y & b_z\end{vmatrix}-\vec{j}\begin{vmatrix}a_x & a_z\\ b_x & b_z\end{vmatrix}+\vec{k}\begin{vmatrix}a_x & a_y\\ b_x & b_y\end{vmatrix}$$

DETERMINANTE DRUGOG I TREĆEG REDA

Izracunati determinantu, drugog I treceg reda

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11} \cdot a_{22} - a_{12} \cdot a_{22}$$

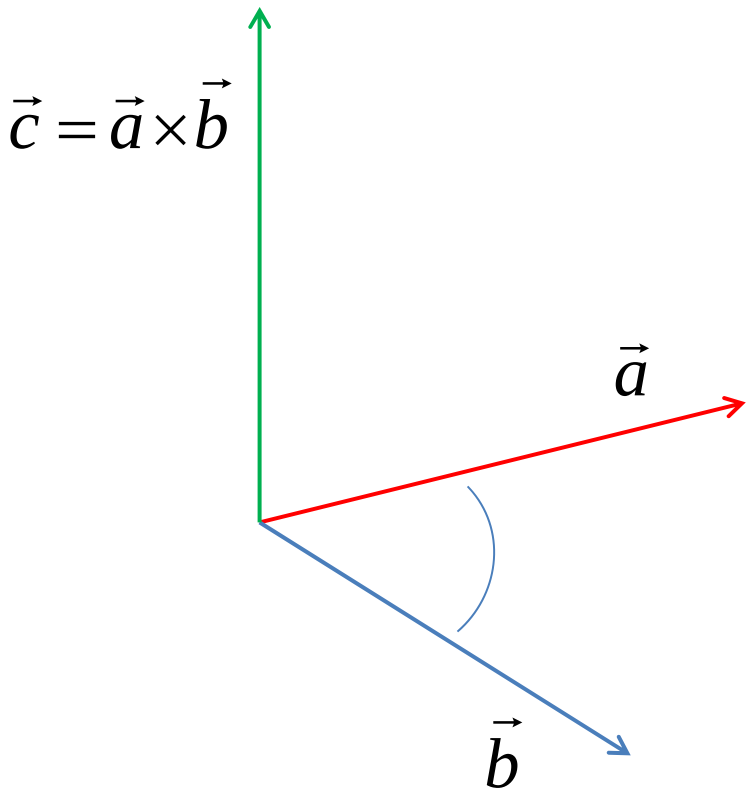
$$\begin{vmatrix} -1 & 3 \\ -3 & 4 \end{vmatrix} = (-1) \cdot 4 - 3 \cdot (-3) = 5$$

$$\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \cdot \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$\begin{vmatrix} 2 & 1 & -2 \\ -1 & 3 & -2 \\ -3 & 4 & 1 \end{vmatrix} = 2 \cdot \begin{vmatrix} 3 & -2 \\ 4 & 1 \end{vmatrix} - \begin{vmatrix} -1 & -2 \\ -3 & 1 \end{vmatrix} - 2 \begin{vmatrix} -1 & 3 \\ -3 & 4 \end{vmatrix} = 22 + 7 - 10 = 19$$

PRIMER



Izračunati vektorski proizvod vektora

$$\vec{a} = (1, -1, 2)$$

$$\vec{b} = (-1, 1, 0)$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 2 \\ -1 & 1 & 0 \end{vmatrix} = \vec{i} \begin{vmatrix} -1 & 2 \\ 1 & 0 \end{vmatrix} - \vec{j} \begin{vmatrix} 1 & 2 \\ -1 & 0 \end{vmatrix} + \vec{k} \begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix}$$

$$\vec{a} \times \vec{b} == -2\vec{i} - 2\vec{j}$$

$$\vec{a} \times \vec{b} == (-2, -2, 0)$$

PRIMER

Za date vektore $\vec{a} = (1, 1, -2)$ $\vec{b} = (1, 0, 2)$

Izračunati $\vec{a} \times \vec{b}$ i $\vec{b} \times \vec{a}$.

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & -2 \\ 1 & 0 & 2 \end{vmatrix} = \vec{i} \begin{vmatrix} 1 & -2 \\ 0 & 2 \end{vmatrix} - \vec{j} \begin{vmatrix} 1 & -2 \\ 1 & 2 \end{vmatrix} + \vec{k} \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix}$$

$$\vec{a} \times \vec{b} = 2\vec{i} - 4\vec{j} - \vec{k}$$

$$\vec{a} \times \vec{b} = (2, -4, -1)$$

$$\vec{b} \times \vec{a} = -\vec{a} \times \vec{b} = -(2, -4, -1) = (-2, 4, 1)$$

PRIMER

Izračunati površinu paralelograma konstruisanog nad datim vektorima:

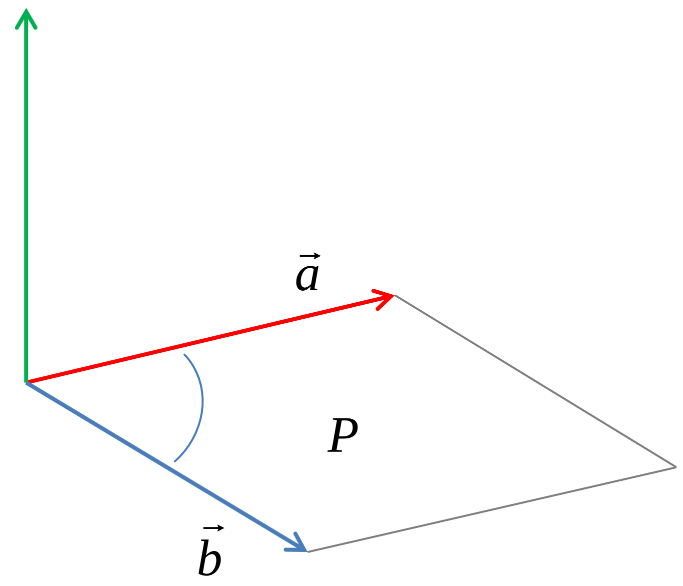
$\vec{a} = (1, 5, -2) \qquad \vec{b} = (1, 0, -2)$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 5 & -2 \\ 1 & 0 & -2 \end{vmatrix}$$

$$\vec{a} \times \vec{b} = \vec{i} \begin{vmatrix} 5 & -2 \\ 0 & 2 \end{vmatrix} - \vec{j} \begin{vmatrix} 1 & -2 \\ 1 & -2 \end{vmatrix} + \vec{k} \begin{vmatrix} 1 & 5 \\ 1 & 0 \end{vmatrix}$$

$$\vec{a} \times \vec{b} = 10\vec{i} - 5\vec{k} \qquad \vec{a} \times \vec{b} = (10, 0, -5)$$

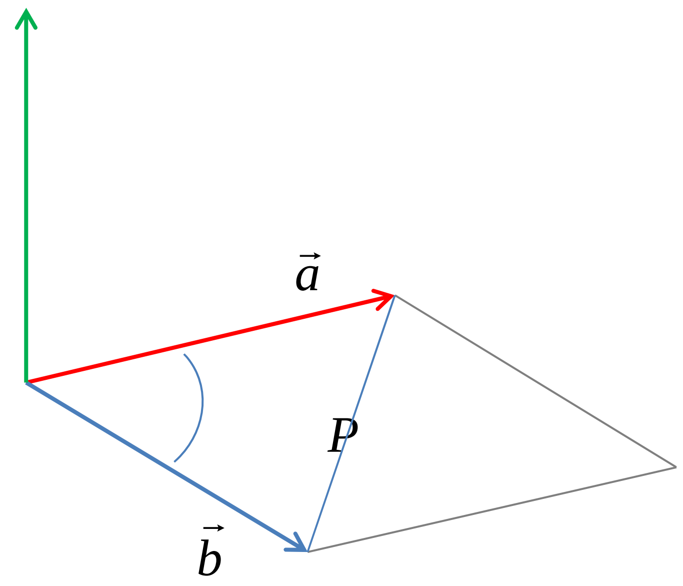
$$P = \left|\vec{a} \times \vec{b}\right| = \sqrt{10^2 + 0^2 + (-5)^2} = \sqrt{125} = 5\sqrt{5}$$



PRIMER

Izračunati površinu trougla konstruisanog nad datim vektorima:

$\vec{a} = (1, 5, -2)$ $\vec{b} = (1, 0, -2)$



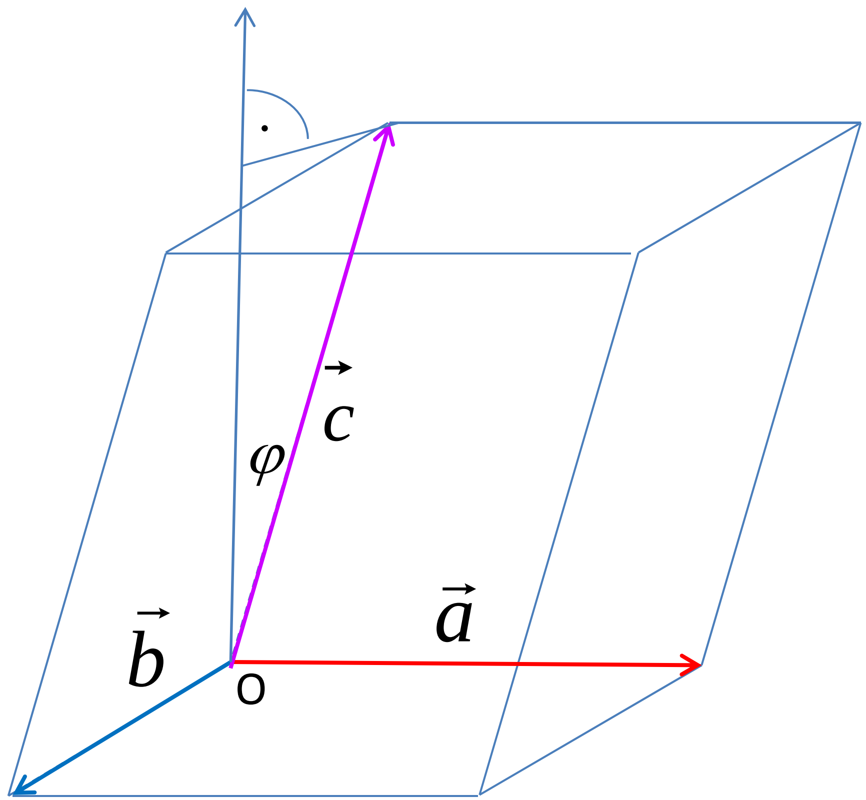
$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 5 & -2 \\ 1 & 0 & -2 \end{vmatrix}$$

$$\vec{a} \times \vec{b} = \vec{i} \begin{vmatrix} 5 & -2 \\ 0 & 2 \end{vmatrix} - \vec{j} \begin{vmatrix} 1 & -2 \\ 1 & -2 \end{vmatrix} + \vec{k} \begin{vmatrix} 1 & 5 \\ 1 & 0 \end{vmatrix}$$

$\vec{a} \times \vec{b} = 10\vec{i} - 5\vec{k}$ $\vec{a} \times \vec{b} = (10, 0, -5)$

$$P = \frac{1}{2} \left| \vec{a} \times \vec{b} \right| = \frac{1}{2} \sqrt{10^2 + 0^2 + (-5)^2} = \frac{1}{2} \sqrt{125} = \frac{5\sqrt{5}}{2}$$

MEŠOVITI PROIZVOD VEKTORA



Mašoviti proizvod tri vektora

$\vec{a}, \vec{b}, \vec{c}$

je skalarni proizvod

vektora $\vec{a} \times \vec{b}$ i vektora $\vec{c}$:

$$[\vec{a}, \vec{b}, \vec{c}] = (\vec{a} \times \vec{b}) \cdot \vec{c}$$

$$\left[\vec{a}, \vec{b}, \vec{c}\right] = \left(\vec{a} \times \vec{b}\right) \cdot \vec{c} = \left|\vec{a} \times \vec{b}\right| \cdot \left|\vec{c}\right| \cdot \cos\varphi = B \cdot \left(\pm H\right) = \pm V$$

MEŠOVITI PROIZVOD VEKTORA

Mešoviti prozvod tri vektora $\vec{a}, \vec{b}, \vec{c}$ je skalar,
čija je apsolutna vrednost jednaka zapremini paralelopipeda konstruisanog nad tim vektoraima.

Znak mešovitog proizvoda označava orjentaciju trijedra koji grade ti vektori u odnosu na orjentaciju koordinatnog sistema. Pozitivan rezultat označava da je trijedar orjentisan isto kao i koordinatni sistem, a negativan rezultat označava da je njegova orjentacija suprotna od orjentacije koordinatnog sistema.

MEŠOVITI PROIZVOD VEKTORA

Cikličkom permutacijom vektora, vrednost mešovitog proizvoda se ne menja.

$$[\vec{a},\vec{b},\vec{c}]=[\vec{b},\vec{c},\vec{a}]=[\vec{c},\vec{a},\vec{b}]$$

$$(\vec{a}\times\vec{b})\cdot\vec{c}=(\vec{b}\times\vec{c})\cdot\vec{a}=(\vec{c}\times\vec{a})\cdot\vec{b}$$

MEŠOVITI PROIZVOD VEKTORA

$$\vec{a} = \left(a_x, a_y, a_z\right)$$

$$\vec{b} = \left(b_x, b_y, b_z\right)$$

$$\vec{c} = \left(c_x, c_y, c_z\right)$$

$$[\vec{a}, \vec{b}, \vec{c}] = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}$$

MEŠOVITI PROIZVOD VEKTORA – USLOV KOMPLANARNOSTI

Tri vektora $\vec{a}, \vec{b}, \vec{c}$ su komplanarni ako i samo ako je njihov mešoviti proizvod jednak nuli.

USLOV KOMPLANARNOSTI TRI VEKTORA

$$\vec{a} = (a_x, a_y, a_z)$$

$$\vec{b} = (b_x, b_y, b_z)$$

$$\vec{c} = (c_x, c_y, c_z)$$

$$[\vec{a}, \vec{b}, \vec{c}] = (\vec{a} \times \vec{b}) \cdot \vec{c} = 0$$

$$[\vec{a}, \vec{b}, \vec{c}] = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix} = 0$$

PRIMER

Vektori $\vec{a} = (1,-2,1)$ $\vec{b} = (2,2,-2)$ $\vec{c} = (0,2,-1)$ $[\vec{a},\vec{b},\vec{c}] = 2$

su nekomplanarni zato sto je

$$[\vec{a},\vec{b},\vec{c}] = \begin{vmatrix} 1 & -2 & 1 \\ 2 & 2 & -2 \\ 0 & 2 & -1 \end{vmatrix} = -2 \cdot \begin{vmatrix} 1 & 1 \\ 2 & -2 \end{vmatrix} - \begin{vmatrix} 1 & -2 \\ 2 & 2 \end{vmatrix} = 8 - 6 = 2 \neq 0$$

Vektori $\vec{a} = (1,-2,3)$ $\vec{b} = (2,3,-2)$ $\vec{c} = (3,1,1)$

su komplanarni zato sto je

$$[\vec{a},\vec{b},\vec{c}] = (\vec{a} \times \vec{b}) \cdot \vec{c} = \begin{vmatrix} 1 & -2 & 3 \\ 2 & 3 & -2 \\ 3 & 1 & 1 \end{vmatrix} = 1 \cdot \begin{vmatrix} 3 & -2 \\ 1 & 1 \end{vmatrix} + 2 \cdot \begin{vmatrix} 2 & -2 \\ 3 & 1 \end{vmatrix} + 3 \cdot \begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix}$$

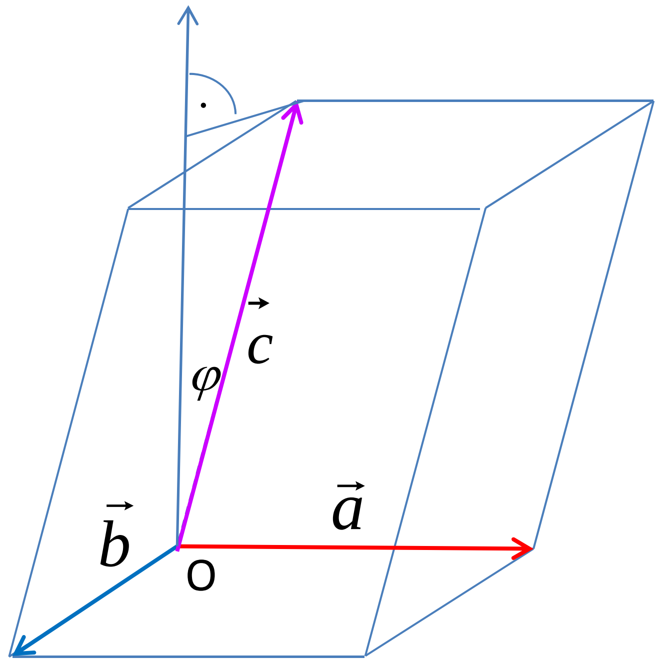
$$[\vec{a},\vec{b},\vec{c}] = 5 + 16 - 21 = 0$$

PRIMER

Izračunati zapreminu paralelopipeda konstruisanog nad vektorima

$\vec{a} = (1,-1,2)$ $\vec{b} = (-1,1,0)$ $\vec{c} = (1,2,0)$

i odrediti orjentaciju u odnosu na orjentaciju koordinatnog sistema.



$$[\vec{a},\vec{b},\vec{c}] = (\vec{a}\times\vec{b})\cdot\vec{c} = \begin{vmatrix} 1 & -1 & 2 \\ -1 & 1 & 0 \\ 1 & 2 & 0 \end{vmatrix}$$

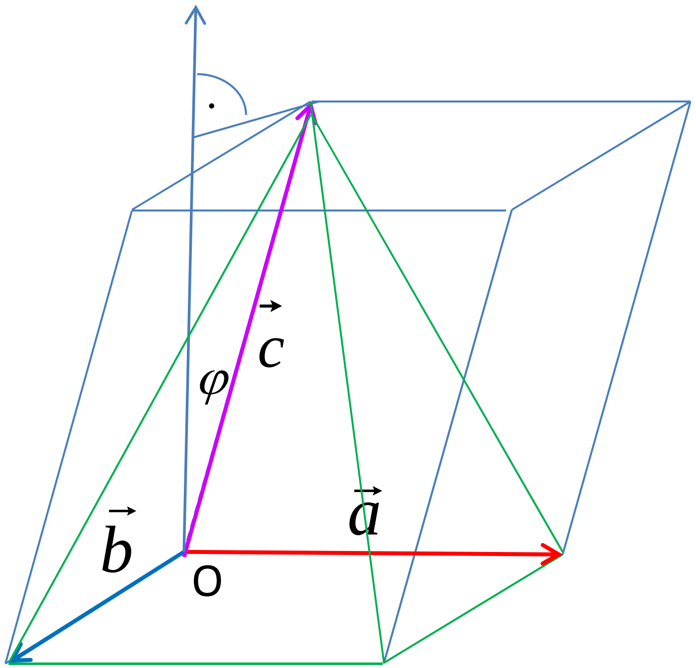
$$[\vec{a},\vec{b},\vec{c}] = 2\cdot\begin{vmatrix} -1 & 1 \\ 1 & 2 \end{vmatrix} = 2\cdot(-3) = -6$$

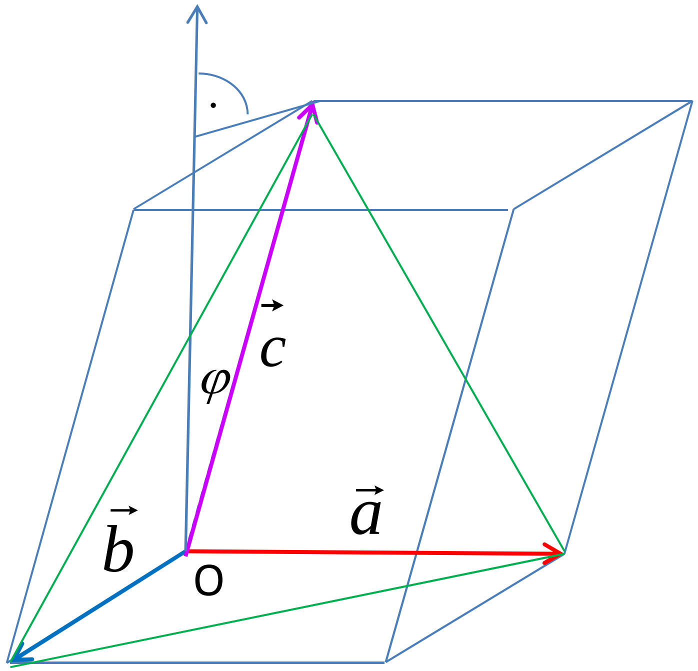
$$V = \left|[\vec{a},\vec{b},\vec{c}]\right| = |-6| = 6$$

$[\vec{a},\vec{b},\vec{c}] = -6 < 0$ Trijedar $\vec{a},\vec{b},\vec{c}$ suprotne orjentacije u odnosu na koordinatni sistem.

PRIMER

$$V_{paralelopipeda} = \left|[\vec{a}, \vec{b}, \vec{c}]\right| \qquad V_{paralelopipeda} = BH$$





$$V_{piramide} = \frac{1}{3}BH = \frac{1}{3}V_{paralelopipeda} = \frac{1}{3}\left|[\vec{a}, \vec{b}, \vec{c}]\right|$$

$$V_{tetraedra} = \frac{1}{3}B_1 H = \frac{1}{3}\frac{1}{2}BH = \frac{1}{6}V_{paralelopipeda} = \frac{1}{6}\left|[\vec{a}, \vec{b}, \vec{c}]\right|$$

PRIMER

$\vec{a} = (1, -1, 2)$ $\vec{b} = (-1, 1, 0)$ $\vec{c} = (1, 2, 0)$

$$V_{paralelopipeda} = \left|[\vec{a}, \vec{b}, \vec{c}]\right| = |-6| = 6$$

$$V_{piramide} = \frac{1}{3} V_{paralelopipeda} = \frac{1}{3}\left|[\vec{a}, \vec{b}, \vec{c}]\right| = \frac{1}{3} \cdot 6 = 2$$

$$V_{tetraedra} = \frac{1}{6} V_{paralelopipeda} = \frac{1}{6}\left|[\vec{a}, \vec{b}, \vec{c}]\right| = \frac{1}{6} \cdot 6 = 1$$